Раздел 2

ПРОЕКТИРОВАНИЕ МАТЕРИАЛОВ И КОНСТРУКЦИЙ

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ANALYTICAL AND NUMERICAL SOLUTION FOR STRESS FIELDS IN PLANE SHEAR PROBLEM OF ANISOTROPIC ELASTICITY

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1.Introduction

To estimate any experimental method in mechanics of materials, it is needed to recognize the relation between stress state in a specimen and loading. In the case of shear tests on flat specimens for anisotropic materials it is difficult to obtain a perfect material reaction to the given loading. The method, known as the Iosipescu shear test, is used most often in the literature and is the one recommended by the standards to the prediction of the shear properties of anisotropic materials. In the Iosipescu test, an existence of pure shear is assumed. Fig 1a shows the scheme of specimen loading in the Iosipescu method.

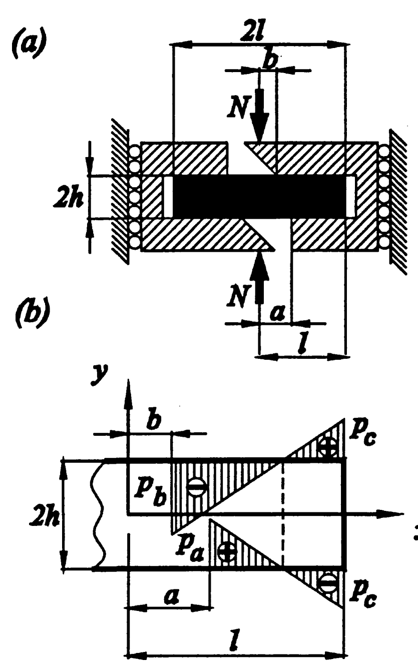

Fig.1. (a) The scheme of loading of the Iosipescu specimen, **(b)** The assumed loading distributions on the loading blocks

On the basis of the simple theory of beams the Iosipescu specimen treated as a rectangular beam, is subjected to four-point asymmetrical bending and therefore shear stress is calculated (by Adams, Wolrath (1987,1983)) in the central part, according to the formula:

$$\tau = N/A \tag{1}$$

where: N -shearing force, A -beam section in the central part.

In fact in the Iosipescu specimen mounted in both sides of the test fixture one side of the fixture is displaced vertically while the other side remains stationary. It is important to note that, there is no uniformly distribution of the reaction forces on loading blocks.

The aim of this paper is to calculate the stress state in the central part of the specimen by an analytical method. The Iosipescu specimen is considered as a flat plate, because the width of the beam is large compared with the thickness and comparable with its length.

2. Analytical Solution

A rectangular beam of length *2l*, width *2h*, and thickness $t << h$, fixed asymmetrical at each end in the loading blocks has been considered. The upper and bottom contact area between the blocks and the flat plate has length $(l-a)$ and $(l-b)$. The additional assumption is made that the ends of the flat plate are unsupported. The presented plane problem of elasticity is solved by the method of stress function *F* in a form of the Fourier series with given stress boundary conditions according to Fig. 1b. Orthotropic constants of elasticity were introduced as for beech wood to the calculations. The axes *x* and *y* in Fig 1b. coincide with the main orthotropy axes of the analyzed material, according to *L* and *T*. Stress field in the flat plate is described by the formula:

$$\sigma_{xx} = \frac{\partial^2 F}{\partial y^2}, \sigma_{yy} = \frac{\partial^2 F}{\partial x^2}, \tau_{xy} = -\frac{\partial^2 F}{\partial x\, \partial y}, \tag{2}$$

where: *F* – stress function. The Stress function is the solution of fourth-order biharmonic differential equation in the form (Lechnickij(1947)):

$$\left(\frac{1}{E_{TT}}\right)\frac{\partial^4 F}{\partial x^4} + \left(\frac{1}{G_{LT}} - \frac{2\nu_{LT}}{E_{LL}}\right)\frac{\partial^4 F}{\partial x^2 \partial y^2} + +\left(\frac{1}{E_{LL}}\right)\frac{\partial^4 F}{\partial y^4} = 0, \tag{3}$$

To simplify the calculations the loading in Fig. 1b is divided into the following parts: symmetrical and asymmetrical, therefore the stress function is built in the form of the formula $F = F_1(x,y) + F_2(x,y)$. In the case of symmetrical loading the stress function $F_1(x,y)$ in the form of the cosine series was assumed. To satisfy the boundary conditions two additional expressions are introduced to the cosine series:

$$\begin{aligned} F_1(x,y) = \sum_{m=1}^{\infty} \cos\alpha_m x[C_1 ch(G\alpha_m y) + C_2 sh(G\alpha_m y) + \\ + C_3 ch(H\alpha_m y) + C_4 sh(H\alpha_m y)] + C_5 xy - f_1(x,y), \end{aligned} \tag{4}$$

where: $\alpha_m = \frac{m\pi}{l}$, $m = 1,2,3...$, C_1, C_2, C_3, C_4, C_5-constants, $f_1(x,y)$-special function is taken from the obtained symmetrical solution, which considers the edge ends of the flat plate as unsupported. For asymmetrical loading stress function $F_2(x,y)$ in the form of the sine series with one additional expression was assumed.

$$F_2(x,y)=\sum_{m=1}^{\infty}\sin\alpha_m x[D_1ch(G\alpha_m y)+D_2sh(G\alpha_m y) \\ +D_3ch(H\alpha_m y)+D_4sh(H\alpha_m y)]-f_2(x,y), \tag{5}$$

$\alpha_m=\frac{m\pi}{l}$, $m=1,2,3...$, D_1,D_2,D_3,D_4-constants, $f_2(x,y)$-special function is taken from the obtained asymmetrical solution, which considers the edge ends of the flat plate as unsupported. In this paper stress distributions in the central part of the flat plate is considered therefore shear stress τ_{xy} can be determined at the asymmetrical loading but normal stress at the symmetrical loading. Functions $f_1(x,y)$ and $f_2(x,y)$ in expressions (4) and (5) can be treated as stress functions, which provide solutions of the stress field at the ends of the plate, the same as in the case of their elimination. In order to determine the constants in symmetrical problem the stress boundary conditions are formulated as following:

$$\begin{aligned} &y=\pm h\rightarrow\tau_{xy}=0,\\ &y=\pm h\rightarrow\sigma_{yy}=+q_1(x)\\ &x=\pm l\rightarrow\sigma_{xx}=0\\ &x=\pm l\rightarrow\tau_{xy}=0 \end{aligned} \tag{6}$$

where: $q_1(x)=\frac{c_0}{2}+\sum_{m=1}^{\infty}c_m\cos\alpha_m x$, $c_0=\frac{2}{l}\left[\int_a^l 0.5y_1(x)dx+\int_b^l 0.5y_2(x)dx\right]$,

$c_m=\frac{2}{l}\left[\int_a^l 0.5y_1(x)\cos\alpha_m xdx+\int_b^l 0.5y_2(x)\cos\alpha_m xdx\right]$. While in asymmetrical problem it is assumed:

$$\begin{aligned} &y=\pm h\rightarrow\tau_{xy}=0,\\ &y=\pm h\rightarrow\sigma_{yy}=\pm q_2(x)\\ &x=\pm l\rightarrow\sigma_{xx}=0\\ &x=\pm l\rightarrow\tau_{xy}=0 \end{aligned} \tag{7}$$

where: $q_2(x)=\sum_{m=1}^{\infty}d_m\sin\alpha_m x$, $d_m=\frac{2}{l}\left[\int_a^l 0.5y_1(x)\sin\alpha_m xdx+\int_b^l 0.5y_2(x)\sin\alpha_m xdx\right]$ The integrand functions $y_1(x),y_2(x)$ in the given above integrals have the following form:

$$\begin{aligned} &y_1(x)=-A_1x+A_2\\ &y_2(x)=A_1x-A_2, \end{aligned} \tag{8}$$

The integrand functions $y_1(x),y_2(x)$ satisfy, according to Fig 1b. the conditions:

$$y_1(x=a)=+p_a,\ y_1(x=l)=-p_c,\ y_2(x=b)=-p_b,\ y_2(x=l)=+p_c,$$
$$\int_a^l y_1(x)dx+\int_b^l y_2(x)dx=N,\ \int_a^l y_1(x)xdx+\int_b^l y_2(x)xdx=0.$$

3.Numerical Solution

Numerical calculations of the stress state in the plate were carried in the range of linear elasticity of material by FEM. The finite element computations were performed using the MSCPatran/Nastran. Model 2-D of the orthotropic plate was built taking into consideration the elastic constants from Tab.1.

Table 1

Elasticity constants for beech wood in the orthotropy plane LT

E_{LL} GPa	E_{TT} GPa	G_{LT} GPa	ν_{LT}	ν_{TL}
14.24	1.07	0.91	0.469	0.035

The finite element mesh with two displacement components, i.e. u,υ consists of isoparametric ,eight-nodes quadrilateral elements. The displacements boundary conditions were introduced for the following nodes:

$$\begin{aligned}
&x=0, y=0 \rightarrow u=0, \upsilon=0\\
&b<x<l, y=+h \rightarrow u=0, \upsilon=\frac{-\upsilon_0}{2}\\
&a<x<l, y=-h \rightarrow u=0, \upsilon=\frac{-\upsilon_0}{2}\\
&-l<x<-a, y=+h \rightarrow u=0, \upsilon=\frac{\upsilon_0}{2}\\
&-l<x<-b, y=-h \rightarrow u=0, \upsilon=\frac{\upsilon_0}{2}
\end{aligned} \tag{9}$$

In this way the total displacement of the plate was equal υ_0

4. Conclusions

The stress fields in the central part of the flat plate, obtained by the analytical and numerical methods are presented in Figs. 2,3. The presented analytical solution contains a sum of only seven terms of the trigonometrical series. The mentioned number of terms, i.e. $m=1,2...7$ gives satisfactory approximation of loading distributions in the form of (8) by the trigonometrical series. The presented in Figs. 2,3 stress distributions $\sigma^*_{xx}, \tau^*_{xy}, \sigma^*_{yy}$ were normalized in respect to the average shear stress $\bar{\tau}_{xy}$, occurring in the central part of the flat plate. The average shear stress $\bar{\tau}_{xy}$ was calculated independent for both solutions, as follows

$$\bar{\tau}_{xy} = \frac{1}{2h}\int_{-h}^{h} \tau_{LT}(0,y)dy \tag{10}$$

where: $\tau_{xy}(0,y)$ - shear stress for $x=0$, *h*- half of width of the flat plate. The value *y*, was normalized in respect to the width *h*. In the Figs 2,3 shear stress τ calculated by formula (1) was also presented.

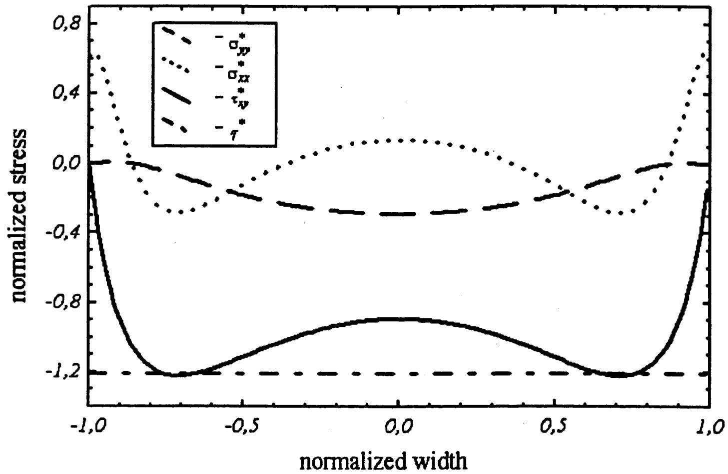

Fig.2. Analytical solution

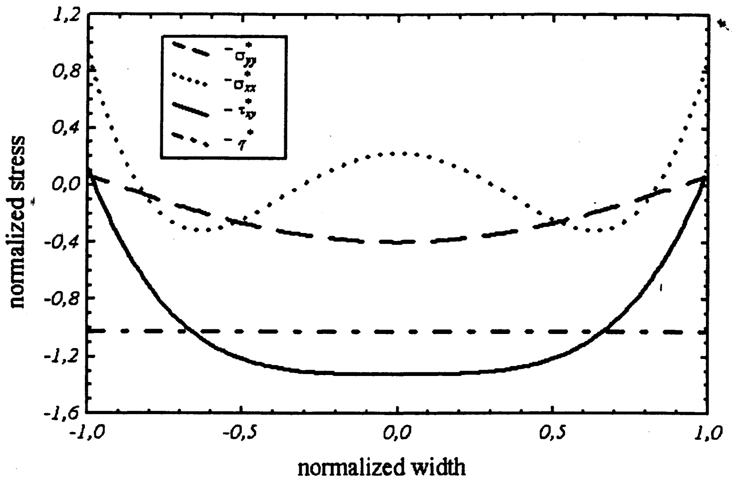

Fig.3. Numerical solution

The conducted analyses shows convergence between analytical and numerical solutions, even for a few number of terms of the trigonometrical series (4) i (5). Additionally, the analyses confirmed the usefulness of formula (1), which correctly describes average shear stress in the central part of the Iosipescu specimen.

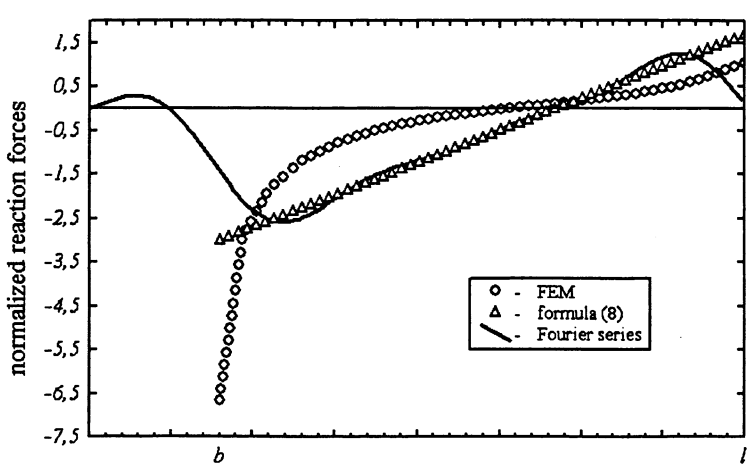

Fig.4. Normalized distribution of the reaction forces at the upper loading block

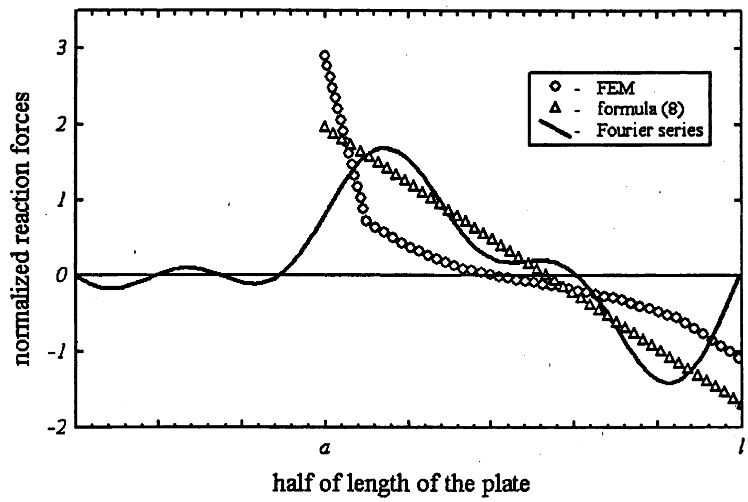

Fig.5. Normalized distribution of the reaction forces at the bottom loading block

Existence of positive values of the reaction forces in the loading distributions on the loading blocks can be treated as a kind of singularity in the analytical solutions. It is important to note that, this singularity is also associated with numerical solutions. The mentioned singularity was presented in Figs 4,5. The presented in Figs. 4,5 loading distributions occurring at the upper and bottom contact area were normalized independent for both solutions in respect to the average shear stress $\bar{\tau}_{xy}$ (10).

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ОБ ЭФФЕКТИВНОСТИ ПРИМЕНЕНИЯ НЕКОТОРЫХ МЕТОДОВ МЕХАНИКИ ДЕФОРМИРУЕМОГО ТВЕРДОГО ТЕЛА К РЕШЕНИЮ ЗАДАЧ ТЕХНИЧЕСКОЙ ДИАГНОСТИКИ

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Основной задачей технической диагностики является повышение надежности объектов на этапе их производства, эксплуатации и хранения. Диагностическое обеспечение позволяет повысить достоверность правильного функционирования объектов, увеличить срок их службы и наработку на отказ. Большое значение для инженерной практики имеют решения задач прогнозирования, в частности, для организации технического обслуживания по состоянию, вместо обслуживания по ресурсу. Непосредственное перенесение методов решения задач диагностирования на задачи прогнозирования невозможно из-за различия моделей, с которыми приходится работать: при диагностировании моделью обычно является описание объекта, в то