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ADJUSTED RICHARD-ABBOTT EQUATION FOR CALCULATING THE STIFFNESS OF STEEL CONNECTIONS

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1. Introduction

In the design process of steel structure, the design of main structural members (beams, columns) must take into account the end connections of these members. As the actual behaviour of a member is dictated by the connection, the design of the connection must be made in accordance with assumptions made in the design. Therefore, the designer must consider the behaviour of connection in both analysis and design.

In analysis and design of steel frames, connections are idealized as either rigid or pinned for simplicity and between these two extremities lies the actual behaviour of steel frame connections which is semi-rigid. It was reported in many publications [1, 2, 3].

In the Load and Resistance Factor Design Specification [4], two categories define the types of construction:

- Type FR (fully-restrained), commonly designated as “rigid-frame” (continuous frame), assumes that beam-to-column connections have sufficient rigidity to hold the original angles between intersecting members virtually unchanged.
- Type PR (partially-restrained) assumes that the connections of beams and girders possess a sufficient rigidity between the intersecting members.

The behaviour of a frame connection subjected to moment is defined by relationship between moment and connection relative rotation (M-θ curve). The rigidity of the connections effects the distribution of negative and positive bending moments along the members, and effects the rotational deformation of the structural elements.

The most practical method to model connection behaviour is to curve fit experimentally-measured moment-rotation data for a given connection to an analytical function. As illustrated in Fig. 1, there are many models to predict the rotation and stiffness of the connections.

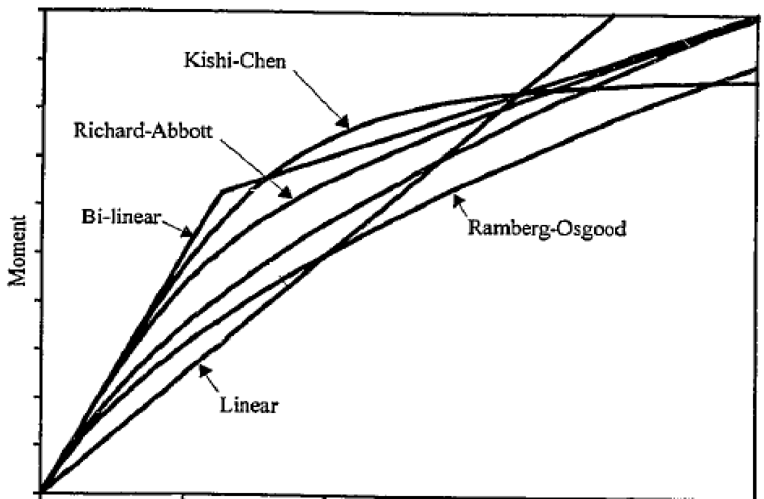

Fig. 1. Curve Fitting Models [5]

Linear model were developed by Lightfoot [6], the stiffness of connection is assumed to be constant throughout its loading range. This model does not represent how the connection stiffness is gradually degraded as load increases. Thus, it does not represent the connection behavior adequately as it approaches its ultimate limit state.

A better representation of the connection behaviour is the bi-linear model developed by Tarpy and Cardinal [7]. The model has a shallower second slope at a particular transition moment M_t to represent the reduced connection stiffness at higher rotations. This model is not suitable for connection types that have non-linear moment-rotation behavior throughout their entire range. To overcome the limitations of the previous models, the multi-linear model was proposed by Moncarz and Grestle [8]. In the multi-linear model, the nonlinear shape of the M-θ curve is approximated by a series of straight lines.

Kishi and Chen [9] developed the following three-parameter power function, plotted in Fig. 1, to model moment-rotation data based on initial stiffness and ultimate moment capacity. The form of the power function is as follows:

$$M = \frac{R_{ki}\phi}{\left(1+\left(\frac{\phi}{\phi_1}\right)^n\right)^{\frac{1}{n}}} \quad (1)$$

Where R_{ki} is the initial connection stiffness, $Ø_1$ is the plastic rotation and n is shape factor.

The plastic rotation is defined as a ratio of the ultimate moment capacity M_u and the initial connection stiffness R_{ki}. The function has no final slope parameter. Therefore, it cannot be made asymptotic to a specific final slope [5].

While Ramberg-Osgood [10] developed their three parameter function to model stress-strain data, the function has been used also to model moment-rotation curves. The form of the Ramberg-Osgood function is as follows:

$$\theta = \frac{M}{S_0}\left[1+\left(\frac{M}{M_0}\right)^{n-1}\right] \quad (2)$$

Where M_o is a reference moment, and S_o is the slope parameter and n is a shape parameter. The function does a poor job of fitting curves as illustrated in Fig. 1 and the initial slope cannot be explicitly specified since the shape parameter affects the initial slope of the curve. Moreover, the Ramberg-Osgood plots with negative curvature (concave down) can have only one point of maximum curvature. Thus, the function cannot accurately fit data that require multiple points of maximum curvature [5].

Frye and Morris [11] represented the Polynomial Models and the form as follows:

$$\theta r = C_1(KM)^1 + C_2(KM)^3 + C_3(KM)^5 \quad (3)$$

Where K is parameter depending on the geometry of the connection and C is curve fitting constant to be determined by the method of least squares.

The primary disadvantage of this model is that the first derivative of the function which indicates the connection stiffness may become negative at some values of M, which is physically impossible.

The multi-parameter exponential models were proposed by Lui and Chen [12] to give good curve-fitting to test data. Some parameters are determined analytically, whilst others are obtained empirically by curve-fitting to experimental data. It has the form:

$$M = \sum_{j=1}^{m} C_j\left[1-\exp\left(-\frac{|\theta_r|}{2j\alpha}\right)\right] + M_0 + R_{kf}|\theta_r| \quad (4)$$

Where M_0 is starting value of connection moment to which the curve is fitted, R_{kf} is strain hardening stiffness of the connection, α is scaling factor and C_j is curve fitting constant.

Del Savio et al. [13] developed a consistent and simple approach to determine moment-rotation curves for any axial force level. Basically, this method works by finding moment-rotation curves through interpolations executed between three required moment-rotation curves, one disregarding the axial force effect and two considering the compressive and tensile axial force effects.

The modified richard-abbott (MRA) function [14] gives better estimation to the connection rotation and stiffness among others as explained in Scerbo [5] but sometimes give stiffness value higher than the experimental result which is unsafe.

This study proposes adjustment to MRA function to give more accurate estimation to the connection stiffness by adding one more parameter $\varnothing_0$ (the initial rotation of the connection) which has a major effect on the behavior, rotation and stiffness of connections.

2. Characteristics of the MRA function

The MRA function, presented in equation 5, define the relation between the moment and the relative rotation in term of the six parameters, S_0, S_p, M_0, $\varnothing_u$, n_1 and n_2 [14]

$$M = \frac{(s_0 - s_p)\phi}{\left[\left(\frac{\phi_u - \phi}{\phi_u}\right)^{n_1} + \left(\frac{(s_0 - s_p)\phi}{M_0}\right)^{n_2}\right]^{1/n_2}} + S_p\phi. \tag{5}$$

Where: M_0 is a reference moment,
S_o is the initial slope of the plot,
S_p is the slope of a final asymptote line,
n_1 is a shape parameter,
n_2 additional shape parameter,
and $\varnothing_u$ is the ultimate rotational deformation.
As shown in Fig. 2, the typical curve fitting plot using MRA function that has initial tangent slope S_0, and final slope S_p, M_0 is the moment at which the final slope line crosses the vertical axis. The final slope is defined by $M = M_0 + S_p\phi$ [14].

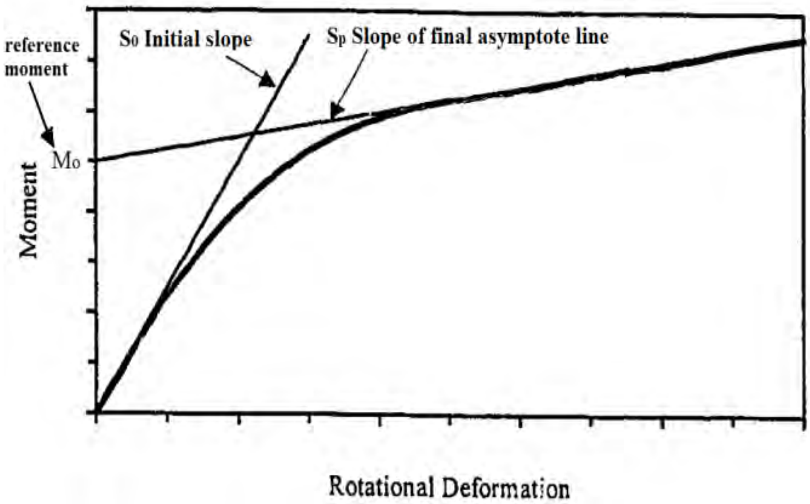

Fig. 2. Typical curve fitting plot using MRA function [14]

The MRA function, presented in equation 6, define the relation between the stiffness of connection and the relative rotation.

$$S(\phi) = S_p + \frac{(s_0 - s_p)}{\left[\left(\frac{\phi_u - \phi}{\phi_u}\right)^{n_1} + \left(\frac{(s_0 - s_p)\phi}{M_0}\right)^{n_2}\right]^{1/n_2}} S_p\phi - \tag{6}$$

$$-\frac{1}{n_2}\left[\frac{(s_0 - s_p)\phi\left(\left(n_2\left(\frac{(s_0 - s_p)}{M_0}\right)\left(\frac{(s_0 - s_p)\phi}{M_0}\right)^{n_2 - 1}\right) - \left(\frac{1}{n_2\phi_u}\right)\left(\frac{\phi_u - \phi}{\phi_u}\right)^{n_1 - 1}\right)}{\left[\left(\frac{\phi_u - \phi}{\phi_u}\right)^{n_1} + \left(\frac{(s_0 - s_p)\phi}{M_0}\right)^{n_2}\right]^{1 + 1/n_2}}\right]$$

Using MRA equation to calculate the stiffness give sometimes higher estimation than the experimental result which is unsafe. So that we proposes adjustment to MRA equation to give better estimation to the stiffness of connections by adding one more parameter $Ø_0$, the initial rotation of the connection, which has a major effect on the behaviour of connections.

3. The Proposed Adjustment on MRA Function

An extensive research has been performed by the authors to adjust the modified Richard-Abbott function using MATLAB software [15]. Computerized research using the curve fitting method guides us to add one more parameter $Ø_0$ (the initial rotation of the connection, Fig. 3) which has a major effect on the behavior, rotation and stiffness of connections and has yielded the following formula:

$$S(\phi) = S_p + \frac{(s_0 - s_p)}{\left[\left(\frac{\phi_u - \phi}{\phi_u}\right)^{n_1} + \left(\frac{(s_0 - s_p)\phi}{M_0}\right)^{n_2}\right]^{1/n_2}} S_p\phi - \tag{7}$$

$$-\frac{1}{n_2}\left[\frac{(s_0 - s_p)\phi\left(\left(n_2\left(\frac{(s_0 - s_p)}{M_0}\right)\left(\frac{(s_0 - s_p)\phi}{M_0}\right)^{n_2 - 1}\right) - \left(\frac{1}{n_2\phi_u}\right)\left(\frac{\phi_u - \phi}{\phi_u}\right)^{n_1 - 1}\right)}{\left[\left(\frac{\phi_u - \phi}{\phi_u}\right)^{n_1} + \left(\frac{(s_0 - s_p)\phi}{M_0}\right)^{n_2}\right]^{1 + 1/n_2}}\right]$$

$$-0.7 \times s_p \times \log\left(\frac{\phi_0 \times 10^4}{\phi}\right)$$

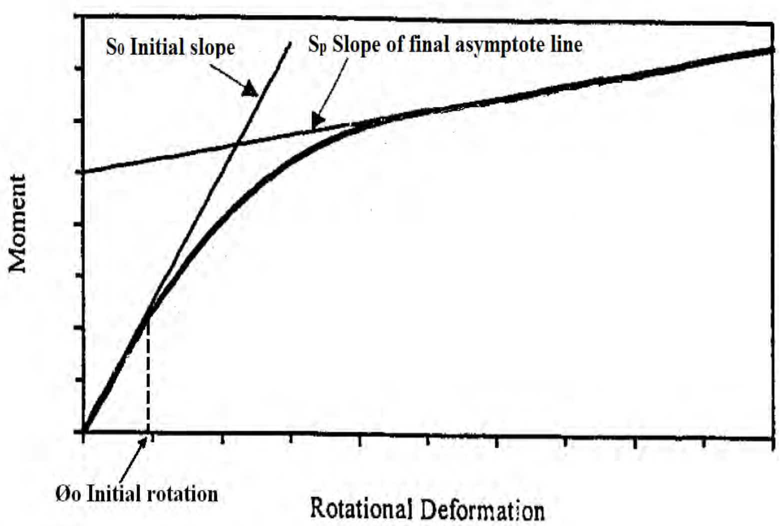

Fig. 3. $Ø_0$ the initial rotation of the connection

4. Verification of the Proposed Adjustment, Result and Discussion

In order to verify the accuracy of the proposed function (equation 7) to estimate the connection stiffness, the following examples are selected from many examples solved by the authors to compare the proposed function and MRA function with the experimental results.

Example 1:

Typical Flush End Plate connection with following characteristics: M_0 = 51 kN.m, S_o = 9040 kN.m/rad, S_p = 205 kN.m/rad, n_1 = 0.752, n_2 = 1.518, $Ø_u$ = 0.055 rad, $Ø_0$ = 0.00028 rad. These values have been taken from Scerbo [5].

By taking different values of rotations from the experimental results [5] and calculating the connection stiffness utilizing the proposed equation (Eq. 7) provides the value of column 3 in table 1 and employing MRA function (Eq. 6) produces the value tabulated in column 4 table1.

Table 1. Connection stiffness in example 1

Rotation (radians)	Stiffness of connection (kN.m/ rad)		
	Experiment result	Proposed equation (Eq. 7)	MRA function (Eq. 6)
0.0003	8663	8347	8921
0.0026	5898	5743	6183
0.0097	1328	1152	1509
0.0201	380	180	492

Figure 4 shows the connection stiffness using proposed method, MRA and experimental results.

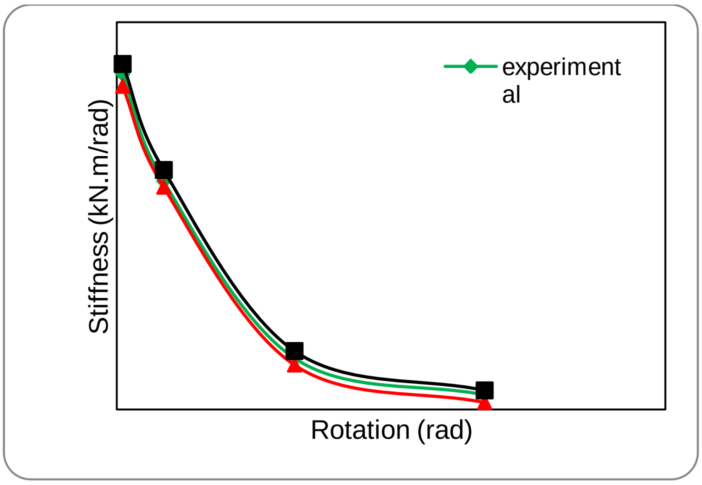

Fig. 4. Stiffness-rotation of the connection in example 1

By comparing the connection stiffness using proposed method, MRA function and experimental results, it can be noticed that the prediction of the proposed method give stiffness values equal or less than the experimental results while MRA function give higher estimation which is unsafe for design purposes. It can be concluded from table 1 that the prediction of the proposed method is more accurate than MRA function.

Example 2:

Typical Double Web Angle connection with following characteristics: M_0 = 20 kN.m, S_o = 3880 kN.m/rad, S_p = 90 kN.m/rad, n_1 = 1.093, n_2 = 1.927, $Ø_u$ = 0.079 rad, $Ø_0$ = 0.0005 rad. These values have been taken from Scerbo [5]

By taking different values of rotations from the experimental results [5] and calculating the connection stiffness utilizing the proposed equation (Eq. 7) provides the value of column

3 in table 2 and employing MRA function (Eq. 6) produces the value tabulated in column 4 table2.

Table 2. Connection stiffness in example 2

Rotation (radians)	Stiffness of connection (kN.m/ rad)		
	Experiment result	Proposed equation (Eq. 7)	MRA function (Eq. 6)
0.0011	3600	3424	3654
0.0046	1483	1504	1696
0.0060	903	980	1165
0.0083	593	486	661

Figure 5 shows the connection stiffness using proposed method, MRA and experimental results.

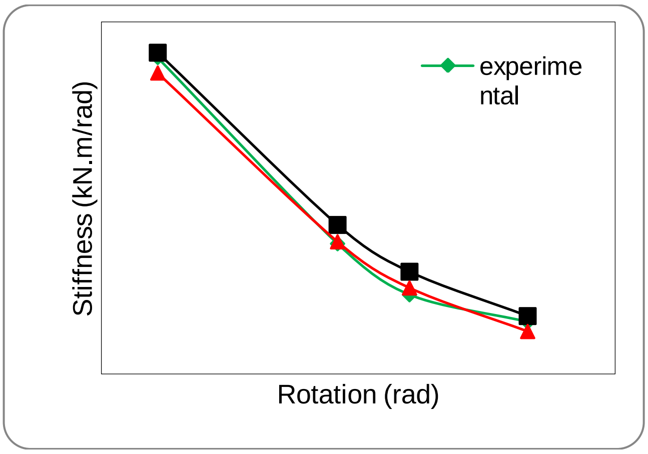

Fig. 5. Stiffness-rotation of the connection in example 2

It can be noticed that the prediction of the proposed method give stiffness values equal or less than the experimental results which is safe, while MRA function give higher estimation than experimental result.

Example 3:

Typical Extended End Plate connection with following characteristics: M_0 = 173 kN.m, S_o = 130000 kN.m/rad, S_p = 2200 kN.m/rad, n_1 = 0.267, n_2 = 1.409, $Ø_u$ = 0.022 rad, $Ø_0$ = 0.00001 rad.

By taking different values of rotations from the experimental results [5] and calculating the connection stiffness utilizing the proposed equation (Eq. 7) provides the value of column 3 in table 3 and employing MRA function (Eq. 6) produces the value tabulated in column 4 in table 3.

Table 3. Connection stiffness in example 3

Rotation (radians)	Stiffness of connection (kN.m/ rad)		
	Experiment result	Proposed equation (Eq. 7)	MRA function (Eq. 6)
0.0001	126270	114600	130000
0.0003	105771	103478	108914
0.0013	41692	41841	46312
0.0035	10054	7664	11445

Figure 6 shows the connection stiffness using proposed method, MRA and experimental results.

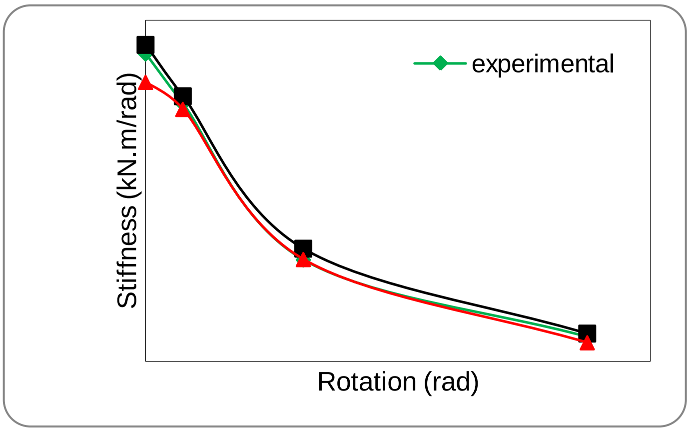

Fig. 6. Stiffness-rotation of the connection in example 3

By comparing the connection stiffness using proposed method, MRA and experimental results, it can be noticed that the prediction of the proposed method give stiffness values equal or less than the experimental results while MRA function give higher estimation which is unsafe. It can be concluded from table 3 that the prediction of the proposed method is more accurate than MRA function.

Conclusion

The design of real structures require the calculation of connection stiffness. A number of analytical functions have been proposed by the researchers to calculate the stiffness of connections but most of them have disadvantages which are explained above. The proposed method (adjustment of MRA function) give a satisfactory approximation of stiffness.

The accuracy of the adjusted equation has been proven by solving several examples. by comparing the result of the adjusted equation with MRA function , It can be noticed that the prediction of the proposed method give stiffness values equal or less than the experimental results while MRA give higher estimation which is unsafe.

SUMMARY

The behavior of steel frame may be significantly affected by the moment versus rotational deformation behavior of its beam to column connections. One approach to the modeling of connection behavior has been to curve fit experimentally-measured moment-rotation data for a given connection to an analytical function. Various models were derived to predict the rotation and stiffness of the connections such as linear model, power models, Polynomial models, Ramberg-Osgood function, exponential model and Richard-Abbott function. The Richard-Abbott function gives better estimation to the connection rotation and stiffness among others. This study proposes an adjustment to the Richard-Abbott equation to give better estimation to the stiffness of connections by adding one more parameter Ø0 (the initial rotation of the connection) which has a major effect on behavior of connections. By comparing the result of the adjusted equation with Richard-Abbott function, it can be noticed that the adjusted equation give stiffness values equal or less than the experimental results which is safe, while the Richard Abbott equation gives sometimes a higher estimation for the connection stiffness than the experimental results which is unsafe. Since the adjusted equation is safe, it is believed to be suited for design purposes.

REFERENCES

[1] Nassani. D. E., (2014). "Behaviour of steel cap plate connections: experimental tests." International Journal of Steel Structures, Vol (14), Issue 3, pp. 649-657.

[2] Al-Jabri, K. S., Burgess, I. W., Lennon, T., and Plank R. J. (2005). “Moment rotation temperature curves for semi-rigid joints.” Journal of Constructional Steel Research, 61, pp. 281–303.

[3] Tahir, M. M. D. (2008). “Experimental tests on composite and non-composite connections using trapezoid web profiled steel sections.” International Journal of Steel Structures, 8(1), pp. 43–58.

[4] AISC. (2005). "Manual of steel construction, Load and Resistance Factor Design". American Institute of Steel Construction, Chicago, IL.

[5] Scerbo, J.S. (1996). "Analysis of steel frames with deformable beam to column connections". M. Sc. Thesis, University of Manitoba, Canada.

[6] Lightfoot, E. & Le Messurier, A.P. (1974). "Elastic analysis of frameworks with elastic connections." Journal of Structural Division, ASCE, 100(6), 1297-1309.

[7] Tarpy, T. S. & Cardinal, J. W. (1981). "Behavior of Semi-Rigid Beam-to-Column End Plate Connections". Proceedings conference, Joints in Structural Steelwork, Halsted Press, London pp. 2.3-2.25.

[8] Moncarz, P.D. & Grestle, K.H. (1981). "Steel frames with nonlinear connections". Journal of Structural Division, ASCE, 107(8), 1427-1441.

[9] Kishi, N. & Chen, W. F. (1990). "Moment-rotation relations of semi-rigid connections with angles". Journal of Structural Engineering, ASCE, 116(7), 1813-1834.

[10] Ramberg, W. & Osgood, W. R. (1943). "Description of Stress-Strain Curves by Three Parameters". NACA Technical Paper, No.902, 1943.

[11] Frye, M.J. & Morris, G.A. (1975). "Analysis of flexibility connected steel frames". Canadian Journal of Civil Engineering". 2(3), 280-291.

[12] Lui, E. M. & Chen, W. F. (1987). "Steel Frame analysis with Flexible Joints". Joint Flexibility in Steel Frames". Journal of constructional steel research, 8, 161-202.

[13] Del Savio, A.A., Nethercot, D.A., Vellasco, P.C.G.S., Andrade, S.A. & Martha, L.F. (2007) "Developments in semi-rigid joint moment versus rotation curves to incorporate the axial versus moment interaction". Third international conference on steel and composite structures. 2007b. pp. 1-7.

[14] Dong, Q. (1994). "Microcomputer Analysis of Reinforced Concrete Flat-Plate Structures Subjected to Lateral Loading". M. Sc. Thesis, University of Manitoba, Canada, 1994.

[15] MATLAB, (1997). The language of technical computing, Version 5.0. The Mathworks Inc., Natick, Mass.

РЕЗЮМЕ

В работе рассматривается один из подходов к расчетному обоснованию связующих элементов конструкций стальных рам. Метод заключается в аппроксимации экспериментально-замеренных кривых о моменте, возникающем для конкретных случаев соединения ответственных элементов конструкций, в виде аналитической функции. Как показывает практика, функция Ричарда-Эбботта дает возможность улученной оценки расчета поворота соединений элементов конструкций и их жесткости по сравнению с другими подходами. В текущем исследовании предложен улучшенный вариант уравнения Ричарда-Эбботта с целью более точной оценки параметра жесткости различных типов соединений. Это достигается путем добавления параметра начального вращения для каждого конкретного соединения. Показано, что параметр начального вращения обладает важным влиянием на последующее поведение соединений и конструктивных элементов. Сравнение результатов показало, что модифицированное уравнение дает улушенные значения для жесткости, в то время как классическое уравнение Ричарда-Эббота дает завышенные результаты. Скорректированное уравнение пригодно для практических целей проектирования элементов конструкций.

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