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INSPECTION ROBOTS WITH PIEZO ACTUATORS: MODELING, SIMULATION AND PROTOTYPES

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Models, simulations and experimental setups of resonant inspection robots are presented. The goal is to show ways to cope with the new requirements and to use the given chances to create novel mobile robots. For the creation of a directed motion the vibration behavior of simple beams and plates is used. It is possible to design robots for 2-dimesnional locomotion which are characterized by a light weight, small size, relative simple design and the ability to create controllable motion using only one actuator. Different types of actuators for micro robots are presented and compared. Furthermore the dynamical behavior of a piezoelectric bending actuator under elastic boundary conditions is investigated and a model for the motion of the locomotion-generating limbs is presented. The comparison with experiments and prototypes shows that the results of the analytical and computational models agree (E-mail : Klaus.Zimmermann@tu_ilmenau.de)

Key Words: resonant robots, piezo-driven systems, vibrations of continua.

Introduction

Inspection mobile robots are dominated by rigid multibody systems with constant mass distribution. The bodies are coupled by kinematic pairs. The modification of relative positions is provided mostly by actuators in the joints. Systems with legs and wheels to perform the locomotion are well known and used in many fields of application.

Actual developments, especially in medicine, biology and inspection technology, require new mobile robots, which are characterized on the one hand by relative small size and weight and one the other hand by small costs for production and application. In this context established robots are not optimal for the new challenges. Joints and typical actuators cannot be minimized arbitrary because of the scaling effects of the used physical principles. Also the costs for manufacturing and assembling would increase disproportionally. To provide multi-dimensional movement several actuators are needed, at least one for every degree of freedom. The energy consumption and effort for controlling for the established systems are relative high.

The objective of this scientific project is to break through these disadvantages and to develop systems which are customized for the new requirements on mobile systems. Using the theory of forced vibration of continua, especially bending vibration of beams and plates, it is possible to create systems which are small, light and cheap with a relative simple design. Following the principle «intelligence in the mechanics» [2] robots are in the design process (figure 1), which are controllable by only one actuator to perform motion in three degrees of freedom.

In the following figures four mobile robots for 2-dimensional locomotion on a flat surface are presented.

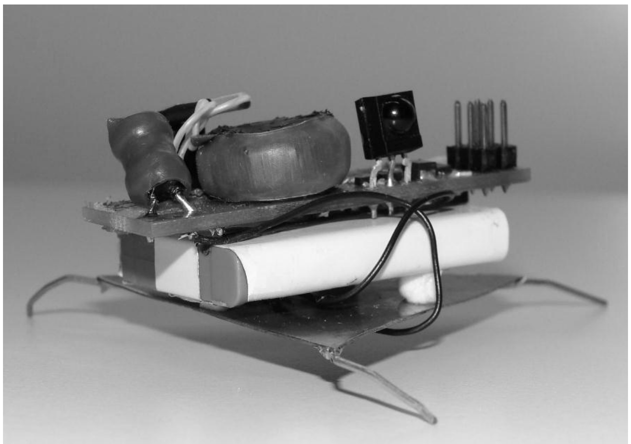


Figure 1 – Beetle-Robot [1] – autonomous, programmable and remote controlled robot driven by a single piezo actuator using resonance characteristics of elastic continua

Design issues

In choosing the right actuator the most critical issue to consider is the speed to excite the resonance characteristics of small mechanical devices. Ideally would be a light and compact actuator with a high speed, low power consumption, large output force and strain. A high potential for the use in small micro robots have actuators made form piezoelectric materials, shape memory alloy (SMA), ionic polymer-metal composite (IPMC) and DC motors [4]. The main disadvantages of SMA and IPMC are the slow speed, the small force and the relative high power consumption. By consuming a large amount of energy DC motors have the possibility to create a very large displacement with a high speed and a moderate force. A problem is the heavy weight and the limits in miniaturization because of the scaling of the electromagnetic force in relation to the friction force in the sliding bearings. Piezo actuators are characterized by light weight, high speed, large force and high energy efficiency. It is possible to convert more than 90 % of the electrical energy into mechanical energy. The main disadvantage is the small strain and the temperature behavior in the dynamic mode.

For a small weight of a robot the piezo element can be used as actuator and base body. This vibratory drive creates the needed ultrasonic excitement. Assuming that the average value of the excitation vibrations equals zero over one period of time an asymmetry in the systems characteristics is needed to create a directed locomotion. The character of the system in the first half period of the vibration needs to be another than in the second. For this the vibration behavior of simple mechanical structures like beams or plates can be used.

In the exemplary robot presented in figure 1 bended metal wires are used as vibration transducers which are excited by circular piezo unimorph drives.

Modeling, simulation and experiments connected to the actuator dynamics

Analytical description of the actuator dynamics

The actuation system is modeled as a thin elastic plate, which can be described with the help of the hypothesis of plates and laminates. A characteristic parameter of a plate is the bending stiffness N. For the introduction of this parameter and the use of the Kirchhoff hypothesis a neutral area is needed. In a homogenous plate this strain- and stress-free area is situated exactly in the middle. The position changes in a laminated plate. This is illustrated in figure 2, where $\vec{e}_z$ and $\vec{e}_r$ are the unit vectors, E_1 and E_2 the Young's modulus of the materials, h_1 and h_2 the thickness of the plates and h_n the distance between the adherend and the neutral area which can be calculated using equations (1) and (2). It is assumed that there is no motion between the adherend surfaces. Furthermore the distributions of strain $\mathrm{e}(z)$ and stress $\mathrm{y}(z)$ are presented.

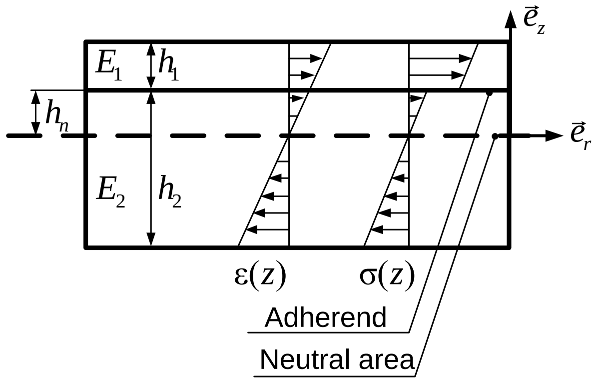


Figure 2 – The distribution of strain and stress in a laminate of two layers

In equation (3) and (4) the calculation of the bending stiffness of a homogenous plate as well as a laminate is shown, where υ is Poisson's ratio [3]:

$$h_n = \frac{h_1}{2} \cdot \frac{ea^2 - 1}{ea + 1}, \tag{1}$$

$$a = \frac{h_2}{h_1}, \; e = \frac{E_2}{E_1}, \tag{2}$$

$$N_{homogenuos} = \frac{Eh^3}{12(1 - v^2)}, \tag{3}$$

$$N_{Laminate} = \frac{E_1 h_1^3}{12(1 - v^2)} \cdot \frac{4ae(1 + a)^2 + (1 - ea^2)^2}{(1 + ae)} \cdot \tag{4}$$

The equations for the bending vibration of a circular plate and the general solution are written in (5) and (6), where ρ is the density, ω the natural angular frequency, λ the eigenvalue, J and I the Bessel function and the modified Bessel function of first kind. The problem is considered to be rotational symmetric so that the function, which is describing the bending of the plate w, depends only on the radius *r* and the time *t*.

$$\Delta\Delta w(r,t) + \frac{\text{с}h}{N}\partial_{tt}^{2} w(r,t) = 0, \tag{5}$$

$$w(r,t) = [c_1 \cos(\text{ш}t) + c_2 \sin(\text{ш}t)] \cdot [c_3 J_0(\text{л}r) + c_4 J_0(\text{л}r)]. \tag{6}$$

The influence of the elastic robot legs to the actuator plate is modeled as linear springs (figure 3). We assume that the stiffness of the plate along the circumference is relatively high so that the springs can be modeled as evenly distributed over the circumference of the plate (figure 4).

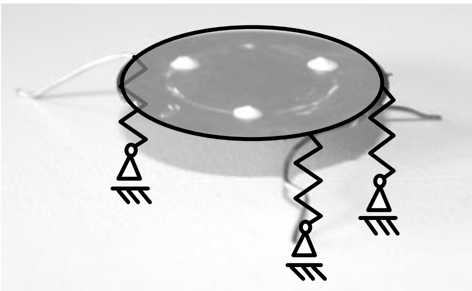


Figure 3 – Boundary conditions of robots' actuator

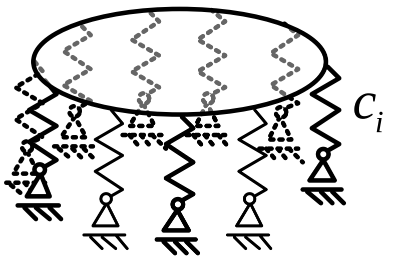


Figure 4 – Boundary conditions of the analytical model

The boundary conditions are presented in (7), where Q_r is the shear force, M_r the bending moment and $\bar{c}$ a characteristic stiffness, which can be calculated using (8).

$$\begin{aligned} 0 &= Q\text{г}(R,t) + \bar{c}w(R,t), \\ 0 &= M\text{г}(R,t), \end{aligned} \tag{7}$$

$$\bar{c} = \frac{\sum_i c_i}{2\text{p}R}. \tag{8}$$

The characteristic equation, to calculate the eigenvalues and natural frequencies of a such plate, is formulated in (9) where F_0, F_1 and F_2 are functions of the material and geometrical properties, as well as of the modified Bessel function of the first kind and different orders. The parameter c could be calculated as $c = \bar{c}R\text{i}/N$ and is the relation between characteristic stiffness parameters. For different boundary conditions, on the circumference of the plate, i.e. fixed or flexible support, the eigenvalues could be found in the literature. They are used to verify the analytical model as well as the FEM model. Some results of the numerical analysis of equation (9) are presented in table 1.

$$0 = F_0 J_0(\text{л}R) + F_1 J_1(\text{л}R) + F_2 J_2(\text{л}R). \tag{9}$$

FEM modeling and simulation

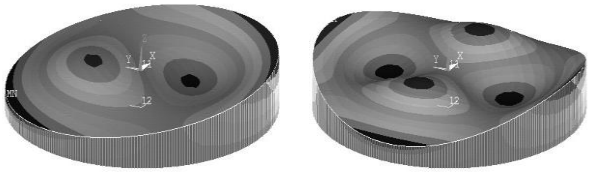
To analyze to vibration behavior of the actuator, under the described boundary conditions, a FEM model is formulated. The plate is modeled to be homogenous. With the help of a modal analysis, the natural frequencies and normal modes are simulated and compared with the results from the analytical calculations. It could be noticed that the results of both modeling methods agree. With the full rotationally symmetric mathematical modeling can be calculated only the natural frequencies of such normal modes. With FEM simulation also the non-symmetric modes are determined. Some examples are given in figures 5 and 6.

Table 1 – Natural frequencies for the rotationally symmetric normal modes of a plate under linear elastic boundary conditions for one set of parameters – analytical and FEM calculations

No.	Analytical [Hz]	FEM [Hz]
1	179	216
2	798	782
3	2062	2012
4	4495	4430
5	7957	7903
6	12415	12432

a *b*

Figure 5 – rotational symmetric normal modes: *a* – 3rd with 2012 Hz; *b* – 4th with 4430 Hz



a *b*

Figure 6 – normal modes: *a* – 9th with 1206 Hz; *b* – 15th with 1862 Hz

Experiment

The natural modes of a circular piezo unimorph actuator are investigated and presented in figure 7. In agreement with the analytical and computational models, it is possible to establish different normal modes. The boundary conditions of this plate are different than the presented models. Also the soldered dots have an influence to the vibration behavior.

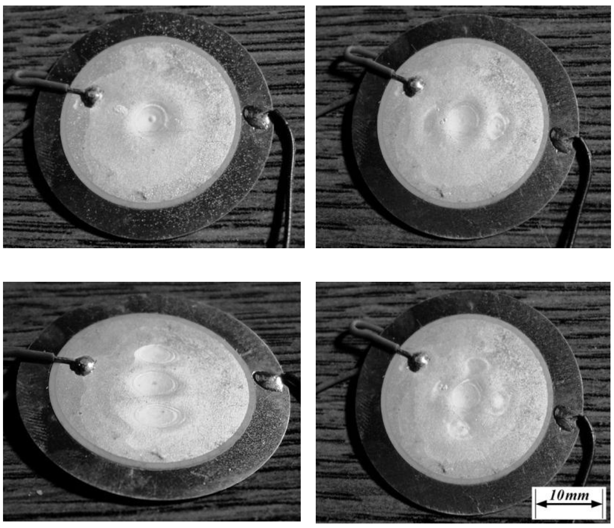


Figure 7 – Natural modes of a robots' actuator

Analytical modeling of the legs motion

The geometry of the robot determines the frequency spectrum of its oscillations. Resonant excitation of the robot body, which is realized by the described actuator, leads to a transfer of the vibration energy to the limbs. It allows the transforming of the periodic motion into a forward one, at a certain synchronization of their oscillations. The main role in the transformation of these motions plays the geometry of the legs, which consist of three links. The length of the links and the angles between the links affect significantly the nature of the transformation of the vibrations. To make first assertions about the movement of the endpoints of the legs, which are the contact points between the robot and the flat surface, an analytical model is presented.

The reference point of the robot leg B performs vibrations in a spatial coordinate system. The origin lies in point A which is connected to the body of the robot (figure 8).

Characteristic is the so called radius vector $\vec{R}(t)$ connecting the points A and B. Then its projections on the reference axes $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$ are $R_x = l_1$, $R_y = l_2$ and $R_z = -l_3$. The angels between and the unit vectors of the axes are $\varphi_1(t)$, $\varphi_2(t)$ and $\varphi_1(t)$. $\varphi_1(0)$, $\varphi_2(0)$ and $\varphi_3(0)$ define the geometry of the robot leg. The relations (10) and (11) are given.

$$\cos^2\varphi_1 + \cos^2\varphi_2 + \cos^2\varphi_3 = 1, \tag{10}$$

$$l_1^2 + l_2^2 + l_3^2 = R^2. \tag{11}$$

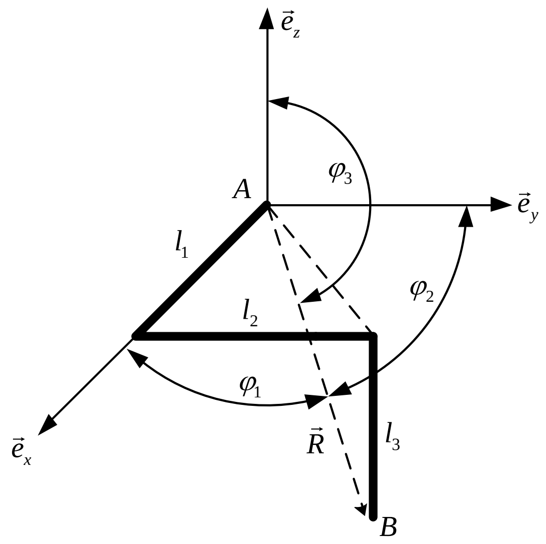


Figure 8 – Estimated model of the robot leg

Using a spherical coordinate system to describe the radius vector $\vec{R}(t)$ three variables can be used: the length $l(t)$ of the vector and the angles $\varphi(t)$ and $\Theta(t)$ between $\vec{R}(t)$ and the unit vectors $\vec{e}_x$ and $\vec{e}_z$ (figure 9).

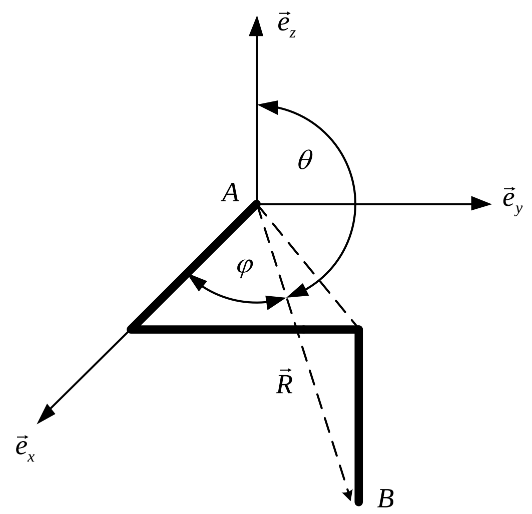


Figure 9 – Model of robot leg in a spherical coordinate system

In a first approximation it is considered that the motion of the reference point B occurs on a sphere of a constant radius $l(t) = l_0 = const$. The equations of motion for φ and Θ are presented in (12) and (13), where the mass m of the leg is modeled to be concentrated in B. The motion of the reference point is considered to be small. The linearized equations are (14) and (15).

$$\frac{d}{dt}(ml_0^2 \varphi \sin^2 \Theta) = 0, \qquad (12)$$

$$\ddot{\Theta} - \dot{\varphi}^2 \sin\Theta \cos\Theta + \frac{g}{l_0} \sin\Theta = 0, \qquad (13)$$

$$\dot{\varphi}\Theta\dot{\Theta} = 0, \qquad (14)$$

$$\ddot{\Theta} - \dot{\varphi}^2 \Theta \cos\Theta + \frac{g}{l_0}\Theta = 0. \qquad (15)$$

From equation (14) we get:

$$\dot{\varphi}\Theta = 0 \text{ or } \dot{\Theta} = 0. \qquad (16)$$

In the first case of (16) the equation (15) has the form of (17) with the solution (18), where g is the gravitational constant, v_0 the initial velocity, t the time and $\text{щ} = \sqrt{g/l_0}$ the angular frequency of the vibration. In the case point B performs an harmonic vibration in the plane with $\varphi(t) = \varphi_0 = const$,

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$$\ddot{\Theta} + \frac{g}{l_0}\Theta = 0, \qquad (17)$$

$$\Theta(t) = \Theta_0 \cos \text{щ}t + \frac{v_0}{\text{щ}} \sin \text{щ}t = 0. \qquad (18)$$

In the second case ($\dot{\Theta} = 0$) equation (15) has the form of (19) with the solution (20). B performs a circular movement in the xy plane with $\Theta(t) = \Theta_0 = const$.

$$\dot{\varphi}^2 = \frac{g}{l_0}, \qquad (19)$$

$$\varphi(t) = \varphi \pm \sqrt{\frac{g}{l_0}}\, t\ . \qquad (20)$$

In general, the motion of the reference point B is described by three coordinates $l(t)$, $\varphi(t)$ and $\Theta(t)$ with $|\vec{R}(t)| = l(t)$. In that case the equations for φ and Θ has the form (21) and (22), where α and β are arbitrary dimensionless constants of the legs' mass geometry. These are the well-known equations of motion of s spherical pendulum.

$$\dot{\varphi} = \frac{ml_0^2 \text{щ}_0 \text{б}}{ml^2(t)\sin^2\Theta}, \qquad (21)$$

$$\begin{aligned} &\frac{1}{2} ml^2(t)\dot{\Theta}^2 + mgl(t)\cos\Theta + \\ &+ \frac{1}{2}\frac{ml_0^4 \text{щ}_0{}^2 \text{б}^2}{l^2(t)\sin^2\Theta} = \frac{1}{2} ml^2 \text{щ}_0{}^2 \text{в}. \end{aligned} \qquad (22)$$

Regarding equations (21) and (22) three cases can be considered.

In the first case when $\alpha = 0$ and $\varphi(t) = \varphi_0 = const$ the plane motion of the pendulum is given, described by the laws $l(t)$ and $\Theta(t)$.

For the second case the constant α has a value $0 < \alpha^2 < f(\beta)$ $f(\beta)$ is presented in equation (23). Then $\Theta_2 \le \Theta \le \Theta_1$ and the motion occurs on a sphere with the variable radius $l(t)$ between the parallels z_2 and z_1 (figure 10). z_1 describes the lifting phase of robots' platform (the leg is in contact to the ground). z_2 gives the phase of lowering, where the leg loses the contact to the ground.

$$f(\text{в}) = \frac{1}{54}\left[(\text{в}^2 + \text{в})^{\frac{3}{2}} + 36\text{в} - \text{в}^3\right]. \qquad (23)$$

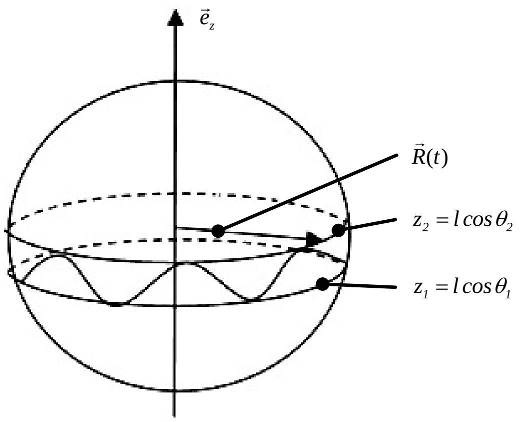

Figure 10 – Scheme of the motion of the robot platform in the second case

In the third case ($\alpha^2 = f(\beta)$) the radius vector $\vec{R}(t)$ moves on a conical surface and $\Theta(t) = \Theta_0 > \frac{p}{2}$. The reference point B moves in a circle of the radius $l\sin\Theta_0$ in the horizontal plane $z_0 = l\cos\Theta_0$ (figure 11).

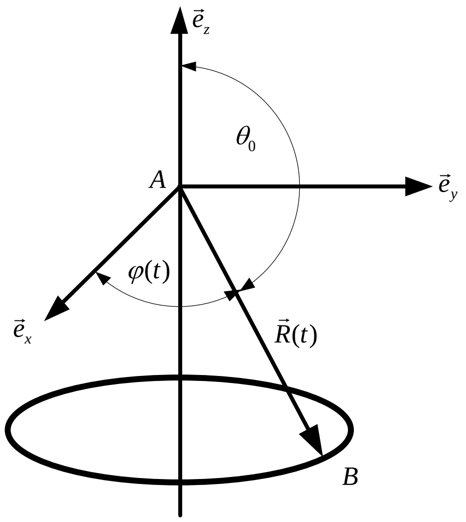

Figure 11 – Scheme of movement in the third case

Prototypes and Experiments

Follow the mentioned goals and using the described models for creating small mobile robots the two prototypes presented in Figure 1 are developed. Structural data are given in Table 2. The actuator is controlled through a sinusoidal electrical signal with amplitude of 20 V.

To analyze the motion of the contact point of the leg and the surface (reference point B) experiments using a scanning electron microscope (SEM) are made.

Table 2 – Data of prototypes

Plate-Robot	
Length × Width × Height	58 × 42 × 10 mm³
Mass	3,5 g
Max. velocity on glass	150 mm/s
Excitation frequency	10–60 kHz
Beetle-Robot	
Length × Width × Height	69 × 80 × 30 mm³
Mass	31,7 g
Max. velocity on glass	20 mm/s
Excitation frequency	12–70 kHz

As experimental setup Plate-Robot is used. In figure 12, the overlay of two photos is presented. The solid scheme represents the endpoint of a leg in a static state.

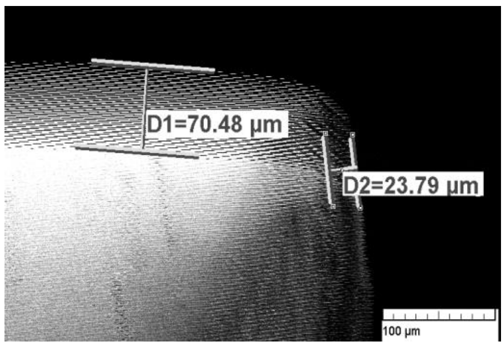

Figure 12 – SEM picture of a vibrating leg

The moveable part shows the systems under excitation. It should be noticed, that with the used microscope, it is possible to take one picture every three seconds. The vibration frequency of the leg is much higher, which means that the marked amplitudes do not represent necessarily the maximum va-lue. The leg is excited by the bending vibrations of the actuator plate (figure 5 to 7). According to the frequency different vibration forms are produced. The endpoint of the leg performs longitudinal and transversal vibrations. The trajectory corresponds to the relation between the amplitude of the longitudinal and transversal vibrations (figure 10). This behavior can be described using the third case $\alpha^2 = f(\beta)$ of chapter 4 (figure 11). The motion of the robot depends on this vibration behavior. During the movement the friction forces in the contact points between robot and environment are changed periodically.The motion direction could be controlled using the resonance shift between the legs. This resonance shift is caused by the asymmetric system properties. Furthermore the robot legs lose the contact to the surface during the vibration, described by the second case $0 < \alpha^2 < f(\beta)$ (figure 10). The motion is influenced by the resulting shock effects.

Conclution

Actuators for small mobile robots are compared. The dynamical behavior of a piezoelectric unimorph actuator was studied using analytical and computational methods as well as an experimental setup. An analytical model for the description of the motion of robot legs is presented and analyzed. Following this ideas experimental investigations and prototypes are shown.

Further investigations will be connected with robots legs. Including the boundary conditions, given by the contact properties between robot and environment, will be determined the resonance characteristics. The objective of the future work is to find the qualitative and quantitative relations between the excitation frequency and the locomotion properties. Models with a lower grad of abstraction are needed. Further micro robots, which are in the design process, will use the described principle of motion. The concentration on the motion of the mobile robots in the different environments will arise in the further work.

Acknowledgments

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Беккер Ф., Циммерманн К., Минченя В., Лысенко В., Чигарев А.

Инспекционные роботы с пьезоприводом: моделирование, симуляция, опытные образцы

Приведены конструкции, представляющие собой симуляции и экспериментальные модели инспекционных микророботов, работающих на основе резонансных колебаний. Использование разработанных подвижных систем позволит создать микророботы, соответствующие новым требованиям разработчиков. Для создания направленного движения инспекционных микророботов используются вибрационные колебания балки на опорах и пластин. Показана возможность создания резонансных инспекционных роботов для двумерного движения и контроля, которые будут иметь небольшой вес, маленькие размеры и относительно несложный дизайн. Измерение параметров резонанса позволяет получить информацию о характерных особенностях опорной исследуемой поверхности Управляемое движение и контроль осуществляется за счет активации только одного элемента микроробота. Сравниваются различные типы механизмов для микророботов; представлены результаты исследования динамических свойства пьезоэлектрического механизма изгиба пластин в границах упругой зоны. Представлена модель движения, порождаемого ножками робота. (E-mail : Klaus/Zimmermann@tu_ilmenau.de)

Ключевые слова: резонансные роботы, системы пьезопривода, вибрационные колебания.

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