мость коэффициента трения f пары ТМ-1 от удельного нагружения контакта при различной концентрации абразива в жидкости.

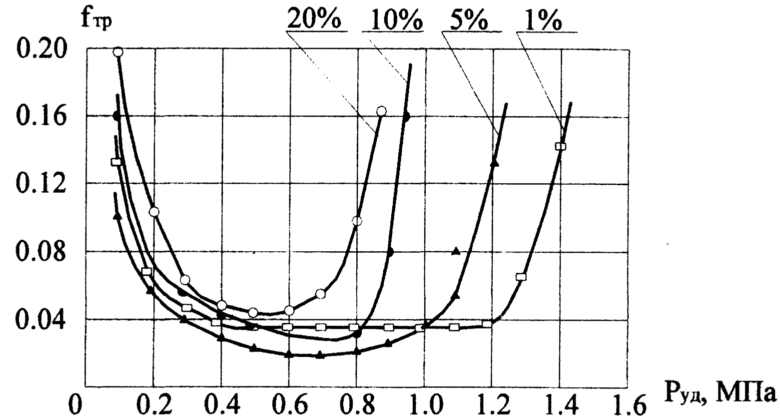

Рис. 2. Зависимость коэффициента трения от удельного давления на контакте пары трения ТМ-1 по ТМ-1 и содержания абразивных частиц в гидросмеси.

Анализируя зависимость $f=\varphi(p_{уд})$ можно сделать выводы:

1. Надежная, в некоторой степени, работа пары трения из твердых материалов в абразивной гидросмеси возможна при pv≤6 МПа·м/с.

2. На износ материалов колец пары трения существенное влияние оказывает тонкая фракция абразивных включений рабочей среды.

3. Защита пары трения возможна путем выбора твердых материалов и установки специальных фильтров в узлах трения.

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ALGORITHM OF INVESTIGATION OF MACHINE ELEMENTS NATURAL OSCILLATIONS

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Among the problems connected with designing and operation of machines with power-operated transmission drives an important place belongs to the problem of study of the frequency range of natural oscillations.

One of the most labour-consuming computing operations is not only the computation of the natural frequencies but also the determination of their quantity in the given frequency range. Well known is the problem when it is necessary to determine the probability of occurrence of resonance conditions at certain frequencies with no need to calculate the natural frequencies.

The system of equations of motion of the mechanical system elements can be obtained by equation:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right)-\frac{\partial E_k}{\partial q_i}+\frac{\partial E_p}{\partial q_i}+\frac{\partial E_d}{\partial \dot{q}_i}=Q_i, \qquad (1)$$

where E_k, E_p, E_d - are kinetic, potential and dissipated energies of the system, respectively, q_i - the generalized coordinate, Q_i - the generalized force.

After substitution in the equation (1) of the expressions for the given mechanical system energies and appropriate transformations, the equation of motion written in the matrix form will change into:

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{F\}, \qquad (2)$$

where $[M]$, $[C]$, $[K]$ - are mass, damping and stiffness matrixes of the system, respectively, $[q]$, $[\dot{q}]$, $[\ddot{q}]$ - the displacement vector of the mechanical system and its derivatives, $\{F\}$ - the vector of external forces.

Since for natural oscillations the vector of load $\{F\}=0$, and vector of motions is described by the periodic function $\{q\}=\{q_a\}e^{j\omega t}$.

Thus, the equation (2) assumed the form:

$$([K]-\omega^2[M])\{q_a\}=0, \qquad (3)$$

where $j=\sqrt{-1}$, ω - the natural frequency, $\{q_a\}$ - the vector of amplitude values of displacements of the system units at oscillations with the system damping to be neglected, i.e. $[C]=0$.

It is evident that the natural oscillations equation (3) has a non-trivial solution in one and the only case when the determinant

$$D=\det([K]-\omega^2[M])=0. \qquad (4)$$

Since the potential energy of transmission E_p is a positively determined quadrant function, the roots of the secular equation (4) are positive and (in accordance to the secular equation roots division theorem) divided by the principal diagonal minors of the determinant D.

Let us form a sequence of the principal diagonal minors of the determinant D, added with $D_n=1$:

$$D,\ D_1,\ D_2,\ D_3,\ \ldots,\ D_{n-2},\ D_{n-1},\ 1. \qquad (5)$$

According to the secular equation roots division theorem (4) when $\omega^2=0$ all the terms of the sequence (5) are positive, and the quantity of changes of sign in the sequence (5) is equal to zero. When $\omega^2\to\infty$ the sequence (5) has n changes of sign. Let us assume that when $\omega=\omega_1$ the quantity of changes of sign by the sequence (5) is equal to s_1, and when $\omega=\omega_2$ is equal to s_2. Then s_1 of natural frequencies are in the interval of the frequency axis $0\le\omega\le\omega_1$, and s_2 - in the interval $0\le\omega\le\omega_2$. It is evident that $s=s_2-s_1$ of frequencies are in the interval $\omega_1\le\omega\le\omega_2$.

With sequential reduction of the interval $[\omega_1,\omega_2]$, we can select frequency ranges with the only natural frequency or their given quantity. Every natural frequency can be determined with a given accuracy. The most simple and easy method of frequency selection is the method of dividing frequency intervals in half.

In most mechanical transmissions the quantity of masses in the system is greater by one than the quantity of degrees of freedom. This is explained by that such systems have no stationary mass closing. Dimensionality of matrixes in the described method is determined by the quantity of masses. In this case when $\omega \to 0$ there is one change of sign in the sequence (5), since $\omega = 0$ is a «fictitious» frequency of the system corresponding to the motion of the system as a solid whole. The quantity of real-valued frequencies for such system is equal to $n-1$.

The suggested method is effective not only for calculation of natural frequencies of transmissions but for determination of their quantity in the given frequency ranges. This allows to access the possibility of occurrence of resonance conditions in operation under real loads.

The assessment of reliability of the results obtained through the described method is given in comparison with the accurate analytical solutions. The suggested algorithm is realized in the calculation of tractor-drawn silage combine harvester. The obtained natural frequencies have satisfactorily agreed with those found through other methods.

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NATURAL OSCILLATIONS OF MODIFIED CONSTRUCTIONS

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Equation of natural oscillations for undamped mechanical system in matrix form is written as:

$$([K]-\omega^2[M])\{q\}=0$$

or

$$([E]-\omega^2[P][M])\{q\}=0 \quad (1)$$

where

$[E]$ - identity matrix,

$[P]=[K]^{-1}$ - compliance matrix inverse of the stiffness matrix $[K]$,

$[M]$ – mass matrix,

$\{q\}$- displacement vector (at the given natural frequency ω_i the vector $\{q\}_i$ corresponds to the i -th form of oscillations).

If the mechanical system has ***small*** constructional modifications with the compliance and mass matrixes changed to $[\Delta P]$ and $[\Delta M]$, respectively, then equation (1) for the new construction modification is written as:

$$\left([E]-(\omega^2+\Delta\omega^2)([P]+[\Delta P])([M]+[\Delta M])\right)(\{q\}+\{\Delta q\})=0 \quad (2),$$

where $\Delta\omega^2$ and $\{\Delta q\}$ – are changes in the frequency square and vector of form of natural oscillations.

As a first approximation, omitting the small increment products, the matrix equaion (2) is written as:

$$\{A\}+\lambda\{B\}=0 \quad (3)$$

where