Real Estate, Asset and Business Valuation by Income Approach: Modern Techniques

Nikolai TRIFONOV[1]

Abstract

For over two centuries, the income approach has been considered one of the most powerful and reasonable tools for income property appraisal (*i.e.* real estate, asset and business valuation) or for real estate-related investment project management. The first in time of its emergence to implement the income approach was the direct capitalisation method used in the 19th century. In the direct capitalisation method, simple analytical expressions are used that can be easily developed, *e.g.* the inclusion of a constant increase in income (Gordon method). About a hundred years ago, the direct capitalisation method began to be supplemented and subsequently replaced by the discounted cash flow method, or DCF analysis. The author of the DCF analysis should be Fisher, who introduced the concept of net present value based on discrete cash flow and constant discount rate. Later, it was supplemented by Solomon with the important idea of terminal value, also interpreted as capital reversion, final return on capital, or final sale. The recent global financial crisis has increased the investor's attention to the valuation of capital subjects of income property and business with values in use instead of values in exchange, primarily market value. In these cases, the DCF method is the basis for the calculation. The paper discusses the problematic aspects of the valuation of a real estate or a related investment project using the DCF method. First, well-known expressions of DCF analysis were refined in the case of the discount rate variability. Further, previously unknown exact formulas of the DCF method with a variable discount rate and different patterns of cash flow (discrete, continuous and discrete-continuous) have been obtained. Applying derived exact formulas may reduce calculation errors from using traditional expressions with constant discount rates and discrete cash flow.

Keyword: DCF Analysis, Discrete-Continuous Cash Flow, Variable Discount Rate, Income Approach, Income Property

[1] Dr.Sc., FRICS, Full member of International Academy of Engineering, Belarus State Economic University, guild@unibel.by, Minsk, Belarus, ORCID iD 0000-0001-6396-3478

1. Introduction

For over two centuries, the income approach has naturally been considered one of the most powerful and reasonable tools for income property appraisal (i.e. valuation of a real estate with commercial potential) or real estate-related investment projects. The first in time of its emergence to implement the income approach was the direct capitalisation method, which was used in the 19th century (Fuhrer, 1944). In the 30s of the last century, the direct capitalisation method was supplemented by the discounted cash flow method or DCF analysis.

The author of the DCF analysis should consider Irwin Fisher, who, among others, introduced the concept of net present value based on discrete cash flow and constant discount rate (Fisher, 1907, 1930). Later, it was supplemented with the important idea of terminal value (Solomon, 1956). The recent global financial crisis has increased the attention of investors to the valuation of capital subjects of income property and business by values in use, such as investment value and user value, as opposed to values in exchange, primarily market value (Trifonov, 2010). In these cases, the DCF method is the basis for the calculation.

2. Simplest Form

At its inception, the DCF method was considered rather difficult due to the need for a large amount of initial data and insufficient computer power. Therefore, it was used as an additional. At this time, a single magnitude of the discount interest rate (or rate of return) was used in the calculations related to different years of the forecast, which greatly simplified the calculations (see, *e.g.* (Adair, Downie, McGreal & Vos, 1996) or more modern (Appraisal Institute, 2013) and even recent (International Valuation Standards Council, 2019). It was in this form that the method was introduced into the real estate valuation practice and is often recommended for use to this day:

$$V = \sum_{t=1}^{n} \frac{I_t}{(1+R\quad)^t}, \qquad (1)$$

where V is the present value of the valuation subject,

I_t is the value of the t-th current periodic (usually annual) cash, while I_n includes the terminal value V_n at the end of the forecast period (or the value of the final return of capital),

R is the discount rate,

n is the number of the last period (usually year).

Note that if the periodic income is constant, the series can be easily summed up, passing into the formula of the direct capitalisation method. This circumstance made it possible to write in International Valuation Standards: "Although there are many ways to implement the income approach, methods under the income approach are effectively based on discounting future amounts of cash flow to present value. They are variations of the Discounted Cash Flow (DCF) method" (International Valuation Standards Council, 2019: 37).

Even in this form (1), the method was difficult to calculate some time ago. For a special but popular case of the application of the DCF to the rent analysis of the property valuation yields, a simplification was put forward, called the short-cut DCF (Baum and Crosby, 1995), which is still applied at present (Ataguba, 2021).

With the development of computer technology, proposals have appeared for using different magnitudes of the discount rate in a single calculation (a particular valuation). This was due to the ideas formed at that time about the dependence of the magnitude of the discount rate on the risks of activities associated with the valuation subject. First, it was noted in CAPM (Sharp, 1964). The idea has caused numerous discussions and developments. Useful literature on the matter is reflected in (Damodaran, 2012).

3. Variable Rate Attempts

Due to the difference in expected risks, using different magnitudes of the respective discount rates would be natural. For example, the use of one rate magnitude to discount the cash flow series and the other one to discount the terminal value (or the so-called capital reversion), which in the general case may differ by an order of magnitude. Naturally, in the case of real estate leases, the first-rate reflects benefits or contractual constraints, while the other is dictated by the terms of a free, open market (Appraisal Institute, 2013). In addition, the magnitude of the discount rate in the general case should change over time, on the one hand, due to changes in the general economic situation, on the other hand, due to possible changes in the state of the valuation subject. An example of the latter can be the assessment of an investment project for real estate development; when money transfers over time to tangible assets, the investment risks (and the respective discount rate) should decrease.

Attempts are known to consider a rate variability in the method. They are reflected in the following formula (Gribovsky, 2016):

$$V = \sum_{t=1}^{n} \frac{I_t}{(1+R_t)^t} + \frac{V_n}{(1+R_n)^n}, \tag{*}$$

Where R_t is the discount rate during the *t*-th period (year) for a periodic cash,

R_n is the discount rate for the terminal value.

In this form, the formula is incorrect from the point of view of financial mathematics, but when one uses a constant discount rate ($R_t = R_n = const$), it goes over to the previous (1) and becomes more correct due to its origin. Getting the correct formula for the DCF method in that case is given below.

4. Exact Expressions with Variable Rate

Let the dependence of the discount rate on the period number be as described in Table 1.

Table 1. Rate designations depending on the period

Period (year, month)	1	2	...	*n*
Rate	R_1	R_2	...	R_n

We will use simple induction to understand the correct form of the expression describing the DCF method with a variable rate. For simplicity, in the first stage, we suppose that periodic (annual) cash payments are made at the end of the period (year), there is no final return on capital (terminal value), and the forecast is made only for 1 period (year). The value of discounted cash in 1st period is

$$V = \frac{I_1}{1+R_1}.$$

In case of periodic cash payments within two years, we will receive

$$V = \frac{I_1}{1+R_1} + \frac{I_2}{(1+R_1)(1+R_2)},$$

Since the cash payment of the second year is discounted sequentially through the second and first years, each with its discount rate. Similarly, for the entire forecast period of *n* years, the sought expression takes the form (Trifonov, 2019, 2021):

$$V = \sum_{t=1}^{n} \frac{I_t}{\prod_{j=1}^{t}(1+R_j)}. \quad (2)$$

where $\prod_{j=1}^{t}(1 + R_j)$ means the product $(1+R_1)(1+R_2)\ldots(1+R_t)$.

Note that similar chain products were obtained when modifying the DCF method for considering the interests of both transaction parties - the buyer and the seller (Michaletz and Artemenkov, 2019).

The magnitude of the terminal value (final return on capital or final sale) generally differs from the magnitude of periodic cash payments by an order. Therefore, their discount rates should also differ. Let's denote the discount rates for the final return on capital by the letter *r*. Then, the formula for the ***case of discrete cash flow with variable rate and inclusion of terminal value*** will take the following form:

$$V = \sum_{t=1}^{n} \frac{I_t}{\prod_{j=1}^{t}(1+R_j)} + \frac{V_n}{\prod_{t=1}^{n}(1+r_t)}. \qquad (3)$$

The resulting expression (3) is correct from the point of view of financial mathematics. It can be used if the periodic cash payment from the subject of valuation arrives at the end of the period. An example would be the annual dividend accrual or the operation of a real estate facility with a final sale.

5. From Discrete Cash Flow to Discrete-Continuous One

In many cases of real estate appraisal, it is more logical to assume that the recurring income is evenly distributed over the period (for example, monthly rent throughout the year). Then, it is more correct to choose the middle of the period as the moment of payment (Pratt, 2002). Considering this remark, we obtain the other formula for the DCF model of the income approach in the ***case of cash flow payments in the middle of the period***:

$$V = \sum_{t=1}^{n} \frac{I_t}{(1+R_t)^{1/2}\prod_{j=0}^{t-1}(1+R_j)} + \frac{V_n}{\prod_{t=1}^{n}(1+r_t)}, \qquad (4)$$

provided $R_0 = 0$.

Continuing the reasoning in this direction, it should be noted that in some cases, for example, in the field of retail business, the cash flow can be both uneven and quite frequent. It is possible to consider daily receipts (and expenses). Then, a more accurate technique would be the replacement of a possible discrete model with a small discrete period (per week or day) described above with a continuous model known from financial mathematics (*e.g.* (McCutcheon and Scott, 1986) of the cash flow with density $\rho(t)$ on some time interval [0, n]. In this ***case of continuous cash flow***, the expression for calculating the DCF looks like this:

$$V = \int_0^n \rho(t)v(t)dt + V_n / \prod_{t=1}^{n} (1+r_t), \qquad (5)$$

where $v(t)$ is discount function. Assuming the discount rate is constant, the force of interest $\delta(t)=\delta=const$, and

$$v(t) = e^{-\delta t}.$$

The last term in expression (5) describes discrete terminal values that differ sharply from the values of the cash flow density. Therefore, it is impractical to embed them under the integral. Note that for convenience of calculation, the cash flow density $\rho(t)$ and the discount function $v(t)$ can be approximated piecewise linearly.

In the general case, the nature of the cash flow should be viewed as dual. On the one hand, it contains expressed periodic payments; on the other, periods with frequent receipts (and expenses), like those described in the previous formula (5). It is convenient to call such a pattern of a cash flow discrete-continuous.

The discrete component, as earlier in (3), is a cash flow specified by a sequence of payment moments (1, 2, ..., n, n), values of payments ($I_1, I_2, ..., I_n, V_n$) made at these moments, as well as the discount rates for periodic payments R_t and the discount rates for the terminal value r_t on the periods (t – 1, t) for $t \in (1, 2,..., n)$.

When describing the continuous component, in addition to the expression (5) given above, it can be noted that, in the general case, in the time interval of the calculation [0, n], there may be periods when there are no continuous payments, that is, in which $\rho(t) = 0$. Therefore, it makes sense to consider reducing a continuous cash flow only on the set T composed of time intervals with a nonzero density of payments. Of course, all these time intervals are on the time interval of the calculation, $T \in [0, n]$.

Note that different cash flow patterns reflect risk levels and require different discount rates (International Valuation Standards Council, 2019).

Considering the above, to describe the discounted value in the ***case of a discrete-continuous cash flow with a terminal value***, it is possible to combine expressions for discrete and continuous cash flows since they are independent. We get the following formula:

n t n

$$V = \sum_{t=1} I_t / \prod_{j=1} (1+R_j) + \int_T \rho(t) v(t) dt + V_n / \prod_{t=1} (1+r_t), \qquad (6)$$

6. Conclusions

Despite the external cumbersomeness, formulas (3)-(6) are easy to program using the financial function PV in the Microsoft Excel package.

When calculating the value of an income property using the DCF method, the use of an inaccurate formula (1) or an incorrect formula (*) can introduce significant errors compared to calculations based on formulas (3)-(6), the choice of which is determined by the pattern of the cash flow.

Note that similar expressions should also be used when calculating the NPV indicator for an investment project with the corresponding cash flow pattern.

References

Adair, A., Downie, M., McGreal, S., and Vos, G. (Eds.). (1996). European Valuation Practice. Theory and technique. London a.o.: E&FN SPON.

Appraisal Institute. (2013). The Appraisal of Real Estate. 14th Edition. Chicago, Ill.

Ataguba, J.O. (2021). Synthesis of short-cut DCF appraisal and spreadsheet iteration of freehold rental growth rates across specific valuation epochs. *Real Estate Management and Valuation, 29* (2), 52–70.

Baum, A. and Crosby, N. (1995). Property Investment Appraisal. 2nd Edition. London: Routledge.

Damodaran, A. (2012). Investment Valuation: Tools and techniques for determining the value of any asset. 3rd Edition. John Wiley and Sons, Inc.

Fisher, I. (1907). The Rate of Interest: Its Nature, Determination and Relation to Economic Phenomena. New York: The Macmillan Company. Retrieved from https://socialsciences.mcmaster.ca/econ/ugcm/3ll3/fisher/RateofInterest.pdf

Fisher, I. (1930). The Theory of Interest Is determined by impatience to spend income and the opportunity to invest it. New York: Macmillan. Retrieved from http://www.econlib.org/Library/ YPDBooks/Fisher/fshToI.html

Fuhrer, M. (1944). Our old friend Hoskold. *The Appraisal Journal, Jan.,* 50–51.

Gribovsky, S.V. (2016). Income Property Valuation. Moscow: Pro-Appraiser. (In Russian).

International Valuation Standards Council. (2019). International Valuation Standards. Effective 31 January 2020. Norwich.

McCutcheon, J. and Scott W. (1986). An Introduction to the Mathematics of Finance. Oxford: Heineman.

Michaletz, V. and Artemenkov, A. (2019). The transactional asset pricing approach: Its general framework and applications for property markets. *Journal of Property Investment & Finance, 37(*3), 255–288.

Pratt, S. (2002). Cost of Capital: Estimation and applications. 2nd Edition. Hoboken, New Jersey: John Wiley and Sons, Inc.

Sayce, S., Smith, J., Cooper, R. and Venmore-Rowland, P. (2006). Real Estate Appraisal. From value to worth, Blackwell Publishing Ltd.

Sharp, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance, 19* (3), 425–442.

Solomon, E. (1956). The arithmetic of capital budgeting decisions. *The Journal of Business, April (29),* 124–129.

Trifonov, N. (2010). Modern condition: market value or user value? In 23-26.06.2010. 17th Annual ERES Conference. Book of Abstracts and Programme, 218. Milano, SDA Bocconi.

Trifonov, N. (2019). The exact formula of the discounted cash flow method in an income approach. *Voprosy Otsenki, (3),* 50-52 (in Russian).

Trifonov, N. (2021). Income approach for real estate valuation. In European Real Estate Society conference. Technical University Kaiserslautern, Germany.