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COMPUTER SIMULATION OF 6-DOF PARALLEL MECHANISM

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The six degree of freedom spatial movement system has been examined. Algorithms and simulation programs for direct and inverse kinematic problems solving of spatial mechanism are presented. As a result of computer simulation the movement system's actuator spatial motion capabilities are obtained and investigated.

INTRODUCTION

The modern phase of science and technology evolution in the field of automatic and robotic systems is characterized by the use of multi-degree-of-freedom movement systems, realizing almost any spatial movements of an executive or machining tool. Additional requirements for simultaneous implementation of all translational and angular motions are in increasing frequency demanded for new developed movement position devices. The multi-degree-of-freedom systems constructed on the basis of 6-DOF parallel mechanisms are among the most perspective solutions for such motions implementation. However, the task of rapid kinematic and dynamic analysis of such parallel mechanisms is one of the main difficulties in the sphere of robotics. Therefore the paper examines the simulation and analysis of the 6-DOF parallel mechanism in MATLAB/Simulink development environment to present a productive and up-to-date way of similar robotic manipulators engineering.

KINEMATIC STRUCTURE OF THE MECHANISM

Kinematic structure of 6-DOF (degree-of-freedom) spatial parallel mechanism is shown on Fig. 1. It is composed of 6 independent legs connecting the mobile platform P13 with the base. Each of these legs is a serial kinematic chain that is controlled by one motor which actuates one of the joints.

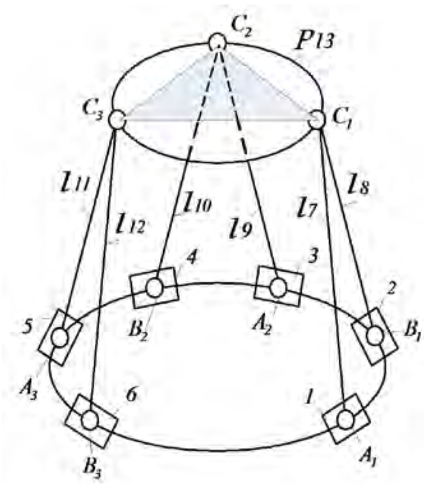

Fig. 1. Mechanism structure

The parallel mechanism, presented on Fig. 1, is intended to the realizing of spatial 3D movements of mobile platform [1]. The points C_1, C_2, C_3 mobile platform P13 is directly connected to direct drives *A1, B1, A2, B2, A3, B3* by shanks $l_7, l_8, \ldots, l_{12}$ (7, 8, 9,10,11,12) with spherical kinematic joints. The 6-DOF spatial parallel mechanism carry out three coordinate axial displacements (x, y, z) and three angular rotations (ψ − yaw, θ − pitch, φ − roll) of the mobile platform.

The number of DOF was calculated by Somov-Malyshev's formula [2]:

$$W = 6n - \sum_{k=1}^{5} k \cdot p_k - \nu = 6,$$

where $n = 13$; $p_5 = 6$; $p_4 = 0$; $p_3 = 12$; $p_1 = p_2 = 0$; $\nu = 6$.

Thus, the mechanism has 6 degrees of freedom.

DIRECT AND INVERSE KINEMATIC PROBLEM

The direct and inverse problems solution implies the implementation of algorithms in simulation software. These algorithms solve both direct and inverse problems for mechanism investigated.

Solution direct and inverse problem kinematics for executive mechanism present important stages of system motion design as a whole Fig.2.

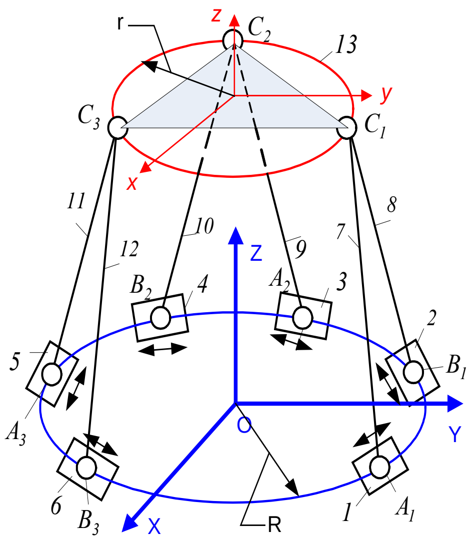

Fig. 2. Geometric representation of the mechanism

DIRECT KINEMATIC PROBLEM

Direct kinematics problem [2] is to determine the mechanism for a given structure and geometry of the moving parts, the laws of movement of executive-level relative to the fixed coordinate system according to the known laws of movement of the input or important links.

Input parameters (as shown on Fig. 2) are the *x* and *y* coordinates of point's pairs A_1, B_1, A_2, B_2 and A_3, B_3, which correspond to drives; coordinate *z* of these points are supposed to equal to zero. We accepted coordinates of points C_1, C_2, C_3 as output parameters Fig. 3.

For the point C_i we'll have:

$$x_{C_i} = x_{N_i} - (N_i C_i)\cos\varphi_i \cos\alpha_i;$$

$$y_{C_i} = y_{N_i} - (N_1 C_1)\cos\varphi_i \sin\alpha_i;$$

$$z_{C_i} = (N_i C_i)\sin\varphi_i,$$

where $i = 1, 2, 3$; N_i − point between A_1 *and* B_1; $\varphi_i - \alpha_i$ − angles; (as shown on Fig. 3) and the distance $N_i C_i$ is calculated by formula

$$N_iC_i = \sqrt{l^2 - (x_{B_i} - x_{A_i})^2 - (y_{B_i} - y_{A_i})^2} .$$

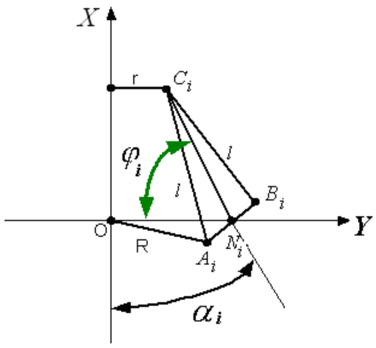

Fig. 3

As we accepted, the distances C_1C_2, C_2C_3 and C_3C_1 are equal, then we can write

$$C_1C_2 = C_2C_3 = C_3C_1 = a = \text{const} .$$

Finally, we'll write equation system, which contain geometric condition for coordinates of points C_1, C_2, C_3:

$$\begin{cases} (x_{C_2} - x_{C_1})^2 + (y_{C_2} - y_{C_1})^2 + (z_{C_2} - z_{C_1})^2 = a^2 \\ (x_{C_3} - x_{C_2})^2 + (y_{C_3} - y_{C_2})^2 + (z_{C_3} - z_{C_2})^2 = a^2 \\ (x_{C_3} - x_{C_1})^2 - (y_{C_3} - y_{C_1})^2 + (z_{C_3} - z_{C_1})^2 = a^2 \end{cases}$$

Solving these expressions together, we can get needed coordinates of platform points C_1, C_2, C_3. Unfortunately, it's necessary to use numerical methods for the solving of equation system due to non-linearity of their equations.

For simulation of mechanism we developed the module in MATLAB environment, which solves the direct problem.

INVERSE PROBLEM

Inverse kinematics problem [2] of the mechanism (Fig. 1) can be formulated as follows. It is necessary for the position and orientation of the movable element of mechanism, which is defined in movable coordinate system *oxyz*, specified by a discrete or parametrical function in relation to a stationary coordinate system *OXYZ*, to find the position of segments 1, 2, ..., 6 (see Fig. 1), defined in the coordinate system *OXYZ* coordinates of points A_i and B_i, respectively, (i = 1, 2, 3).

In the simulation software position and orientation of the movable coordinate system *oxyz* in relation to the fixed coordinate system *OXYZ* are characterized by, respectively, the Cartesian coordinates of platform center $O(x_0, y_0, z_0)$ and Euler angles φ, θ, ψ. The position and orientation matrix is given by

$$M_{\varphi,\theta,\psi} = M_\varphi \cdot M_\theta \cdot M_\psi =$$

$$= \begin{bmatrix} \cos\varphi\cos\theta & -\sin\varphi\cos\psi + \cos\varphi\sin\theta\sin\psi & \sin\varphi\sin\psi + \cos\varphi\sin\theta\cos\psi \\ \sin\varphi\cos\theta & \cos\varphi\cos\psi + \sin\varphi\sin\theta\sin\psi & -\cos\varphi\sin\psi + \sin\varphi\sin\theta\cos\psi \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi \end{bmatrix}.$$

In the uniform coordinates matrix $M_{\varphi,\theta,\psi}$ shall transformed to the fourth-order matrix:

$$M = M_0 \cdot M'_{\varphi,\theta,\psi} = M'_{\varphi,\theta,\psi} \cdot M_0 = \begin{bmatrix} & \left[M_{\varphi,\theta,\psi}\right]_{3\times 3} & & \begin{matrix} x_0 \\ y_0 \\ z_0 \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

which are used for calculating of a platform point coordinates. Given matrix fully describes orientation and position of movable platform of parallel mechanism.

Converting point coordinates C_1, C_2, C_3 in stationary coordinate system *OXYZ* can be realized by formula

$$R^{(C_i)} = \begin{bmatrix} x^{(C_i)} \\ y^{(C_i)} \\ z^{(C_i)} \\ 1 \end{bmatrix} = M \cdot \begin{bmatrix} x_{C_i} \\ y_{C_i} \\ z_{C_i} \\ 1 \end{bmatrix}.$$

To find the points A_i and B_i appropriate to the location of the point C_1:

$$\begin{cases} \left(x - x^{(C_1)}\right)^2 + \left(y - y^{(C_1)}\right)^2 = l^2 - \left(z^{(C_1)}\right)^2, \\ x^2 + y^2 = R^2. \end{cases}$$

$$y = \pm\sqrt{R^2 - x^2},$$

$$R^2 + (x^{(C_1)}) + \left(y^{(C_1)}\right)^2 + \left(z^{(C_1)}\right)^2 - l^2 - 2 \cdot x \cdot x^{(C_1)} = \pm 2 y^{(C_1)} \cdot \sqrt{R^2 - x^2}.$$

When we denote

$$R^2 + (x^{(C_1)}) + \left(y^{(C_1)}\right)^2 + \left(z^{(C_1)}\right)^2 - l^2 = A,$$

the previous expression takes a form

$$A - 2 \cdot x \cdot x^{(C_1)} = \pm 2 y^{(C_1)} \cdot \sqrt{R^2 - x^2}.$$

Now we raise both parts of this expression into a second power:

$$ax^2 - 2bx + c = 0,$$

where $a = 4\left(\left(x^{(C_1)}\right)^2 + \left(y^{(C_1)}\right)^2\right)$, $b = 2Ax^{(C_1)}$, $c = A^2 - 4\left(y^{(C_1)}\right)^2 R^2$.

From last quadratic equation we can find coordinates of points $A_1, B_1, A_2, B_2, A_3, B_3$.

Thus results solution the inverse kinematic problem, that implies the determination of the input variables (motors shafts rotation angles) out of the output variables (platform position x, y, z and orientation ψ, θ, φ) [2]. We developed the module in MATLAB environment, which solves the inverse problem.

HARDWARE-IN-THE-LOOP CONTROL MODEL SIMULATION IN MATLAB/SIMULINK

The 6-DOF spatial parallel mechanism control system computer simulation is implemented as the hardware-in-the-loop control model simulation in MATLAB/Simulink modeling environment. MATLAB/Simulink in conjunction with Real-Time Workshop can automatically generate, package, and compile source code from Simulink models to create real-time software applications that can immediately be executed on a variety of systems and hardware platforms thus enabling the 6-DOF spatial parallel mechanism control system hardware-in-the-loop simulation as well. [3] The 6-DOF spatial parallel mechanism control system hardware-in-the-loop simulation structure is presented on Fig. 4. As the result of hardware-in-the loop simulation (Fig. 4), the 6-DOF spatial parallel mechanism control system hardware-in-the-loop simulation model has been developed in MATLAB/Simulink for simulation on TI TMS320C2000 DSP debug hardware platform (target board).

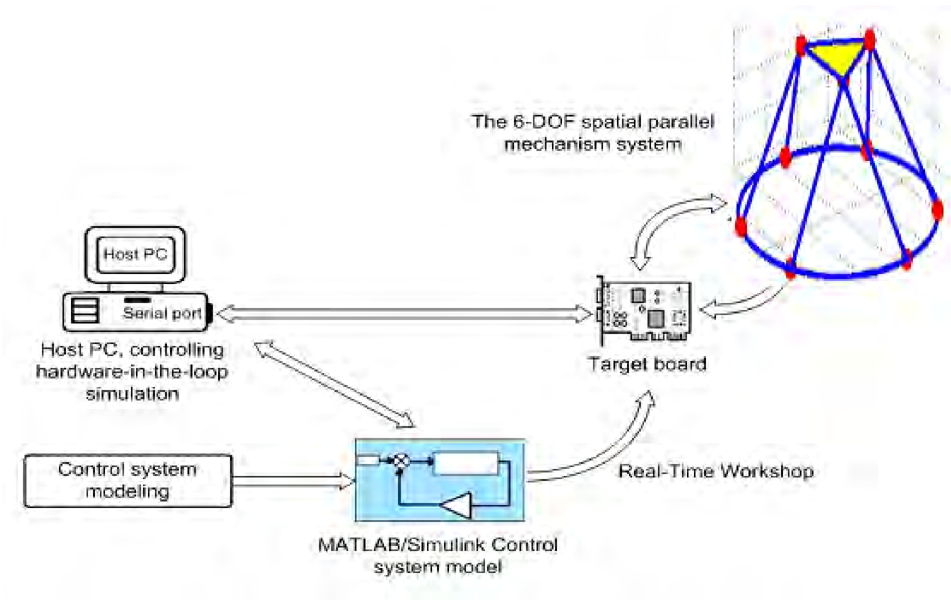

Fig. 4. Structure of mechanism model

The developed hardware-in-the-loop simulation model enables the execution of the MATLAB/Simulink control system model directly on physical hardware debug platform, giving that way the possibility of rapid control prototyping for the 6-DOF parallel mechanism control system.

Hardware-in-the-loop simulation approach permits testing hardware components and debugging of controller software by means of connecting hardware to the program models which were developed and simulated within modeling environment.

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