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Modeling of spraying with time-dependent material feed rate

Dmitry A. Stepanenko *

Department of Construction and Production of Instruments, Belarusian National Technical University, 65 Independence Ave., Minsk 220027, Belarus

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Abstract

This paper presents a mathematical model for predicting the thickness of coatings deposited by means of spraying onto rotating parts with circular symmetry, for the case of time-dependent material feed rate. A procedure for calculating the material feed rate control law providing production of coatings with uniform thickness or coatings with a predefined law of thickness variation is developed. The proposed procedure was used to analyze the process of spraying onto the surface of a rotating disc. A material feed rate variation law providing production of uniform coatings and coatings with linear thickness variation is calculated. It is demonstrated that for the first case the optimal law of material feed rate variation is described by a linear function and for the second case it can be successfully approximated by a quadratic function. The proposed calculation technique can be easily used for the case of coatings with a more complex law of thickness variation.

Keywords: Uniform coating; Spray deposition; Material feed rate

1. Introduction

Today spraying is widely used for depositing coatings onto various articles. Thermal spraying is based on dispersion of liquid metal or ceramic material stream produced by melting continuously supplied wire or powder with the help of an electric or plasma arc [1]. Dispersion is performed with the help of a high-speed gas stream. Colliding with the substrate, droplets of the dispersed melt spread and solidify, forming so called "splats". So, in order to create an integrated model of the spraying process, the following phenomena are to be considered:

(1) spray formation [2,3];
(2) dynamics and thermal state of droplets during flight [2–4];
(3) collision of droplets with the substrate and subsequent cooling and solidification [2,3,5];
(4) evolution of the article shape during spraying [6].

* Present address: 54/3-72, Kalinovsky St., Minsk 220086, Belarus. Tel.: +375 17 263 40 71.
E-mail address: stepd@tut.by

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Nomenclature

A	deposition rate in the center of the spraying area, m/s
d	size of the sprayed material particles, m
$f(\rho,t)$	exponential part of the deposition rate function
$F_j(\rho)$	function obtained as a result of numerical integration of function $f(\rho,t)$ on the time interval $(\tau_{j1}(\rho);\tau_{j2}(\rho))$, s
h	coating thickness, m
M	total number of crossings of the radial section under consideration with the spraying area
M_{j1} and M_{j2}	numbers of the time intervals to which the time instants τ_{j1} and τ_{j2} belong
N_1	number of time samples
N_2	integer number defining frequency of spatial sampling
N_3	number of spatial sampling intervals
r	effective radius of the spraying area, m
R	radius of the article, m
t	time variable, s
t_Σ	total time of spraying, s
v_h	coating deposition rate, m/s
v_s	speed of gun motion, m/s
β	porosity factor
γ	density of the sprayed material, kg/m^3
γ'	apparent density of the coating, kg/m^3
δ	distance from the center of the spraying area to the surface point under consideration, m
Δt	increment of time sampling, s
$\Delta\rho$	increment of spatial sampling, m
ε	small positive number
μ	mass feed rate of the sprayed material, kg/s
ρ	distance from the article axis to the surface point under consideration, m
σ	root-mean-square deviation of the coating deposition rate function, m
τ_{j1} and τ_{j2}	instants of time at which the point under consideration comes into the spraying area and leaves it at the jth crossing of the section under consideration with this area, s
ω	angular speed of the article rotation, rad/s

This paper is focused on the problem of optimization of the spraying process for obtaining the required final profile of the surface.

The process of coating formation by means of spraying can be modeled in one of the following ways:

(1) Discrete modeling. In this case individual particles of a melt are traced. The method allows prediction of microcharacteristics of coatings, particularly the surface roughness and microporosity. The main drawback of the method is its high computational cost. As a result the method can be applied only in the case of thin coatings, when the total number of deposited droplets is relatively small.
(2) Continuous modeling. In this case a spraying device is represented as a continuous source of mass and heat. The method allows only prediction of macrocharacteristics, particularly thickness.

An important characteristic of coatings produced by means of spraying is consistency of their thickness on the surface of an article. In this case minimal mechanical post-processing is needed. Another important problem for practical applications is the problem of producing coatings with a predefined law of thickness variation. Particularly, this problem occurs when geometrically-complex fixture for casting or metal forming is manufactured by spraying layers of mechanically durable and thermally stable material onto templates made of materials with lower mechanical properties [7]. The uniformity of coating thickness has been provided traditionally by trial and error method. However, this method is time-consuming and expensive and requires a skilled operator. In this connection, numerical methods have been proposed in a number of papers [8–10], which can be used to study possibility of deposition of coatings with uniform thickness by means of programmable control of the spraying process parameters. To define coating thickness, Goedjen et al. [8] use adaptive integration of the function describing the coating deposition rate. Integration step is selected taking into account the fast decrease of the deposition rate, taking place as the distance between the point under consideration and the center of the spraying area increases. This distance is considered as the parameter determining the step size. However, the modeling technique is not discussed in sufficient detail. Analysis is limited to the case of flat surfaces, since the deposition rate is considered to be given for a certain fixed distance between the spraying gun and the substrate, and no methods are proposed to account for change of this distance which will evidently occur for more complex surfaces. Hansbo and Nylén [9] proposed a technique for modeling and optimizing the coating thickness for rotationally symmetric surfaces. A model of the spraying cone is suggested. The model makes it possible to calculate the dependence of the deposition rate on the distance between the spraying gun and the substrate. Conner et al. [10] proposed a model of spray painting of geometrically-complex surfaces. A novel projective method of calculation of the deposition rate for curved surfaces is described. Modeling of the spraying cone is also discussed by Djurić and Grant [6,11]. To determine the dependence of the deposition rate on the distance between the spraying gun and the substrate, they use the law of conservation of matter and the assumption that the deposition rate distribution width at the half maximum is independent on this distance [6]. Calculation of the deposition rate treated as a function of the material mass flow and the spray apex angle is also considered [11]. However, the proposed model is partially based on heuristic relationships. In all described papers the trajectory of spraying gun motion is considered as the main factor determining the uniformity of coating deposition. At the same time, control of this factor is associated with dynamic errors resulting from the deviation of the actual law of gun motion from the calculated one [8]. Furthermore, control of gun motion requires application of complex and expensive manipulators. Today commercial software for generation of trajectories of manipulators during spraying is available. However, it was developed for the case of arbitrary trajectories and surfaces with no simplification of calculations for more trivial cases. Moreover, this software involves a number of additional features (for example, realistic models of robotic equipment) which complicate the modeling process. This paper discusses the possibility to control the uniformity of coating deposition by changing the material feed rate. Data from the literature on the use of such control method are not available, but there is information on application of control of powder mixture components feed rate for producing coatings with continuous spatial variation of physicochemical properties (functionally-graded coatings). This method is used, for example, in LENS™ (Laser Engineered Net Shaping) technology [12]. The mathematical model used to describe the spraying process was initially developed by the author for investigating the process of diamond-bearing slurry spraying onto the surface of a scaife (an abrasive tool for grinding and polishing of diamonds) [13,14], but it can be applied for analyzing other processes related to mass transfer and having similar kinematics, especially thermal spraying of coatings and spray painting, as well as subtractive processes of surface shaping by concentrated energy flows (for example, ion-beam machining). The drawback of the suggested models is the simplified assumption about uniform distribution of abrasive particles across the section of a slurry jet. In this case, the density of distribution of particles over the substrate surface is solely defined by the time of stay of individual surface elements in the spraying area. To calculate this time, a kinematic approach can be used [13], thus reducing the problem to solution of transcendental equations. However, such an approach is applicable only for surfaces with simple shape. Besides, it requires a considerable computational resource, while the convergence of the numerical method used for solving the equations is essentially dependent on initial approximations. In another model [14], the total time of spraying is divided into a number of small intervals. The coating thickness in the surface point under consideration is indirectly defined by the number of time intervals for which the point stays in the spraying area. The position of the point relative to the spraying area is determined by checking simple geometrical conditions. The drawback of this model is its limitation to the specific law of distribution of particles. The mentioned drawback is eliminated in this paper.

2. Description of the modeling technique

2.1. Deposition model for the case of stationary feed rate

Let us define the coating deposition rate v_h as the thickness of the material deposited in the surface point under consideration per a unit of time. As the coating thickness h is defined as the total thickness of the material deposited in the point under consideration, the following relation is satisfied:

$$v_h = \frac{\mathrm{d}h}{\mathrm{d}t}.$$

To calculate the coating thickness, we have to define the function describing the spatial distribution of the coating deposition rate for a given spray section. Uniform distribution is not acceptable for describing the rate of mass transfer across the spray section for thermal spraying and spray painting. As it is known [8–10], the coating deposition rate can be most adequately described with the use of Gaussian function, but other functions, for example, quadratic function [15], Cauchy distribution [16] and beta-distribution [17], can also be used. The results obtained for different functions agree well. The difference is only in parameters of these functions and in methods of their determination. For subtractive processes, the material removal rate can also be described with the use of Gaussian function [18]. In the case when the spraying gun is stationary relative to the article with a flat surface (stationary spraying) and spray axis is normal to the article surface, the coating deposition rate will be described with the help of the Gaussian function:

$$v_h(\delta) = A\exp(-\delta^2/\sigma^2), \tag{1}$$

where A and σ are the amplitude and the root-mean-square deviation and δ is the distance from the center of the spraying area to the surface point under consideration (center-to-point distance).

The amplitude A can de defined as the value of the deposition rate in the point of intersection of the spray axis with the surface of the article (in the center of the spraying area). The root-mean-square deviation can be defined as the center-to-point distance δ for which the deposition rate decreases by e times as compared with the amplitude. This deviation describes the diameter of the spraying area.

To determine the parameters of the coating deposition rate function, an experimental method [6,8] or stochastic modeling [19] can be used. In the first case, spraying onto a flat stationary substrate is done with the help of a spraying gun moving at a constant speed along a linear path which is parallel to the substrate surface. Then the coating thickness in the section normal to the gun path is measured. When the deposition rate is described with the help of Gaussian function, the coating profile in this section will be also described by Gaussian function. Having defined its parameters on the basis of experimental data, we can calculate the parameters of the deposition rate function. Another strategy is to use a so called "footprint experiment" [6]. In this case, stationary spraying described above is used.

In stochastic modeling, statistical distributions of size, speed, temperature and impact angle of droplets of molten material are assumed to be known. During cooling edges of splats become distorted under the action of thermal stresses. This distortion is assumed to be the main source of porosity. Splat parameters can be calculated using droplet parameters. Thus, if statistical distributions of droplets parameters are known, it is possible to treat formation of the coating microstructure as a random process of splats accumulation and to calculate the coating roughness, porosity and thickness, as well as the deposition rate function.

The dependence between the coating deposition rate function and the center-to-point distance is schematically shown in Fig. 1.

Let us consider the process of coating deposition onto the flat surface of a rotating article with circular symmetry. Every point of the surface will be described by two coordinates: radial coordinate ρ (distance from the article axis to the surface point under consideration) and angular coordinate θ counted off from the plane passing through the article axis and the spray axis. Let $2r$ be the effective diameter of the spraying area, that is diameter of the circular area on the surface of the article, with the deposition rate outside this circular area considered negligible, ω be the angular speed of the article rotation, v_s be the speed of gun motion, R be the radius of the article. The introduced nomenclature and the diagram of the spraying process are shown in Fig. 2.

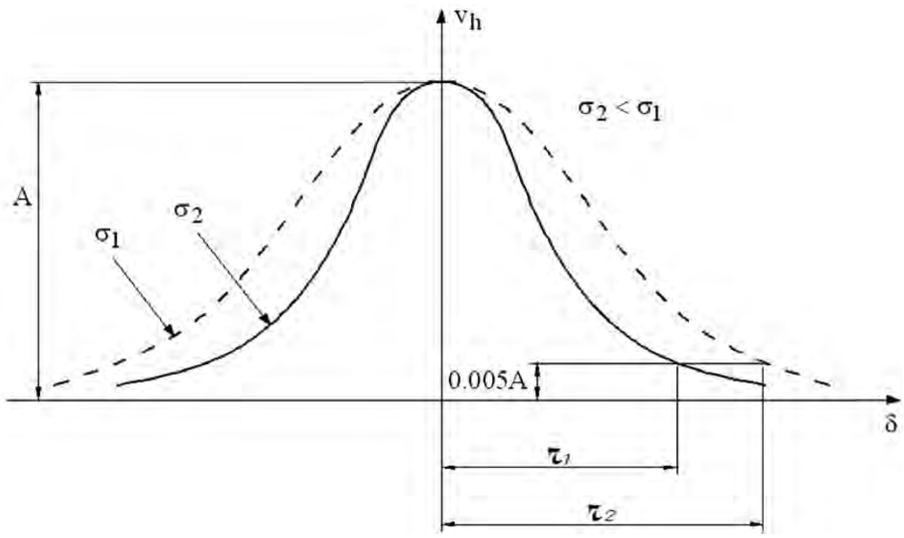

Fig. 1. Dependence between the coating deposition rate function and the center-to-point distance.

As it is shown in the picture, the gun is moving in the radial direction. The process modeling technique depends on the relation between the value of gun displacement per one revolution of the article ($2\pi/\omega \cdot v_s$) and the effective diameter of the spraying area ($2r$). During spraying, a spiral track of the sprayed material appears on the surface of an article. Adjacent turns of the track may partially overlap or may have no common points, depending on the relation of the above mentioned parameters. When $r < \pi v_s/\omega$, adjacent turns of the track do not overlap and uncoated areas appear on the surface. In this case, distribution of the coating thickness along the radial section of the article depends essentially on the angular coordinate and is strongly non-uniform. Small fluctuations of thickness are observed even in the case of a rather uniform coating, what may be attributed to the discrete nature of coating build-up in the radial section of the article. Goedjen et al. [8] visually observed fluctuations of thickness in the form of a spiral pattern on the surface of an article. When $\pi v_s/\omega r \ll 1$, the dependence between the coating thickness and the angular coordinate can be neglected and the coating thickness can be considered to be only the function of the radial coordinate. This allows to make a conclusion about the coating uniformity for the article in whole if the coating thickness is known for a

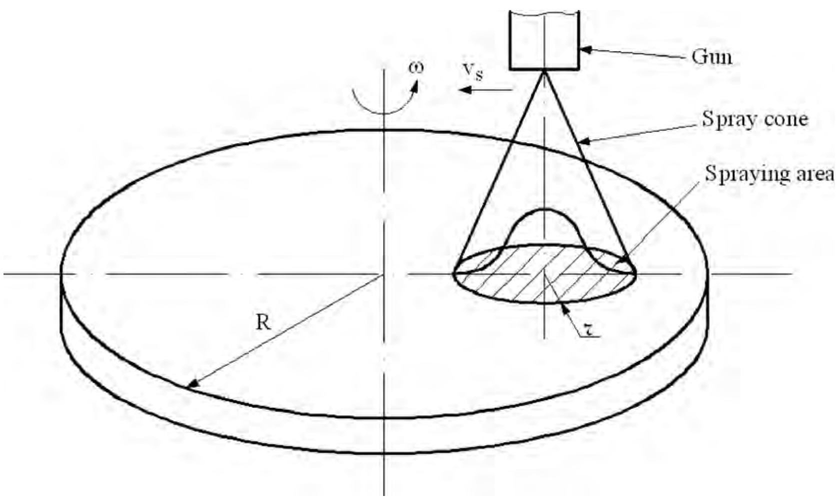

Fig. 2. Diagram of the spraying process.

number of points in an arbitrarily chosen radial section. In the case when the mentioned dependence cannot be neglected, the coating thickness has to be calculated for a number of points evenly distributed over the whole surface. It is not difficult algorithmically, but it takes a lot of computing time. Such an approach is used in some models where the surface of an article is divided into a number of small elements [20,21].

In the case of a moving spraying gun, the function of the coating deposition rate is given by the equation

$$v_h(\rho,t) = A\exp(-\delta^2(\rho,t)/\sigma^2). \tag{2}$$

A similar relation is used by Goedjen et al. [8].

The center-to-point distance can be determined by the cosine rule (Fig. 3):

$$\delta^2(\rho,t) = (R-r-v_s t)^2 + \rho^2 - 2\rho(R-r-v_s t)\cos\theta.$$

The angular coordinate θ is equal to ωt accurate to an integer number of revolutions.

It is assumed here that at the initial instant of time the angular coordinate of the point under consideration is equal to zero. The initial position of the spraying area is shown in Fig. 3 by dashes. It should be taken into account that in fact a spraying gun moves along a linear path as depicted in Fig. 2. However, in order to make Fig. 3 more understandable, a pair of motions (radial feed of the gun and rotation of the article) is substituted with a single compound motion of the gun along a spiral path.

Eq. (2) follows directly from expression (1) if we take into account that the center-to-point distance δ for a moving gun will be dependent on the time t and the radial coordinate ρ. This dependence results from the motion of the point under consideration relative to the spraying area during rotation of the article and radial feed of the spraying gun.

As it follows from the relation between the coating thickness and the deposition rate, the dependence of the coating thickness on the radial coordinate can be determined by means of integration:

$$h(\rho) = \int_0^{t_\Sigma} v_h(\rho,t)\,\mathrm{d}t,$$

where t_Σ is the total time of spraying.

For each jth revolution of the article there may exist a time interval for which the point under consideration moves away from the centre of the spraying area to such an extent that the deposition rate in this point

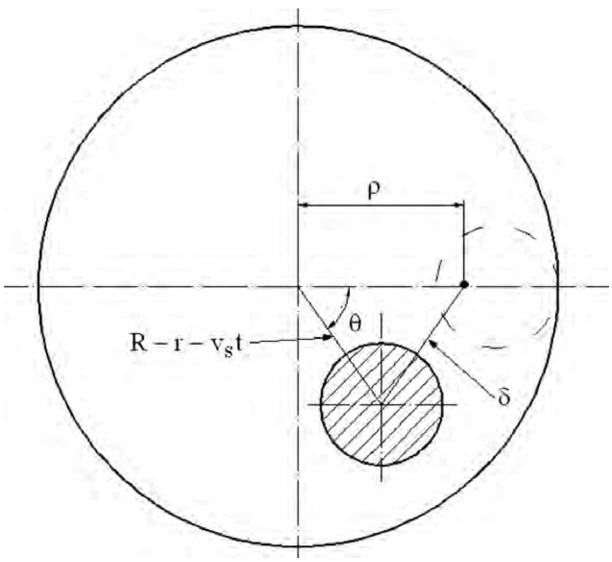

Fig. 3. Geometrical construction used for calculation of the center-to-point distance.

becomes negligible. Mathematically it can be expressed by the inequality $\delta(\rho,t) > r$. Contribution of these intervals into the total value of the coating thickness can be neglected. This is equivalent to representing the deposition rate function in the following truncated form:

$$\tilde{v}_h(\rho,t) = \begin{bmatrix} v_h(\rho,t), & \delta(\rho,t) \leqslant r, \\ 0, & \delta(\rho,t) > r. \end{bmatrix}$$

The superscript will be omitted hereinafter and the deposition rate function will be used in the truncated form if not specified otherwise.

Such truncation is reasonable from the practical point of view, because in practice the deposition rate is non-zero only for a finite region.

The effective radius of the spraying area may be defined from the condition $v_h(r) = \varepsilon A$, where ε is a small positive number whose value describes the error appearing when truncation is used. In the numerical examples given in the third section it was assumed that $\varepsilon = 5 \times 10^{-3}$.

After truncation of the deposition rate function, the dependence $h(\rho)$ can be presented in the following form:

$$h(\rho) = \sum_{j=1}^{M} \int_{\tau_{j1}(\rho)}^{\tau_{j2}(\rho)} v_h(\rho,t)\mathrm{d}t, \tag{3}$$

where M is the total number of crossings of the radial section under consideration with the spraying area and $\tau_{j1}(\rho)$ and $\tau_{j2}(\rho)$ are the instants of time at which the point with the coordinate ρ comes into the spraying area and leaves it at the jth crossing of the section under consideration with this area.

For a fixed point of the surface, the integral appearing in expression (3) possesses non-zero values only for certain values of the index j because for some revolutions the point under consideration does not come into the spraying area. It will be taken into account below when sampling by the variable ρ (see expression (6)). In expression (3), in order to generalize it for applying to any values of the variable ρ, summation is performed for all possible values of the index.

Summation in expression (3) reflects the discrete nature of the coating thickness build-up: for each revolution of the article the coating build-up in the point under consideration takes place only during the time interval corresponding to its stay in the spraying area.

2.2. Deposition model for the case of time-dependent feed rate

Let us consider the possibility of producing coating with a uniform thickness by controlling the mass feed rate μ of the sprayed material. The feed rate μ can be defined as the mass of the material produced by the spraying gun per a unit of time. In the case of time-dependent material feed rate the amplitude of function (1) will be dependent on the feed rate value and, therefore, on time:

$$A = A(\mu(t)).$$

If it is assumed that the root-mean-square deviation of function (1) is independent of the feed rate value, then for the stationary spraying we obtain the following relation between the material feed rate and the deposition rate function:

$$\mu = \gamma(1 - \beta(d,\mu)) \int_0^{+\infty} v_h(\delta) \cdot 2\pi\delta\,\mathrm{d}\delta.$$

Here γ is the density of the sprayed material, β is the porosity factor, $2\pi\delta\,\mathrm{d}\delta$ is the element of the spraying area and the improper integral gives the value of the volume of the material sprayed onto the surface of an article per a unit of time. The deposition rate function is used in the non-truncated form. The porosity factor is defined by the expression $\beta = 1 - \gamma'/\gamma$, where γ' is the apparent density of the coating, and is a function of the material particles size d and the feed rate μ. The dependence $\beta(d,\mu)$ can be determined from experimental data or by means of stochastic modeling. Hereinafter it will be assumed that $\beta = 0$.

The integration yields

$$\mu = \pi\gamma\sigma^2 A,$$

hence it follows that

$$A(\mu) = \frac{\mu}{\pi\gamma\sigma^2}. \tag{4}$$

Thus, for the above assumption, the amplitude of the deposition rate function is directly proportional to the material feed rate.

If the feed rate of the sprayed material is considered to be sensibly constant for the time interval $\tau_{j2}(\rho) - \tau_{j1}(\rho)$, expression (3) can be written as follows:

$$h(\rho) = \sum_{j=1}^{M} A_j \int_{\tau_{j1}(\rho)}^{\tau_{j2}(\rho)} \exp(-\delta^2(\rho,t)/\sigma^2)\mathrm{d}t,$$

where $A_j = A(\mu(t_j))$ and $t_j = 2\pi(j-1)/\omega$ is the time during which the article makes $j-1$ complete revolutions.

As under the assumption accepted above the angular coordinate of the point under consideration is equal to zero at the initial instant of time, the following inequality is satisfied: $\tau_{j1}(\rho) \leqslant t_j \leqslant \tau_{j2}(\rho)$.

Intending to calculate the integral by the rectangular quadrature formula, let us divide the total time of spraying into N_1 equal time intervals with the duration $\Delta t = t_\Sigma/N_1 = (R-2r)/v_s N_1$. Then, denoting the integrand as $f(\rho,t)$, we obtain

$$h(\rho) = \Delta t \sum_{j=1}^{M} A_j \sum_{i=0}^{M_{j2}(\rho)-M_{j1}(\rho)} f(\rho,(M_{j1}(\rho)+i)\Delta t) = \Delta t \sum_{j=1}^{M} A_j F_j(\rho), \tag{5}$$

where $M_{j1}(\rho) = [\tau_{j1}(\rho)/\Delta t] + 1$, $M_{j2}(\rho) = [\tau_{j2}(\rho)/\Delta t]$ are the numbers of the time intervals to which the time instants $\tau_{j1}(\rho)$ and $\tau_{j2}(\rho)$ belong.

The function $F_j(\rho)$ describes the increment of the coating thickness during the jth revolution.

The coating thickness was determined for the range of discrete values of the radial coordinate, defined by the equation

$$\rho_k = R - k\Delta\rho, \quad k = 0,1,\ldots,N_3,$$

where $\Delta\rho = 2\pi v_s/\omega N_2$ is the sampling increment, $N_3 = [R/\Delta\rho]$ is the number of sampling intervals and N_2 is the integer number defining the sampling frequency.

Having sampled expression (5) by the variable ρ, the following expression is obtained:

$$h_k = \Delta t \sum_{j=j_{\min}(k)}^{j_{\max}(k)} A_j F_{jk}, \tag{6}$$

where $h_k = h(\rho_k)$, $F_{jk} = F_j(\rho_k)$ and $j_{\min}(k)$ and $j_{\max}(k)$ are the minimal and maximal values of the index j, for which the condition $F_{jk} \neq 0$ is satisfied (revolutions for which the point with the radial coordinate ρ_k comes into the spraying area).

According to expression (6), calculation of samples of the function $h(\rho)$ comes to the weighted summation of samples of the functions $F_j(\rho)$. The weighting coefficients describe the time variation of the amplitude of the coating deposition rate function, related to the variation of the material feed rate. This allows to make a conclusion that the problem of calculation of the material feed rate control law providing uniformity of thickness of the sprayed coating can be reduced to the construction of weighting coefficients ensuring the constancy of the sum (6) for different values of the index k. Let us demonstrate that the following weighting coefficients meet this condition:

$$A_j = \begin{cases} \dfrac{\max_k F_{2,k}}{\max_k F_{j,k}}, & 3 \leqslant j \leqslant M-1, \\ 1, & j=1 \text{ or } j=2, \\ \dfrac{\max_k F_{2,k}}{\max_k F_{M-1,k}}, & j=M. \end{cases} \tag{7}$$

To prove this, numerical examples for the case of a circular disc will be considered in the next section.

3. Numerical examples

The calculations were performed using a specially developed program in the built-in programming language of the Mathcad® computer algebra system. The following parameters were used for the calculations:

- The disc radius $R = 150$ mm.
- The angular speed of the disc rotation $\omega = 10$ rad/s.
- The gun motion speed $v_s = 1.75$ mm/s.
- The amplitude of the deposition rate function $A = 0.017$ mm/s.
- The root-mean-square deviation of the deposition rate function $\sigma = 4.127$ mm.
- The density of the sprayed material $\gamma = 8.9$ mg/mm^3 (nickel).

The calculated value of the effective radius of the spraying area was found to be $r = 9.5$ mm.

Fig. 4 shows the dependence between the coating thickness and the radial coordinate for the case of variation of the amplitude of the coating deposition rate function, defined by weighting coefficients (7).

For comparison, the dependence between the coating thickness and the radial coordinate for the case of spraying with stationary parameters is shown in Fig. 5.

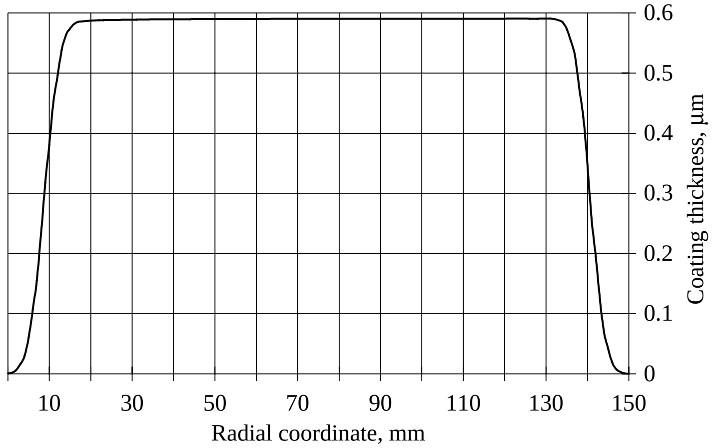

Fig. 4. Dependence between the coating thickness and the radial coordinate for the case of spraying with time-dependent material feed rate.

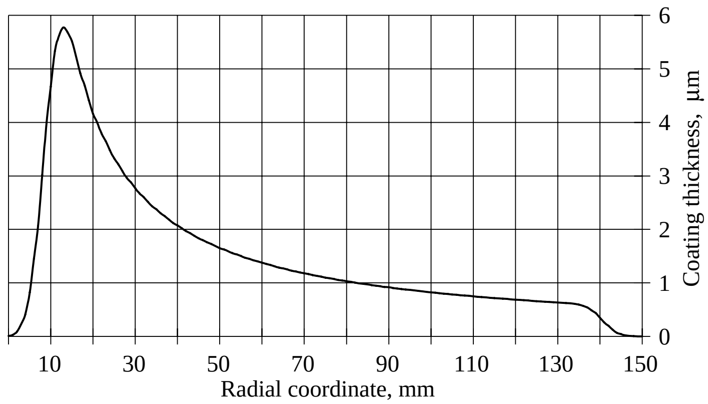

Fig. 5. Dependence between the coating thickness and the radial coordinate for the case of spraying with stationary parameters.

As follows from the analysis of the diagram shown in Fig. 4, the coating thickness is nearly constant for the whole radius of the disc except for the areas in which the edge effect related to start and end of gun motion appears. The width of the edge effect areas can be considered to be equal to the effective diameter of the spraying area. In the case of spraying with stationary parameters (Fig. 5), the coating thickness increases towards the center and then goes down to zero. The peripheral edge effect can be eliminated if at the beginning of gun motion the boundary of the spraying area exceeds the bounds of the disc outer contour. The value of deviation from the uniformity of spraying in the disc center can be reduced when the disc center is partially or fully crossed by the spraying area. However, complete elimination of the spraying non-uniformity was not achieved. The attempts to eliminate the spraying non-uniformity in the disc center were also made by Goedjen et al. [8]. Variation of the gun motion speed and offset of its motion trajectory relative to the center were considered as possible means. The values of parameters, providing considerable reduction of deviation from the uniformity, were found; however, fluctuations of the coating thickness in the disc center were also observed for these values. It should be taken into consideration that many articles with circular symmetry have a central opening. In this case the edge effect in the disc center can be eliminated with the spraying area exceeding the bounds of the inner contour of the article.

Samples of the function describing the time variation of the material feed rate can be calculated from known values of the weighting coefficients, using Eq. (4). The time variation of the material feed rate, plotted on the basis of these samples, is shown in Fig. 6.

The analysis of the diagram shown above makes it possible to conclude that the material feed rate variation law providing the distribution of the coating thickness along the radius as shown in Fig. 4 can be described by a linear function. This result seems to be evident enough, but for the case of predetermined variation of the coating thickness direct determination of the material feed rate control law is impossible and mathematical modeling is needed. Modeling also allows to determine specific control parameters necessary for its practical implementation.

To estimate the calculation accuracy, let us check the condition expressing the law of conservation of matter:

$$2\pi\gamma\int_0^R h(\rho)\rho\mathrm{d}\rho = \int_0^{t_\Sigma}\mu(t)\mathrm{d}t.$$

Calculation of the integrals by the trapezium quadrature formula shows that their values are congruent to 0.368%.

As an example of application of the developed technique for the case of coating with a predefined law of thickness variation, the material feed rate variation providing deposition of coating with linear thickness variation is presented in Fig. 7.

The analysis shows that the diagram in the figure above can be successfully approximated by a quadratic function. The distribution of the coating thickness, corresponding to the material feed rate variation law approximated by a quadratic function, is shown in Fig. 8.

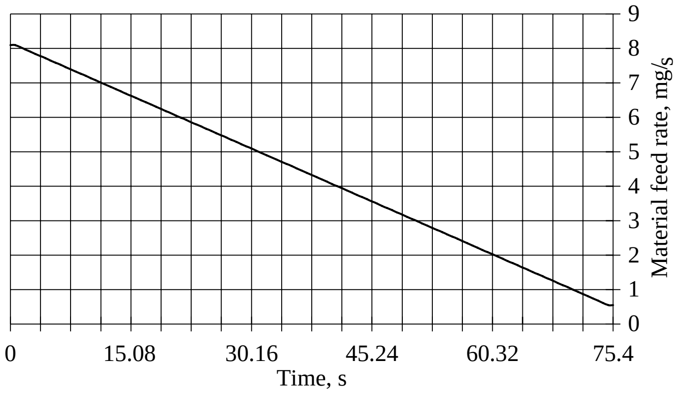

Fig. 6. Time variation of the material feed rate.

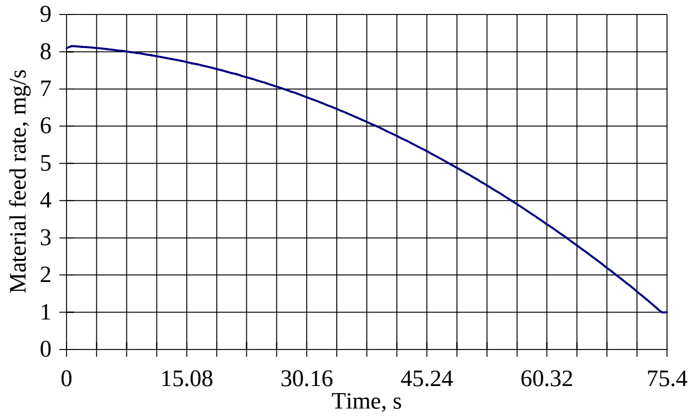

Fig. 7. Material feed rate variation providing deposition of coating with linear thickness variation.

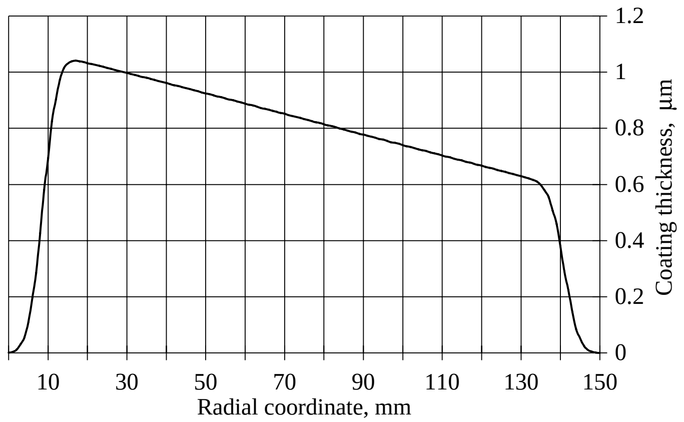

Fig. 8. Distribution of the coating thickness, corresponding to the material feed rate variation law shown in Fig. 7.

As it follows from the analysis of the diagram, the coating thickness for the selected approximation varies according to the linear law.

4. Conclusions

The paper presents a mathematical model of the spraying process. This model can be used for calculating technological parameters providing deposition of coatings with a predefined law of thickness variation, including coatings with uniform thickness. For control of the coating thickness it is proposed to implement spraying with time-dependent material feed rate which may be calculated using a specially developed technique. The efficiency of this technique is proved with numerical examples given for the simple case with a circular disc. In particular, the material feed rate variation law providing deposition of uniform coatings and coatings with linear thickness variation is calculated. The proposed calculation technique can be easily used for modeling spraying of coatings with a more complex thickness variation law and for developing software products which would facilitate the work of mechanical engineers designing spraying processes.

Future research can be concentrated on generalization of the proposed model for the case of articles with curved surfaces, particularly, surfaces of revolution. To account for deviation of the spray axis from the normal to the surface of an article, the model of off-angle spray, suggested by Leigh and Berndt [22], may be useful. The technique described by Conner et al. [10] is also interesting. According to this technique, for calculation of the coating deposition rate in a specified point of the surface, this point is mapped into the point of the basic plane, for which distribution of the deposition rate is specified. Mapping is constructed so that the surface point of interest and its image belong to the same trajectory of the particles of the sprayed material. Then a coefficient is introduced, describing the relation of the area of the small element of the surface, containing the point of interest, to the area of the small element of the basic plane, containing the image of this point. The relation of deposition rates in the mentioned points is determined from the law of conservation of matter.

Now preliminary results on modeling of spraying of coatings onto surfaces of revolution are available [23]. The modeling technique is based on adaptive integration [8] and the projective method described by Conner et al. [10].

The advantage of the proposed method of coating thickness control is the simplicity of its implementation: it does not require any complex manipulators since spraying is done with simple kinematics of gun motion relative to an article. The material feed rate can be changed with the help of simple tools such as a screw feeder with variable-frequency electric drive [14]. The advantage of the proposed technique of calculation of spraying parameters is the simplicity of its algorithmic implementation and reduced computing time required for calculation.

References

[1] E.J. Lavernia, Y. Wu, Spray Atomization and Deposition, Wiley, NY, 1996.

[2] S.P. Kundas, A.P. Dostanko, A.Ph. Ilyuschenko et al., Computer Modeling of Processes of Coatings Plasma Spraying, Bestprint, Minsk, 1998 (in Russian).

[3] U. Fritsching, Spray Simulation: Modeling and Numerical Simulation of Sprayforming Metals, Cambridge University Press, Cambridge, 2004.

[4] P.S. Grant, B. Cantor, L. Katgerman, Modelling of droplet dynamic and thermal histories during spray forming. Part I: Individual droplet behaviour, Acta Metall. Mater. 41 (1993) 3097–3108.

[5] M. Pasandideh-Fard, S. Chandra, J. Mostaghimi, A three-dimensional model of droplet impact and solidification, Int. J. Heat Mass Transfer 45 (2002) 2229–2242.

[6] Z. Djurić, P.S. Grant, Two dimensional simulation of liquid metal spray deposition onto a complex surface, Modell. Simul. Mater. Sci. Eng. 7 (1999) 553–571.

[7] P. Grant, Spray forming, Prog. Mater. Sci. 39 (1995) 497–545.

[8] J.G. Goejen et al., A simulation technique for predicting thickness of thermal sprayed coatings, NASA Technical Memorandum TM-106939, 1995.

[9] A. Hansbo, P. Nylén, Models for the simulation of spray deposition and robot motion optimization in thermal spraying of rotating objects, Surf. Coat. Technol. 122 (1999) 191–201.

[10] D.C. Conner et al., Paint deposition modeling for trajectory planning on automotive surfaces, IEEE Trans. Automat. Sci. Eng. 2 (2005) 381–392.

[11] Z. Djurić, P. Grant, An inverse problem in modelling liquid metal spraying, Appl. Math. Modell. 27 (2003) 379–396.

[12] M.T. Ensz, M.L. Griffith, D.E. Reckaway, Critical issues for functionally graded material deposition by laser engineered net shaping (Online). <http://mfgshop.sandia.gov/MPIF02me.pdf>.

[13] M.G. Kiselev, D.A. Stepanenko, The theoretical estimation of the regularities of diamond particles distribution on the surface of cutting disc in the case of their deposition by means of spraying, Theor. Pract. Mech. Eng. 1 (2005) 34–38 (in Russian).

[14] M.G. Kiselev, D.A. Stepanenko, Investigation of uniformity of coatings deposition by the example of abrasive slurry spraying on the surface of cutting disc, Bull. Belarusian National Tech. Univ. 3 (2006) 51–56 (in Russian).

[15] W. Sheng et al., Automated CAD-guided robot path planning for spray painting of compound surfaces, In: Proc. IEEE/RSJ Int. Conf. Intelligent Robots and Systems, Takamatsu, 2000, pp. 1918–1923.

[16] R. Ramabhadran, J.K. Antonio, Fast solution techniques for a class of optimal trajectory planning problems with applications to automated spray coating, IEEE Trans. Robot. Autom. 13 (1997) 519–530.

[17] M.A. Sahir, T. Balkan, Process modeling, simulation and paint thickness measurement for robotic spray painting, J. Robot. Syst. 17 (2000) 479–494.

[18] P.M. Shanbhag et al., Ion-beam machining of millimeter scale optics, Appl. Opt. 39 (2000) 599–611.

[19] R. Ghafouri-Azar et al., A stochastic model to simulate the formation of a thermal spray coating, J. Therm. Spray Technol. 12 (2003) 53–69.

[20] J. Rastegar et al., On the optimal motion planning for the solid freeform fabrication by thermal spraying, in: Thermal Spray Industrial Applications: Proc. of the 7th National Thermal Spray Conference, Boston, MA, 1994, pp. 463–468.

[21] B. He, F. Tangerman, G. VanDerWoude, Net shape simulation and control, in: Thermal Spray Industrial Applications: Proc. of the 7th National Thermal Spray Conference, Boston, MA, 1994, pp. 415–419.
[22] S.H. Leigh, C.C. Berndt, Evaluation of off-angle thermal spray, Surf. Coat. Technol. 89 (1997) 213–224.
[23] M.G. Kiselev, D.A. Stepanenko, Modeling of process of coatings spraying on the surfaces of revolution, in: Mechanical Engineering and Technosphere of the 21st Century: Proc. of the 13th International Scientific and Technical Conference, Sebastopol, 2006, pp. 154–157 (in Russian).