

# MOTION FORECASTING OF HIGH-SPEED TRACKED VEHICLES ACCORDING TO THEIR DYNAMIC PROPERTIES

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A high-speed tracked vehicle mobility technique is offered considering curvilinear motion according to their dynamic properties – their phase frequency behaviour. The ways of increasing efficiency of implementing potential high-speed qualities of the vehicle.

While developing and modernizing designs of transport vehicles one of the important tasks is prediction of their high speed ability estimated in the average speed. While driving in a road mode along specific roads the prediction techniques based on studying steady-state processes are used, and the average speed is determined as a random variable on the basis of the velocity distribution function along the way considering limitation on towing performance, on drift control along curvilinear road sections, on ride control parameters along rough road sections.

These methods are developed and provide quite accurate results for relatively low-speed vehicles and for driving along deformable roads. For high-speed vehicles, mobility on the roads with slightly deformable covering is limited to a large extent by controllability. This feature characterizes all the dynamic aspects of the system “Human being – Vehicle – Environment” and is estimated by dynamic, kinematic and traction characteristics. The travel speed on the route with intensive trajectory curvature change is limited by a phase lag response leading to the fact that the longitudinal axis of the frame fails to turn in relation to a tangent to the required trajectory. For sizing the vehicle into required curves it is necessary to reduce the speed. In the present paper the dynamic characteristics are defined by the tracked vehicle response to a harmonic control input under predetermined initial conditions, i.e. by the phase frequency characteristics.

The phase frequency characteristic is determined by the differential equation of the torque movement (1) and by hydraulic actuator dynamics in the following form:

$$\omega_{\phi} = \frac{2r_{BK}\omega_{\partial}K_{\omega}\alpha_{um}(S)C_M}{K_R B(1+k)U_{BP}U_{MC}(a_1S^2 + a_2S + C_M)}, \quad (1)$$

where  $r_{BK}$  – drive sprocket radius;

$K_{\omega}$  – amplification factor according to angular speed;

$\alpha_{um}(S)$  – steer angle in Laplace transform  $S$ ;

$C_M$  – modified drive shaft stiffness and ascending or upper track run stiffness;

$K_R$  – factor considering caterpillar drive skidding;

$B$  – track gauge;

$k$  – kinematic parameter of summing planetary gear set;

$U_{BP}, U_{MC}$  – gear ratio of the track gear-box and gear ratio of the hydraulic motor up to summing differential gear;

$a_1$  – reduced inertia moment (with allowance for track drive time response);

$a_2$  – moment of resistance to turning:  $a_2 = \frac{\omega_{\phi}}{V_X} \sum_{i=1}^n C_{Yi}(l_i - \chi)l_i$ .

$V_X$  – line speed;

$C_{Yi}$  – directional trouble coefficient of the  $i$ -th pair (axles) of track wheels ( $i=1 \dots n$ ),  $n$  – number of pairs of track wheels;  $l_i$  – axial coordinate of the  $i$ -th bearing axle in reference to the axle going

through the center of mass;  $\chi$  – axial displacement of the turning center  $\chi = \frac{vL\omega_{\phi}}{2\mu g}$ .

$L$  – length of the ground contact area;  $\mu$  – coefficient of resistance to turning;  $g=9,81 \text{ m/c}^2$ .

The subsystem, describing the driver's actions on controlling a translation speed and route trajectory (direction) and as well as the driver being a feedback link, is included in the model of controlled movement of the vehicle.

The model analysis shows that a vehicle response corresponds to the linear system supplemented with a pure time delay link.

Usually in estimating intensity of driving activity a look-ahead time, during which accuracy of the vehicle trajectory is provided, is considered as the basic indicator. With reference to the process under consideration the time, corresponding to a required look ahead time for a control input in relation to angular speed modulation decreases with a speed increase.

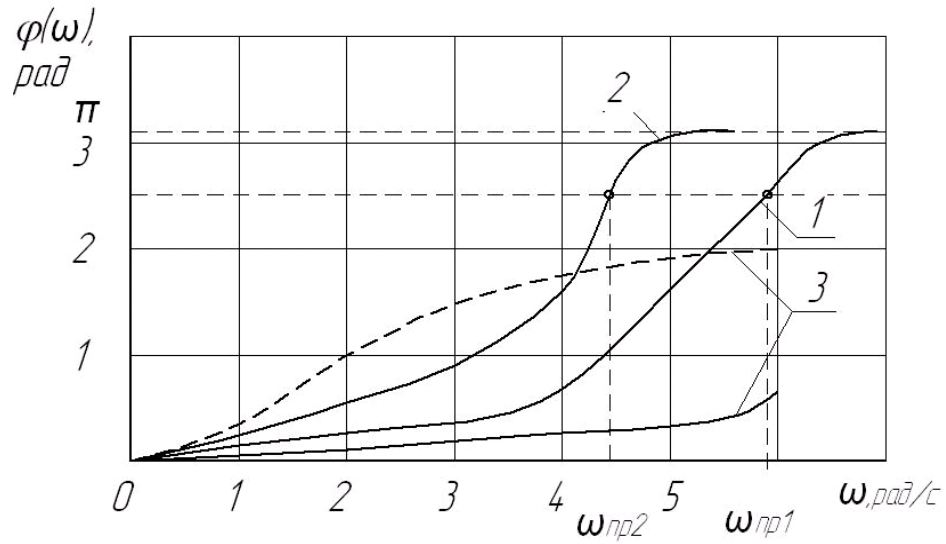
If fast response is considered as an indicator of route control complexity, then it should be recognized that the higher is speed the easier is driving the vehicle along the assigned snakelike trajectory with a constant wavelength. Such a conclusion runs contrary to experience since the time of going through one cycle by a vehicle decreases with growth of its speed.

Researches show that it is difficult for a person, being a loop element of the driving process, to compensate phase lagging  $\psi_\phi$  of an output signal. In the steering drive there is no lead network which could provide an angular velocity change, an azimuth angle (a driving direction) with a required look ahead time; this function is assigned to the driver. Therefore, the more phase shift is, the more rigid requirements of the dynamic system "Vehicle –Environment" to the driver and actuating mechanisms of swing control are. At high values of phase lagging the driver can fail to negotiate the change in direction without speed decrease. In this context the coefficient of phase intensity of controlling travel direction, determined by the relation of a response phase to  $\pi$  number, is accepted as a criterion of driving

complexity as follows:  $k_\phi(V) = \frac{\Psi_\phi}{\pi}$ . The phase frequency characteristic of the system is determined according to the differential equation of motion (1):

$$\varphi_M(\omega) = \left\{ \begin{array}{l} -k\pi - \operatorname{arctg} \frac{\left( V_x^{-1} J_Z^{-1} \sum_{i=1}^n C y_i l_i^2 + C_{MEX}^{-1} \nabla_M C_{np} \right) \omega}{-\omega^2 + \frac{C_{np}}{J_Z} \left( 1 + \frac{\sum_{i=1}^n C y_i l_i^2 \nabla_M}{V_x C_{MEX}} \right)} \end{array} \right. , \text{wherein } k=0,1,2,3,\dots \quad (2)$$

For the tracked vehicle with parameters corresponding to the vehicle of 14 tons in weight the phase frequency characteristic is shown in Figure1.



**Figure 1: Phase Frequency Characteristic of the Vehicle**

As a result of this research, considering motion as a continuous Markov process, the recurrence of engaging the steering gear system with continuous properties is determined as a positive number of overshoots of stationary random function of the trajectory curvature of a "zero" level while driving on the route, which curvature is set by a determine or stochastic function.

The correlation function of the trajectory curvature  $k$  is approximated with exponentially cosinusoidal dependence  $R_k(\tau) = \sigma_k^2 \exp(-\alpha \cdot |\tau|) \cos \beta \tau$  or with twice differentiable function

$R_k(\tau) = \sigma_k^2 \exp(-\alpha |\tau|) (\cos \beta \tau + \frac{\alpha}{\beta} \sin \beta \cdot |\tau|)$ , wherein  $\alpha$  and  $\beta$  are approximation parameters.

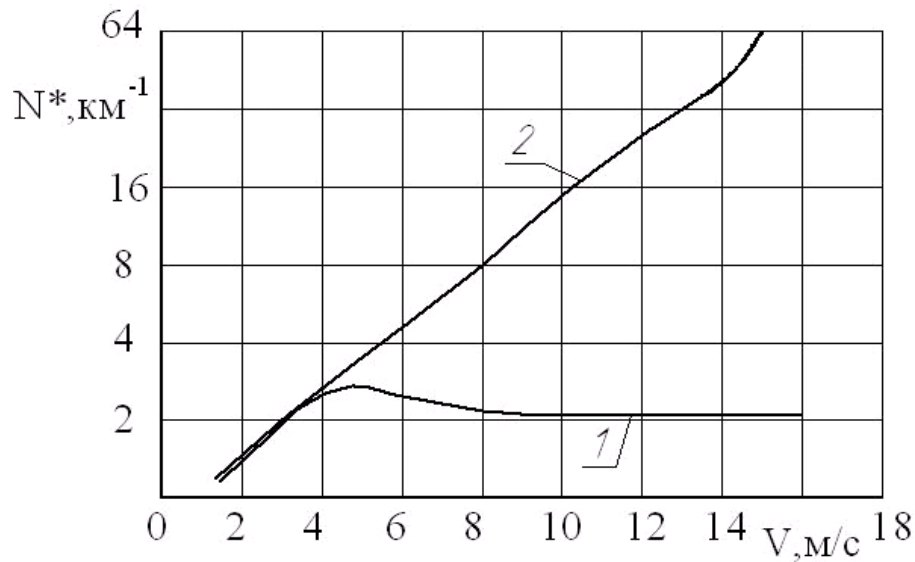
The latter allows engaging the set of Markov processes for studying the system dynamics. In compliance with direct Fourier transformation it is possible to determine the spectral density  $\Phi(\omega)$ , in the form of the fractional rational function of frequency. For the vehicles with pump-controlled hydraulic transmission the average number of engagements makes [1]:

$$N^* = (2\alpha_k^2 + \beta_k^2)^{0,5} / 4\pi\Phi_0(k_\phi / \sigma_k) \quad (3)$$

According to the modified equations it seems possible to predict velocity assuming the following:

- Conformity of the curvature of a route trajectory, at this, the number of engagements of the steering gear is determined by intensity of the change of direction;
- driving is fully controlled, i.e. the longitudinal axis of the vehicle coincides with a tangent to a certain required trajectory;
- the driver sets preemptive control, though accomplishment of his functions as a feedback link on compensation of a deflection in the trajectory isn't considered.

According to the executed studies it is established that recurrence of engagements of the steering gear exceeds essentially the estimated values (fig. 2) and at the maximum speed reaches 40 per a kilometer at a speed of 60 ... 65 km/h.



**Figure 2: Dependence of the recurrence of engagement of the steering gear with continuous properties on speed when changing a route direction accidentally:**

- 1 – predicted number of engagements of SG (steering gear) according to the existing techniques;
- 2 – predicted number of engagements of SG considering the required accuracy of a route direction.

In many respects it is connected with the accepted assumptions. The parameters of the actual driving process differ from the estimated. The recurrence of a steering gear engagement depends not only on stochastic properties of a route curvature but also on the required accuracy of a trajectory, therefore, the number of driver inputs and their frequency differs essentially from the number of road curves.

The driver carries out not only feedforward adjustment of the travel direction but also a feedback function on compensation of deviations, providing sizing to the limited corridor in width  $H$ . The deflection in the trajectory from the estimated, which arises while driving, due to insufficient controllability, requires limiting the speed. The frequency of the process and maximum permissible speed are estimated on the basis of studying the dynamics of the system “Tracked Vehicle – Driver – Environment” in the course of direction control. While controlling the vehicle the driver considers various aspects of the environment without knowing the character of their interactions. He is guided by such parameters which can be estimated quantitatively and which have a dominating value. The key parameters of the information about the current condition of the dynamic system are an azimuth angle or a direction angle, its lowest derivatives, engine operating condition, speed, sideways position of the vehicle with respect to external reference points and etc. In a number of works it is shown that in assignments on driving vehicles the controlled coordinates are a direction angle and sideways position of the vehicle. While analyzing control of a route direction it is considered that the driver, on the basis of his driving experience, optimizes the process of a curvilinear movement in some way. The optimization criterion is the requirement of maintaining the highest possible speed assuming a limited number of engagements of the steering gear. The following driving conditions satisfy to these requirements: straight-line motion within the width of the roadway reserve on the greatest possible length of the route and turning about some  $\beta$  angle.

Due to the sideways shift it is necessary to decrease an actual width of the roadway reserve by the value

$$\Delta Y = \int_0^t V \cdot \theta(V, t) dt.$$

Besides, due to skidding and track slipping an actual curve differs from the estimated:

$k_\phi = k_p \cdot B/L$ . With regard to detogeneity of vectoring the route trajectory parameters limited by operation speed of the system, the actual steering angle is much less than the estimated  $\beta(k_\phi, \alpha_{um})$  and makes:

$$\beta_\phi = \beta(k_\phi, \alpha_{um}) - \theta - \Delta\beta_3 - \Delta\theta. \quad (4)$$

The calculation of deviation components is performed according to the dependences shown in Table 1.

**Table 1: Dependences for calculating deviation components of vectoring the route trajectory parameters**

Parameter	Designation	Calculating formula
1. Steering angle	$\beta(k_\delta, \alpha_{um})$	Is set by the driver on the basis of the traffic situation and prediction of a route trajectory
2. Lateral angle value: - determinate  - stochastic lateral angle component	$\theta$  $\Delta\theta$	$\theta_c = \omega_\phi \left( \frac{mV - V^{-1} \sum_{l,n} C_{Yl} l_i}{\sum C_{Yi}} \right) + \frac{m}{\sum C_{Yi}} \frac{dV_Y}{dt}$  $\Delta\theta = \pm 3\sigma_{\Delta\theta}$  $\sigma^2_{\Delta\theta} = \frac{1}{\pi\omega^2} \int_0^\infty S_\omega d\omega$
3. Trajectory deviation rate compensating system lag	$\Delta\beta_3$	$\Delta\beta_3 = k_\phi \int \dot{V}(t - \sum t_i) dt$

The extreme geometrical position of the tracked vehicle according to the condition of sizing to the limited corridor is determined by the fact that the point maximum possibly removed from the mass center (its angular coordinate in respect of the longitudinal axis makes  $\theta_\Gamma$ ), would not fall outside the limits of the road width  $\theta \leq [\theta]$ :

$$[\theta(v, \mu, H)] = \arccos(k(t) \cdot \left( \frac{B}{2 \sin(\theta_\Gamma)} - H \right)) \pm \theta_\Gamma \quad \theta_\Gamma = \arctg(B + b_\Gamma) / (L_1 \pm \chi) \quad (5)$$

wherein  $L_1$  is overall length.

An additional number of engagements of the steering gear, providing for the required accuracy of the trajectory according to the conditions of sizing to the curve, is possible to specify as an expected value of the stochastic function of the specific number of turns  $N_{y\delta}(k_\delta)$ :

$$N^*_\delta(k_\phi) = m_s [N_\delta(k_\delta)] = \int_0^{k_\phi} \frac{k_\delta}{\beta_\delta} \varphi_s(k_\delta) dk_\delta \quad (6)$$

wherein  $\varphi_s(k_\delta)$  is probability density of the road curve module  $k_\delta = |k_\delta|$  ( $0 \leq k_\delta \leq k_\phi$ ), determined by spectral intensity or correlation function of a road curve  $k_\delta = |k_\delta|$ ;  $\beta_\delta$  is an actual steering angle, its definition is given above.

The expected speed value may be estimated according to the equation:

$$V = \begin{cases} \frac{\omega_{IP} \Delta S}{\pi} & \text{– if it is the determinate function of the curve,} \\ \frac{\omega_{IP}}{2\pi(N^* + N^*_\delta)} & \text{– if it is the stochastic function of the curve} \end{cases} \quad (7)$$

wherein  $\omega_{IP}$  is frequency value corresponding to the accepted value of the phase intensity coefficient  $k_\phi(V) = 0,75$ .

Correctness of assumptions while determining the recurrence of engagements of the steering gear and adequacy of the mathematical model to the real process are observed.

The object of this experimental research is the high-speed tracked vehicle of 14 tons in weight with the increased power-to-weight ratio up to 24.2 kW/tons. The installed capacity of pump-controlled hydraulic transmission of the differential steering gear makes 208 kW, providing vehicle turning with angular speed up to 1.1radian/sec. In the course of this experimental studies the following parameters were registered: steering angle and fuel distribution pedal movement, angular speed, azimuth angle, engine shaft angular speed, linear acceleration of front and rear section of the vehicle. Testing was conducted according to the requirements of the industry regulatory documents. The parameter measurement of the dynamics of the controlled course of movement is performed while testing the vehicle for snakelike maneuvering. Statistical manipulation of the experimental data, their comparison with the results of calculations allows drawing a conclusion that the physical process of the controlled course of vehicle movement is described adequately by the mathematical model. However, a real process is more complicated. An essential vehicle response lag to driving input during transient processes of going into and out of a turn is experimentally confirmed. On the basis of the statistical manipulation of the results dependence of the phase intensity coefficient on speed is plotted  $K_\varphi(V)$  with fixed values of wavelength  $\Delta S$ . The observed results about response time of the steering gear system are shown in Table 2.

**Table 2: The results of estimating response time of the steering gear system of the tracked vehicle and duration time of transient processes**

Driving conditions		Duration time, sec	
Number of transmission	Speed, m/sec	of the driving system response	of transient processes
1	2.3...4.1	0.14...0.26	0.6...1.2
2	4.0...6.7	0.20...0.24	0.5...0.6
3	6.0...11.5	0.20...0.26	0.32...0.43
4	10.6...17.1	0.18...0.22	0.30...0.35

Response lag duration time  $t_{3\omega}$ , i.e. response time of SGS [Steering gear system] does not depend on speed, and the duration time of a transient process  $t_{1\omega}$  with growth of speed decreases, i.e. with speed growing it is easier to control the route trajectory, which conflicts with the experimental evidence. In this connection, the coefficient of phase intensity of route trajectory control  $K_\varphi(V) = \frac{\Psi_\varphi}{\pi}$  is reasonably accepted as the criterion of driving complexity.

From the observed results it follows that on roads with intensive change of trajectory curvature, on test "snakelike routes", implementation of potential high-speed qualities is limited to the condition of sizing to the required corridor. This deflection in the trajectory from the assigned one results from phase lag of response to a driving input. In order to drive without speed decrease it is necessary for a driver to create a relative feed forward driving input. Therefore, the speed of the first attempt of driving along the snake path (without the driver's training) makes 30...40 % from the average speed, which is determined after 4 – 6 test drives.

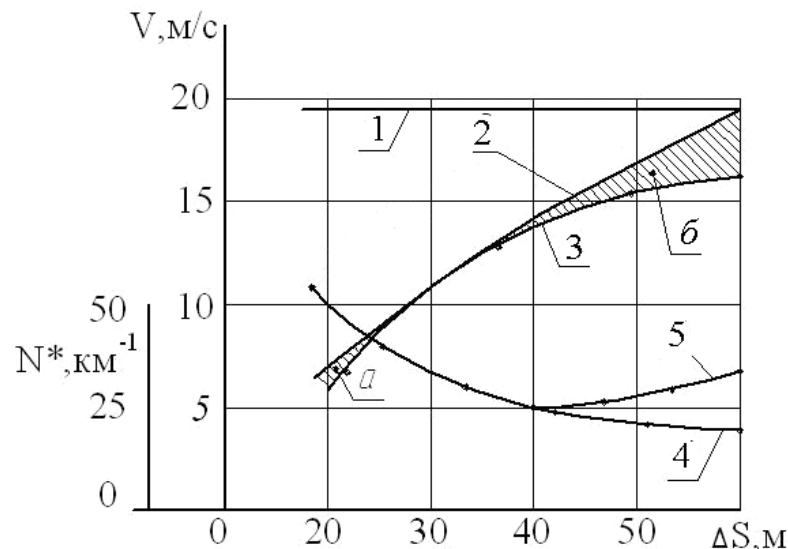
Top speed corresponds to the value  $K_\varphi(V)=0.75$ , at which a driver can technically control the route trajectory. Based on these results dependence of top speed on wavelength  $\Delta S$  of a test snakelike route is estimated. From comparison of this dependence on  $\Delta S$  with the observed data of average speed of driving over concrete and over soil road it follows that vehicle mobility is quite accurately prognosticated according to the phase intensity coefficient.

Experimental studies proves a theoretical conclusion about the necessity of evaluating controllability according to the vehicle response to steering considering a fuel distribution pedal position. It is connected with the fact that a certain angular steering speed, depending on a torque rate and hydraulic ratio, may correspond to a certain position of the pump-controlled hydraulic transmission drive.

On the basis of comparison of theoretical and experimentally observed results the conclusion is made about the necessity of adjustment of the mathematical model by the way of introducing a pure time delay link.

One of the important parameters determining vehicle mobility is recurrence of engagement of the steering gear system. From the observed results it is evident that at speed  $V \leq 5$  km/sec the actual number of engagements corresponds to the estimated. At high scale of speed values the number of engagements increases essentially and at  $V=20$  km/sec makes 42 per a kilometer (0.8 Hz), and is limited to psycho-physiological qualities of a driver as a link of the feedback system. The essential difference in comparison with the known results is caused by the fact that parameters of the actual driving process differ from the estimated. Recurrence of engagement of the steering gear depends not only on probability properties of a road curvature but also on the required accuracy of a route trajectory, therefore, the number of driving inputs, their frequency differs essentially from the number of road curves. While calculating recurrence considering provision for the required trajectory accuracy, the estimated value of the function of engagement recurrence of speed coincides with the experimental data.

On the basis of the studies undertaken the following sequence of predicting tracked vehicle mobility when in motion along curvilinear trajectory is offered. The initial data are: the geometrical and elastic-inertial parameters of the vehicle determining the phase frequency characteristic, power-to-weight ratio, traction performance and steering shift system; spectral density of road and soil traffic conditions: road curvature  $S_k(\omega)$  and its width  $S_H(\omega)$ , as well as resistance-to-turn coefficient  $S_\mu(\omega)$ . On the basis of inverse Fourier transform the parameters of road and soil traffic conditions are estimated  $k(s), H(s), \mu(s)$  in the function of distance.



**Figure. 3: Analysis of the observed data (test snakelike route)**

- 1 – kinematically virtual velocity by the engine  $V$ ;
- 2 – top speed limit according to the phase frequency response curve SGS;
- 3 – observed results  $V$ ; 4 – estimated value of recurrence of engagements of SGS  $N^*$ ;
- 5 – observed value of recurrence of engagements of SGS  $N=N^*+N^*$ , providing trajectory accuracy.
- a – speed limit zone according to power-to-weight ratio and SGS;
- 6 - limit zone according to the dynamic properties of SGS.

Further on we determine parameters of nonlinearity of a driving system, dynamic properties of the tracked vehicle and the driver, of phase intensity coefficient  $K_\phi(\omega)$ , components of steering angle  $\beta(k_D, \alpha_{III T})$  according to the equation (4) and Table 1, recurrence of steering gear engagement (3, 6) and frequency of the process of trajectory control  $\omega_{II P}$ . Besides, restrictions are introduced: sizing-to-fit conditions (5), psychological properties of the driver carrying out a function of a feedback link, extra duty

PHT (Pump-controlled hydraulic transmission), track skidding of the slower running tracks, according to the traction performance and sidewise skidding. On the results of test snake driving the diagrams of kinematic parameters change while driving along a test snakelike route (Fig. 3) and along a route with a random curvature trajectory change are created. On the basis of these data and simulation modeling of vehicular movement estimation of speed as the least possible in compliance with the above-stated restrictions is made.

Implementation of such an approach allows not only predicting high speed of the vehicle according to its dynamic properties but also solving an inverse problem of increasing high-speed properties by differentiated reduction of certain restrictions.

### **Conclusions**

1. On the stage of development and modernization it is reasonable to perform appraisal of the vehicle mobility along a curvilinear trajectory according to the dynamic properties the phase frequency characteristics determined on the basis of the mathematical driving model complemented with new experimental data.
2. The estimated functions of the number of steering gear engagements depending on probability properties of a road curvature and required trajectory accuracy according to sizing to fit the conditions allow predicting high-speed vehicle qualities more precisely.
3. Implementation of the newly developed method allows not only predicting vehicle high speed characteristics according to its dynamic properties more precisely but also solving an inverse problem of increasing high-speed properties by way of differentiated reduction of certain restrictions.

### **References**

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