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IMPROVEMENT OF ACCURACY OF RADIATIVE HEAT TRANSFER DIFFERENTIAL APPROXIMATION METHOD FOR MULTI DIMENSIONAL SYSTEMS BY MEANS OF AUTO-ADAPTABLE BOUNDARY CONDITIONS

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Differential approximation is derived from radiation transfer equation by averaging over the solid angle. It is one of the more effective methods for engineering calculations of radiative heat transfer in complex three-dimensional thermal power systems with selective and scattering media. The new method for improvement of accuracy of the differential approximation based on using of auto-adaptable boundary conditions is introduced in the paper. The efficiency of the named method is proved for the test 2D-systems. Self-consistent auto-adaptable boundary conditions taking into consideration the nonorthogonal component of the incident to the boundary radiation flux are formulated. It is demonstrated that taking into consideration of the non- orthogonal incident flux in multi-dimensional systems, such as furnaces, boilers, combustion chambers improves the accuracy of the radiant flux simulations and to more extend in the zones adjacent to the edges of the chamber.

Test simulations utilizing the differential approximation method with traditional boundary conditions, new self-consistent boundary conditions and “precise” discrete ordinates method were performed. The mean square errors of the resulting radiative fluxes calculated along the boundary of rectangular and triangular test areas were decreased 1.5–2 times by using auto-adaptable boundary conditions. Radiation flux gaps in the corner points of non-symmetric systems are revealed by using auto-adaptable boundary conditions which can not be obtained by using the conventional boundary conditions.

Keywords: radiative heat transfer, numerical simulation, differential approximation, boundary conditions.

Fiq. 4. Tab. 1. Ref.: 11 titles.

УЛУЧШЕНИЕ ТОЧНОСТИ ДИФФЕРЕНЦИАЛЬНОГО ПРИБЛИЖЕНИЯ РАСЧЕТА ТЕПЛООБМЕНА ИЗЛУЧЕНИЕМ В МНОГОМЕРНЫХ СИСТЕМАХ ПРИ ИСПОЛЬЗОВАНИИ САМОСОГЛАСОВАННЫХ ГРАНИЧНЫХ УСЛОВИЙ

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Дифференциальное приближение, получаемое путем усреднения по телесному углу уравнения переноса излучения, является одним из наиболее эффективных методов инженерного расчета лучистого теплообмена в сложных многомерных теплоэнергетических системах с селективной и рассеивающей средой. Представлен подход для улучшения точности расчета лучистого теплообмена методом дифференциального приближения в многомерных системах за счет использования самосогласованных граничных условий. Продемонстрирована эффективность предложенного подхода на примере модельных двумерных систем. Записаны самосогласованные граничные условия, учитывающие неортогональность падающего потока излучения к поверхности границы и алгоритм их использования. Показано, что учет неортогональности падающего потока повышает качество расчета радиационного теплообмена в многомерных системах, особенно вблизи угловых зон.

Расчеты, проведенные с использованием традиционных и самосогласованных граничных условий, сравниваются с «точным» расчетом, выполненным методом дискретных ординат. Показано, что использование нового подхода позволяет уменьшить среднеквадратичную погрешность расчета результирующего потока излучения на стенку в 1,5–2 раза. Использование самосогласованных граничных условий дает возможность выявить скачок результирующего потока в угловых точках многомерной системы, который невозможно получить при расчетах с использованием традиционных граничных условий.

Ключевые слова: лучистый теплообмен, численное моделирование, дифференциальное приближение, граничные условия.

Ил. 4. Табл. 1. Библиогр.: 11 назв.

Introduction. There are many combustion, power engineering, atmospheric heat transfer and other problems which demand prompt calculation of radiation fluxes with relatively low accuracy. The differential approximation (DA) is the fast and effective method for radiation transfer calculation if desired accuracy of radiation fluxes evaluation is of the order of 15–20 % and there is no highly anisotropic radiation fluxes in the system [1]. It takes usually 10–100 times less computational time than "precise" methods of radiation transfer equation integration for non-uniform medias. The economy of CPU time becomes increasingly significant when two and three- dimensional systems with selective media are considered.

At higher optical densities $\tau > 10$ the "precise" methods meet with difficulties connected with increase of the computations for each direction and increase of the iterations to reach the given accuracy. The resulting flux, calculated by discrete ordinate method, may get significant errors when the difference between values of the incident to the boundary and irradiated by the boundary fluxes is small. The DA-method is free of this privation because it calculates the resulting flux directly.

Another advantage of the DA-method – is that it can be easily incorporated into the computational routines of heat and mass transfer and gas dynamics problems. Standard computer codes able to resolve second order steady state differential equations may be utilized.

Differential approximation is a basic approach for reducing integro-differential equation for radiative flux to differential form. It is used since the 40^{th} [2–4]. A common misjudge about the DA, is that it is invalid for optically thin systems [5]. Actually, the only assumption grounding the DA is sufficient uniformity of the radiation field, or more specifically, of the angular distribution of radiation intensity $I(r, l)$. The adequate boundary conditions for the DA equation (4) guarantee good accuracy of the DA method for the arbitrary optical densities and non-homogeneous media [6, 7]. The boundary conditions are formed with radiation energy conservation equations, formulated with specific assumption about the emissivity and reflectivity of boundaries and radiation energy field near the walls (for example, diffusive gray walls and isotropic semispherical irradiation of to the walls may be considered).

Taking in mind importance of the boundary conditions for DA, Olfe [7] proposed modified differential approximation method (MDA). The main idea of the MDA is consideration of radiation from the boarders together with the absorbing media and the media self radiation separately. As reported [5, 7], MDA provides high accuracy, close to the accuracy of the higher order spherical harmonics approximations (P_3, P_5) and “precise” discrete ordinate method. At the same time the integro-differential radiation transfer equation for enclosure appears in MDA, which complicates utilization of this method. Another approach improving the quality of the DA is, so-called, quazi differential approximation (QDA) [8]. According to this method the local adjustment coefficients for the DA are calculated on the basis of exact solution of radiation transfer equation. This makes DA more accurate for the fixed thermodynamic situation. After the thermodynamic situation is changed in a system a new adjustment is necessary. A higher order Spherical harmonics methods (P_3, P_5) are most often used to increase accuracy of radiation heat transfer calculation [8, 9]. This is particularly effective for calculation of radiation fields within the relatively thick optical media. The accuracy of the P_n methods in vicinity of the boundaries is similar to the standard DA and depend on adequacy of the boundary conditions.

The Marshak’s [1, 5] boundary conditions or other BC based on the physical assumptions regarding the angular distribution of the intensity near the boundary are usually applied to solve governing equation. Practically, the boundary points are not symmetric in the multi-dimensional systems and application of the conventional boundary conditions may lead to a loss of accuracy. For example, the calculations of the resulting radiative flux along the boarders of rectangular and triangular volumes, Fig. 1, demonstrate that DA routine do not reveal the radiation flux gaps in the corner points, see Fig. 2. Inaccuracy of the BC is one of the main sources of errors for the DA-method modifications mentioned above [10].

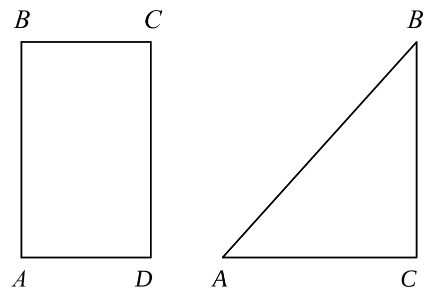

Fig. 1. The geometry of the two-dimensional test areas

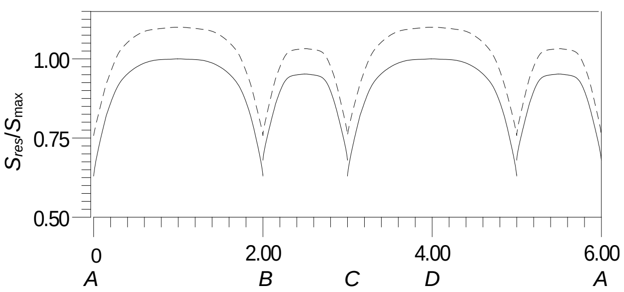

Fig. 2. The relative resulting flux along the boarder of the rectangular, calculated by DA with standard BC – dashed line; discrete ordinates “precise” method – solid line; effective optical depth τ = 1; temperature of the media T = 1500 K; walls temperature T_0 = 1300 K

To improve the boundary conditions it is necessary to know the angular distribution of radiation intensity in each point of the boundary. Here we propose to utilize conventional solution of the DA equation for adjusting the boundary conditions and, consequently, obtaining the higher accuracy of DA solution at the next step. It means that information about the geometry, local radiation properties of the media and temperature fields is used to improve the boundary conditions. The new auto-adaptable (AA) boundary conditions are formulated by using this idea. Though our formulation of the auto-adaptable boundary conditions is not the only possible, the numerical simulation and tests demonstrate that this approach improves the solution for radiation fluxes qualitatively and quantitatively. All the analysis is presented for the non-scattering media and monochromatic radiation, although it can be easily extended for more practical situations.

Theory of the method. The equation of the radiation transfer in selective scattering and absorbing media may be written in the form

$$\frac{dI_\omega(r,l)}{dr} = -(\kappa_\omega + \sigma_\omega)I_\omega(r,l) + \kappa_\omega B_\omega(T,r) + \frac{\sigma_\omega}{4\pi}\int_0^{2\pi} d\varphi' \int_0^{\pi} p(l,l')I_\omega(r,l')\sin\theta' d\theta', \quad (1)$$

where $I_\omega(r,l)$ – radiation intensity at spectral frequency ω, in the point defined by radius vector r in the direction defined by vector l; κ_ω – absorption coefficient; $B_\omega(T,r)$ – black body spectral radiation intensity at temperature T and radiation frequency ω; σ_ω – scattering coefficient; $p(l,l')$ – scattering indicatrix.

The differential approximation equations may be derived by averaging (1) over the solid angle [1–3]. For non-scattering media one can obtain the system of equations for spectral radiation energy density $U_\omega(r)$ and radiation flux $\vec{S}_\omega(r)$:

$$\nabla\vec{S}_\omega = 4\pi\kappa_\omega B_\omega(T,r) - c\kappa_\omega U_\omega; \quad (2)$$

$$\vec{S}_\omega = -\frac{c}{a\kappa_\omega}\nabla U_\omega, \quad (3)$$

where a – parameter characterizing anisotropy of the radiation intensity $I_\omega(r,l)$ angular distribution

$$a(r) = \frac{\int_{4\pi} \nabla I_\omega(r,l)d\Omega}{\int_{4\pi} (l\nabla I_\omega(r,l))l d\Omega}. \quad (4)$$

The spectral radiation energy density and radiation flux are defined as follows:

$$U_\omega(r) \equiv \tfrac{1}{c}\int_{4\pi} I_\omega(r,l)d\Omega; \quad \vec{S}_\omega(r) \equiv \int_{4\pi} I_\omega(r,l)l d\Omega.$$

The first equation of the system (2), (3) is the exact radiation energy continuity equation. The second equation is approximate as far as coordinate dependent anisotropy parameter a is fixed (a = 3 corresponds to the P_1 spherical harmonics approximation and a = 4 – to the two-flux Schwartzchild-Schuster approximation).

Combining equations (2) and (3) and taking, for instance, a = 4, we obtain a second order differential equation for $U_\omega(r, t)$

$$\nabla\left(\frac{1}{\kappa}\nabla U_\omega\right) - 4\kappa U_\omega + 16\pi\kappa/c\, B_\omega(T, r) = 0. \tag{5}$$

Assuming the incident radiation is hemispherically isotropic and the flux vector is orthogonal to the boundary, one can obtain the following correlation for incident radiation [2]

$$\left.\frac{c}{2}U_\omega\right|_\delta = \left|\vec{S}_\omega\right|. \tag{6}$$

Here subscript δ indicate the value at (in the vicinity of) the boundary, column parentheses define the module of vector.

The radiation intensity near the boundary may be considered as a superposition of incident radiation with intensity I_{in}, emission of the wall $\varepsilon B_\omega(T, r)$, and scattered- reflected part of incident radiation with intensity (1 – ε)I_{in}. Simple possible schematics of the radiation field, which may be titled as “isotropic” (a) and “exocentric” (b) are presented on the Fig. 3.

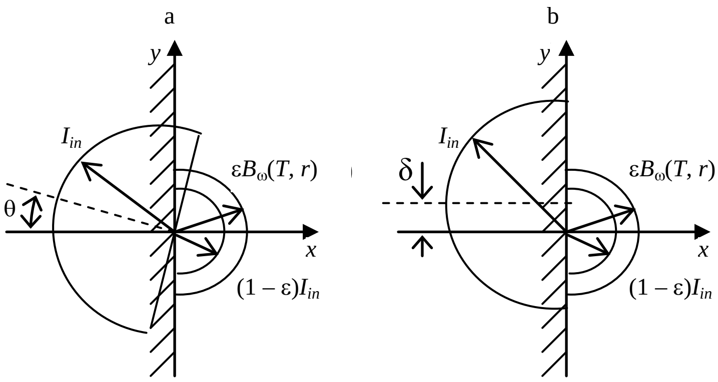

Fig. 3. Schematics of radiation field near the boundary

Let us write down the boundary conditions for the simpler “isotropic” model of non orthogonal to the boundary radiative flux. The total radiant energy density for the grey boundary may be written as follows

$$\left.\frac{c}{2}U\right|_\delta = \pi(2 - \varepsilon)I_{in} + \pi\varepsilon B(T, r), \tag{7}$$

where and below we omit index ω near radiation intensity symbols for better readability of formulas.

The components of the resulting flux near the boundary may be written as:

$$\vec{S}_x = \pi I_{in}(\varepsilon + \cos\theta - 1) - \pi\varepsilon B(T, r); \tag{8}$$

$$\vec{S}_y = \pi I_{in}\sin\theta.$$

Combining (3), (7) and (8) one can obtain the following BC for the equation (5):

$$-\frac{c}{a\kappa}\frac{\partial U}{\partial x}=\left[\frac{c}{2}U-\pi\varepsilon B(T,r)\right]\frac{\varepsilon+\cos\theta-1}{2-\varepsilon}-\pi\varepsilon B(T,r); \tag{9}$$

$$-\frac{c}{a\kappa}\frac{\partial U}{\partial y}=\left[\frac{c}{2}U-\pi\varepsilon B(T,r)\right]\frac{\sin\theta}{2-\varepsilon}.$$

The value of the incident flow vector angle θ, as well as cosθ and sinθ can be estimated by using the system (8) if the radiation energy density $U\big|_{\delta}$ is known in the vicinity of the boundary. Excluding I_{in} from (8) one can obtain:

$$\vec{S}_y\cos\theta-\vec{S}_y(1-\varepsilon)=[\vec{S}_x+\pi\varepsilon B(T,r)]\sin\theta;$$

$$\vec{S}_y\cos\theta-\vec{S}_y(1-\varepsilon)=[\vec{S}_x+\pi\varepsilon B(T,r)]\sqrt{1-\cos^2\theta}.$$

Square equation for cosθ will have the form

$$[\vec{S}_y^2+(\vec{S}_x+\pi\varepsilon B(T,r)]\cos^2\theta-2\vec{S}_y^2\rho\cos\theta+\vec{S}_y^2\rho^2-(\vec{S}_x+\pi\varepsilon B(T,r))^2=0. \tag{10}$$

Solution of (10)

$$\cos\theta=\frac{\vec{S}_y^2\rho}{\vec{S}_y^2+\tilde{U}_x^2}+\sqrt{\frac{\vec{S}_y^4\rho^2}{[\vec{S}_y^2+\tilde{U}_x^2]^2}+\frac{\tilde{U}_x^2-\vec{S}_y^2\rho^2}{\vec{S}_y^2+\tilde{U}_x^2}}, \tag{11}$$

where $\rho=1-\varepsilon$ – reflectivity; $\tilde{U}_x=\vec{S}_x+\pi\varepsilon B(T,\ r)$; $\vec{S}_x=-\frac{c}{a\kappa}\frac{\partial U}{\partial x}$; $\vec{S}_y=-\frac{c}{a\kappa}\frac{\partial U}{\partial y}$.

The formulas (9) define the self – consistent auto-adaptable boundary condition. Equation (5) together with boundary conditions (9), (11) can be solved by iterations starting from "zero-approach" at θ = 0.

Numerical simulation. To test differential approximation method with the Auto-Adaptable BC, we calculated the resulting flux near the boundaries by using equation (5) with boundary conditions (9), (11). This solution was compared to the results obtained by means of deferential approximation with standard boundary conditions and "precise" discrete ordinate solution [11]. The computations were performed for wide range of optical depths and temperature distribution. Serious attention was paid to the benchmark method verification as far as discrete ordinate method may give serious errors at situations of high optical density, "ray" effect and some others. For example, the resulting flux, which is the difference between incident to the boundary flux and irradiated by the boundary flux, may get dramatic errors when the values of incident and boundary irradiating fluxes are close. The DA-method is dealing with the resulting flux directly and is free of this disadvantage.

All the calculations were realized by the finite elements method. One iteration was sufficient for auto adjusting of boundary condition (9), (11) in our case. The second and following iterations did not contribute to the accuracy of the solution within 1 %. Consequently, the averaged computation time for DA with auto-adaptable BC is 2 times more than with fixed BC and about

50–100 times less than “precise” solution implemented by discrete ordinates method [11]. The resulting flux near the boundary of equilateral triangular and rectangular test areas as related to its maximum value is presented on Fig. 4. The temperature of the irradiating media is 1500 K, the walls have the temperature 1300 K. The effective optical depth τ, which is presented in the notes to the Figures and Table, was calculated by the square of the two-dimensional test area S, as follows $\tau = \kappa\sqrt{S}$. At the optical depth $\tau \approx 1.0$ the accuracy of the DA is the worst. At higher $\tau > 5.0$ and lower $\tau < 0.2$ optical depth the accuracy becomes better and approaches the accuracy of the discrete ordinate solution.

For calculation of the resulting flux normal component $\partial U/\partial x$ and tangential component $\partial U/\partial y$ (which is necessary for obtaining $\cos\theta$, Eq. (11)), the finite element triangles adjacent to the corner points were taken symmetric and congruent, otherwise the computational inaccuracy may corrupt the positive effect of using of the AA BC.

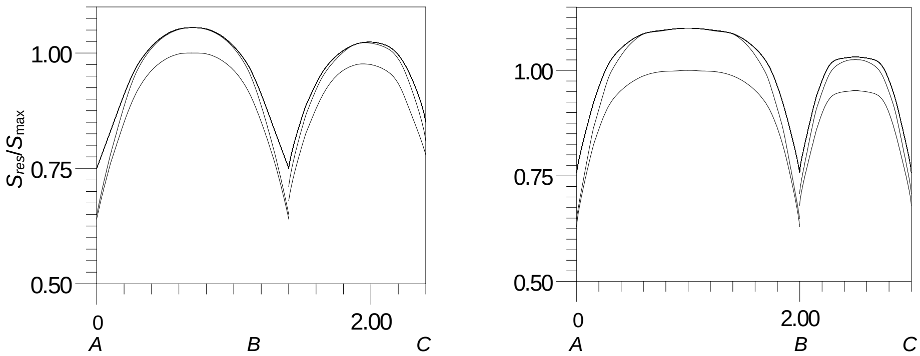

Fig. 4. The relative resulting flux along two boarders of the rectangular and triangular areas, calculated by the standard DA-method – dashed, by DA with auto-adaptable BC – pointed and discrete ordinates “precise” method – solid line; effective optical depth $\tau = 1$

It is follows from the calculations, Fig. 4, that new approach gives more accurate solution. Though the maximum absolute discrepancy is not reduced essentially, the averaged over the perimeter square root discrepancy of the DA with auto-adaptable BC is 1.5–2 times lower than one of the DA with conventional BC. The same tendencies are preserved for different optical depth and temperature distributions inside the volume.

The values of maximum absolute and averaged over the perimeter square root discrepancies are presented in the Tab. 1.

Table 1

Absolute and mean square averaged discrepancies for resulting radiation flux calculated by the differential approximation method with standard and auto-adaptable BC. Optical depths $\tau = 0.5$, $\tau = 1.0$ and $\tau = 2.0$

Geometry	Method	Discrepancy, %					
		$\tau = 0.5$		$\tau = 1.0$		$\tau = 2.0$	
		Abs.	Mean sq.	Abs.	Mean sq.	Abs.	Mean sq.
Rectangular	AA BC	16	5.6	9	4.9	7	3.6
–»–	Standard BC	23	9.3	17	7.8	8	5.0
Triangular	AA BC	20	8.8	12	7.3	12	6.0
–»–	Standard BC	29	14.0	19	11	18	8.0

CONCLUSIONS

Utilization of self-consistent auto-adaptable BC let one improve accuracy of the differential approximation method. This is particularly relates to evaluation of the resulting radiation flow at the boundary.

The numerical simulation performed for “worst” optical depth conditions ($\tau = 1$) show that the mean square errors of the resulting radiative fluxes calculated along the boundary of the rectangular and triangular test areas were decreased 1.5–2 times by using auto-adaptable boundary conditions. Utilization of the mentioned approach could be recommended for the radiation fluxes determination in non-homogeneous non-symmetric two- and three-dimensional systems, such as furnaces, boilers, combustion chambers.

Good results, obtained by using the AA BC encourage one to develop new methods of auto-adaptable differential approximation basing on numerical algorithms by recalculation of anisotropy parameter a. Specific meshing may be utilized for this aim. Improvement of the DA requires standard, internationally approved benchmarks for 3D-radiation transfer problem solution. Profound testing of the auto-adaptable BC, particularly for different schematics of radiation field near the boundary, is a matter of further investigations.

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