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CALCULATION OF DIRECT CURRENT ELECTRICAL CIRCUITS

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Abstract: *The article discusses two basic methods for calculating DC electrical circuits, their features and applications. One method is most effective for circuits with up to three nodes, the other is most effective for circuits with more than three nodes.*

Keywords: *Kirchhoff's laws, loop method, equation, node of electrical circuit, direction of current, loop, loop current, circuit branch.*

Introduction

One of the most common tasks in electrical engineering is the calculation of a DC electrical circuit. And its solution is based on Kirchhoff's and Ohm's laws. There are several basic methods for calculating DC electrical circuits two of which we will consider.

The first is a direct application of Kirchhoff's laws to a circuit according to which equations are made according to the first and second laws. Almost any circuit can be calculated using this method, but the more nodes there are in the circuit, the more equations have to be made and the longer it takes to solve them [1]. For this reason, it is most efficient to calculate circuits through direct application of Kirchhoff's laws when there are no more than three nodes.

If the circuit will have more than three nodes, it makes sense to apply the loop method. The method is based on Kirchhoff's second law and the calculation of true currents through loops and allows the overall number of equations to be reduced [2].

Main body

Calculation of a direct current electrical circuit through direct application of Kirchhoff's laws

Consider the sequence of circuit calculations through Kirchhoff's laws on the example of circuit 1.1 [3], fig.1. Resistances and EMF of all elements are known. It is required to find out the currents in all the branches of the circuit.

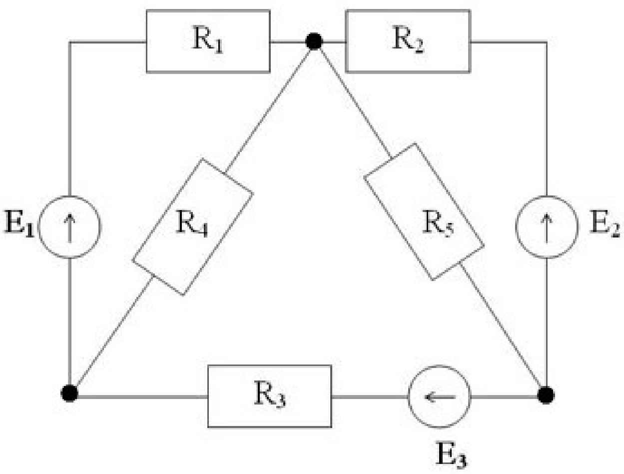


Fig.1 — Circuit 1.1

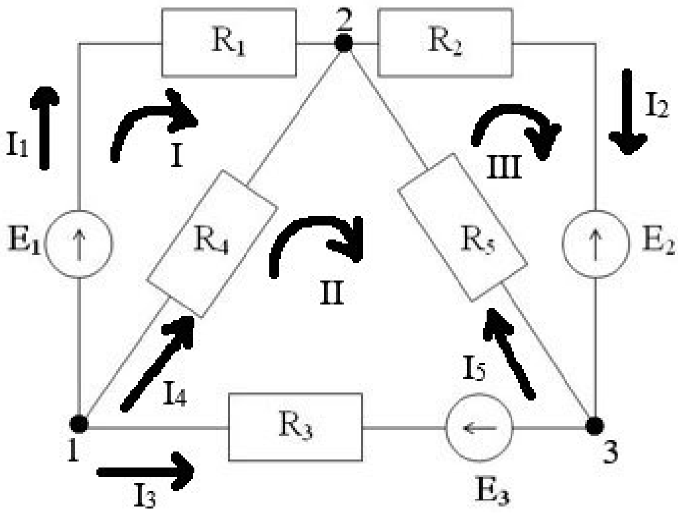
The number of unknown currents is equal to the number of branches (sections of circuit starting at one node and ending at another) in the circuit. The circuit 1.1 shows five branches, hence it follows that the unknown currents are also five, I_1, I_2, I_3, I_4, I_5.

The number of independent loops (a closed circuit section containing at least one new branch) is three.

According to Kirchhoff's first law, the number of equations to be made up is n-1, where n is the number of nodes. In this case the number of nodes is three and the number of equations according to Kirchhoff's first law is two [2].

Let's designate the nodes and the directions of currents and loop bypasses, fig.2. The directions are chosen arbitrarily as this has no effect on the end result. If we get a negative result for the currents, this will only indicate that the true direction of the currents is opposite to the one chosen.

Fig.2 — Circuit 1.2



Now make the equations according to Kirchhoff's first law. The currents leaving the node will be negative and those entering the node will be positive [3].

$$\text{Node 1: } -I_1 - I_3 - I_4 = 0$$

$$\text{Node 2: } I_1 - I_2 + I_4 + I_5 = 0$$

Next, we have to make equations according to Kirchhoff's second law. The number of equations in this case is equal to the number of independent loops, we have three of them [2]. The EMFs and currents whose directions match the chosen direction of the loop bypass will be positive, and those that do not match will be negative [3].

$$\text{Loop I: } I_1R_1 - I_4R_4 = E_1$$

$$\text{Loop II: } I_4R_4 - I_5R_5 - I_3R_3 = E_3$$

$$\text{Loop III: } I_2R_2 + I_4R_4 = -E_2$$

In the end, the resulting system of equations will look as follows:

$$\begin{cases} -I_1 - I_3 - I_4 = 0 \\ I_1 - I_2 + I_4 + I_5 = 0 \\ I_1R_1 - I_4R_4 = E_1 \\ I_4R_4 - I_5R_5 - I_3R_3 = E_3 \\ I_2R_2 + I_4R_4 = -E_2 \end{cases}$$

The system can be calculated in matrix form using the Cramer method.

It is useful to use the power balance [1] to check the correctness of the calculations. If the directions of the EMF and the current do not coincide, the power will be negative.

$$\sum EI = \sum I^2 R$$

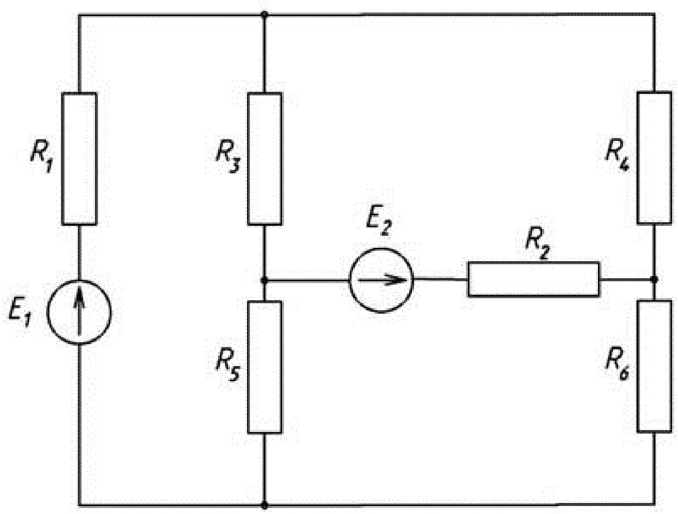
$$E_1I_1 - E_2I_2 - E_3I_3 = R_1I_1^2 + R_2I_2^2 + R_3I_3^2 + R_4I_4^2 + R_5I_5^2$$

Calculation of direct current electrical circuits using the loop method

When calculating complex circuits with more than three nodes, it is not practical to make equations according to Kirchhoff's laws, because the resulting system of equations is much too complex. In this method, the equations are only compiled for loop currents. Thus we exclude from the system the equations which are composed according to the first Kirchhoff's law [2].

Consider circuit 2.1 [4], fig. 3. Resistances and EMF of all elements are known. It is necessary to determine the currents in all branches of the circuit.

Fig.3 — Circuit 2.1



Circuit 2.1 has six branches and six unknown currents I_1, I_2, I_3, I_4, I_5, I_6, as well as four current sources and three independent circuits.

Let's number the nodes, mark the directions of currents and chose directions for bypassing the loop currents, circuit 2.2, fig.4.

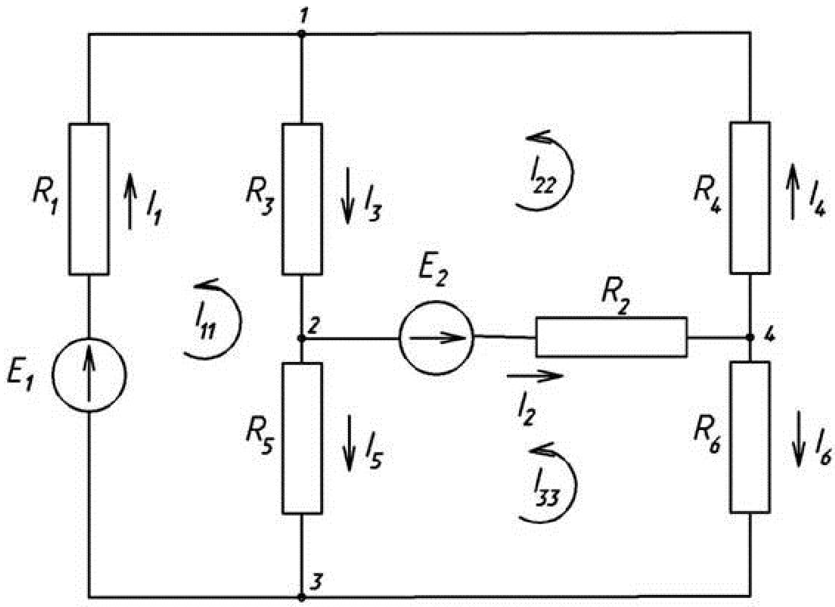


Fig.4 — Circuit 2.2

Since there are three independent loops in circuit 2.2, there will be an equal number of equations made according to Kirchhoff's second law for the loop currents I_{11}, I_{22}, I_{33}. These equations will be made in overall form, which requires calculating the eigen-resistances of all loops R_{11}, R_{22}, R_{33}, they are equal to the sum of all loop resistances [1]. Also it should be determined their relative resistance R_{12}, R_{21}, R_{23}, R_{32}, R_{13}, R_{31}, they are equal to the resistance of the overall branches of the two loops and are negative if the directions of the loop currents coincide [4].

$$R_{11} = R_1 + R_3 + R_5$$

$$R_{12} = R_{21} = -R_3$$

$$R_{22} = R_2 + R_3 + R_4$$

$$R_{23} = R_{32} = -R_2$$

$$R_{33} = R_2 + R_5 + R_6$$

$$R_{13} = R_{31} = -R_5$$

Now we should find the EMFs of loops E_{11}, E_{22}, E_{33}, they are equal to the sum of the EMFs in their loops, with those EMFs whose direction coincides with the direction of bypassing of the loop currents being positive. However, due to the specificity of the circuit the EMFs of the sources will only change their sign depending on the loop in question but will not change the result [1]. Therefore, in the system of equations, instead of the EMF of the loops E_{11}, E_{22}, E_{33} the EMF of the sources E_1 and E_2 will be written.

The system of equations will have the following overall form:

$$\begin{cases} R_{11}I_{11} + R_{12}I_{22} + R_{13}I_{33} = -E_1 \\ R_{21}I_{11} + R_{22}I_{22} + R_{23}I_{33} = E_2 \\ R_{21}I_{11} + R_{32}I_{22} + R_{33}I_{33} = -E_2 \end{cases}$$

Since the resistances and EMFs on are known from this system we will get the values of the loop currents I_{11}, I_{22}, I_{33}. It is also possible to use the matrix form, Cramer's method, for calculations here. Assuming that the values of the loop currents are the same as the real currents in the outer branches, but equal to the difference of the adjacent loop currents in the inner branches, we find the branch currents [2].

If the direction of the real current does not coincide with the loop current, the latter is written with a minus sign [4].

$$I_1 = -I_{11}$$
$$I_4 = I_{22}$$
$$I_6 = -I_{33}$$
$$I_2 = I_{22} - I_{33}$$
$$I_3 = I_{22} - I_{11}$$
$$I_5 = I_{33} - I_{11}$$

It remains to check the power balance.

$$\sum EI = \sum I^2R$$

$$E_1I_1 + E_2I_2 = R_1I_1^2 + R_2I_2^2 + R_3I_3^2 + R_4I_4^2 + R_5I_5^2 + R_6I_6^2$$

Conclusion

Having considered the direct application of Kirchhoff's laws and the loop method for the calculation of a direct current electrical circuit, the following conclusions can be made:

1) The two methods are based on Kirchhoff's laws and are suitable for the calculation of direct current circuits;

2) The loop method makes it possible to calculate the currents of all branches with less time than the direct application of Kirchhoff's laws.

Since the direct application of Kirchhoff's laws results in systems with a large number of equations, for the calculation of complex branched circuits with many nodes and independent loops it is more rational to use the loop method.

Independently of which method to use for calculating the circuit, the correct answer will be obtained.

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