GREENS FUNCTION FOR CLASS $A(z)-$ ANALYTIC FUNCTIONS

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Let $A(z)$ be an antianalytic function, i. e. $\frac{\partial A}{\partial z}=0$ in the domain $D\subset\mathbb{C}$; moreover, let $|A(z)|\le C<1$ for all $z\in D.$ The function $f(z)$ is said to be $A(z)-$analytic in the domain D if for any $z\in D,$ the following equality holds:

$$\frac{\partial f}{\partial \bar z}=A(z)\frac{\partial f}{\partial z} \qquad (1)$$

We denote by $O_A(D)$ the class of all $A(z)-$analytic functions defined in the domain $D.$

According to, the function

$$\psi(z;a)=z-a+\overline{\int_{\gamma(a;z)}\overline{A(\tau)}d\tau}$$

is an $A(z)-$analytic functions.

The following set is an open subset of D:

$$L(a;r)=\left\{|\psi(z;a)|=\left|z-a+\overline{\int_{\gamma(a;z)}\overline{A(\tau)}d\tau}\right|<r\right\}.$$

For sufficiently small $r>0,$ this set compactly lies in D (we denote this fact by $L(a;r)\subset\subset D$) and contains the point $a.$ This set $L(a;r)$ is called the $A(z)-$lemniscate centered at the point $\zeta.$ The lemniscate $L(a;r)$ is a simply – connected set (see [1]).

Let $f=u+iv.$

Definition 1 [4]. A double differentiable function $u \in C^2(D), u : D \to R$ is called $A(z)-$harmonic in the domain of $D,$ if it satisfies the differential equation in D:

$$\Delta_A u = \frac{\partial}{\partial z}\left(\frac{1}{1-|A|^2}\left((1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z}\right)\right) + \frac{\partial}{\partial \bar{z}}\left(\frac{1}{1-|A|^2}\left((1+|A|^2)\frac{\partial u}{\partial z} - 2\bar{A}\frac{\partial u}{\partial \bar{z}}\right)\right) = 0. \qquad (2)$$

The class of $A(z)-$harmonic functions in the domain of D is denoted as $h_A(D).$ Thus, the real part and hence the imaginary part, of the $A(z)-$harmonic function in the domain of $D.$

For $A(z)-$harmonic functions, the following Dirchlet problem is nat-

urally considered:

Dirichlet problem [4]. A bounded domain of $E \subset D$ is given and a continuous function of $u(\zeta)$ is the boundary of $\partial E.$ It is required to find $A(z)-$ harmonic in the domain of $E,$ continuous on the closure of $\bar{E},$ the function of $u(z) \in h_A(E) \cap C(\bar{E})$: $u(\zeta)|_{\partial E} = u(z).$

Definition 2. The Green function of the Dirichlet problem for $A(z)-$ analytic functions in the domain of $D,$ we will call $A(z)-$analytic function of two complex variables $G(z;\zeta),$ which has the following properties:

1. $G(z;\zeta) = \frac{1}{2\pi}\ln|\psi(z;\zeta)| + g(z;\zeta),$ where the function $g(z;\zeta)$ is continuous over a set of variables at $z;\zeta \in D,$ $A(z)-$harmonic over z in D at any $z \in D$: $G(z;\zeta) \in h_A(D).$

2. $A(z)-$analytic function $g(z;\zeta) \in C(\bar{D}_\zeta)$ is continuous in $\zeta \in \bar{D}$ for any $z \in D.$ Thus, the $G(z;\zeta) = 0$ for any z luing on the border ∂D:

$$\lim_{z\to\zeta} G(z;\zeta) = 0.$$

With the help of the Green function class $A(z)-$ analytic functions, you can write a solution to the Dirichlet problem. First, we will show one result in this direction, which has an auxiliary value. Let be a measurable set $M \subset \partial D$ of positive Lebesgue measure.

Theorem. Let the $G(z;\zeta)$ Greens function of the Dirichlet problem for class $A(z)-$ analytic functions in the domain of D be continuous up to the boundary of D with its first order partial derivatives in $\operatorname{Re}\zeta = \xi$ and $\operatorname{Im}\zeta = \eta$ (with the exception of the point $z \in D$). Then any $u(z)$ function, $A(z)-$ harmonic in the D domain and is continuously differentiable up to its values on the ∂D boundary curve by the formula

$$u(z) = \int_{\partial D} u(\zeta)\frac{\partial}{\partial n}G(z;\zeta)\left|d\zeta + A(\zeta)d\overline{\zeta}\right|, \qquad (3)$$

where
$$u(\zeta) = \begin{cases} 1, & in\ M, \\ 0, & in\ \partial D \setminus M. \end{cases}$$

If the $E \subset D$ domain is a lemniscate of $E = L(a;r)$ then

$$\omega_\zeta(z) = \frac{r\left(\psi(z;a) - \psi(\zeta;a)\right)}{r^2 - \psi(z;a)\overline{\psi}(\zeta;a)}, \quad G(z;\zeta) = \frac{1}{2\pi i}\ln\left|\omega_\zeta(z)\right|.$$

References

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