Приборы и методы измерений
2019. – Т. 10, № 1. – С. 7–13
G.G. Bondarenko et al.

Devices and Methods of Measurements
2019, vol. 10, no. 1, pp. 7–13
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Modeling of an Impact of Thin Insulating Film on the Electrode Surface on Discharge Ignition in Mercury Illuminating Lamps at Low Ambient Temperatures

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Received 17.12.2018
Accepted for publication 28.02.2019

Abstract

The mixture of argon and mercury vapor with temperature-dependent composition is used as the background gas in different types of gas discharge illuminating lamps. The aim of this work was to develop a model of the low-current discharge in an argon-mercury mixture at presence of a thin insulating film on the cathode and to investigate the influence of film on the discharge ignition voltage at low ambient temperatures.

When discharge modeling, we used the obtained earlier expression which describes dependence of the mixture ionization coefficient on temperature. When there was a thin insulating film on the cathode the model took into account that positive charges are accumulated on its surface during the discharge. They generate an electric field in the film sufficient for the field emission of electrons from the metal substrate of the electrode into the insulator and some of them can overcome the potential barrier at the film outer boundary and go out in the discharge volume improving emission characteristics of the cathode.

Calculations showed that at a temperature decrease the electric field strengthes in the discharge gap and the voltage in it are increased due to reduction of the saturated mercury vapor density in the mixture followed by the decrease of its ionization coefficient. Existence of a thin insulating film on the cathode surface results in an increase of the cathode effective secondary electron emission yield which compensates the reduction of the mixture ionization coefficient value.

The results of discharge characteristics modeling demonstrate that in case of the cathode with an insulating film the discharge ignition becomes possible at a lower inter-electrode voltage. This ensures outdoor mercury lamp turning on at a reduced supply voltage and increases its reliability under low ambient temperatures.

Keywords: mercury illuminating lamp, low-current gas discharge, insulating film on cathode, field electron emission, discharge ignition voltage.

DOI: 10.21122/2220-9506-2019-10-1-7-13

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УДК 537.525:621.327.53

Моделирование влияния тонкой диэлектрической пленки на поверхности электрода на зажигание разряда в ртутных осветительных лампах при низких температурах окружающей среды

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Поступила 17.12.2018
Принята к печати 28.02.2019

Смесь аргона и паров ртути с зависящим от температуры составом используется в качестве рабочего газа в различных типах газоразрядных осветительных ламп. Целью данной работы являлось построение модели слаботочного разряда в смеси аргон-ртуть при наличии на катоде тонкой диэлектрической пленки, а также определение ее влияния на напряжение зажигания разряда при низкой температуре окружающей среды.

При моделировании разряда мы использовали полученное ранее выражение, описывающее зависимость ионизационного коэффициента рассматриваемой смеси от температуры. В случае наличия на катоде тонкой диэлектрической пленки в модели учитывали, что в разряде на ее поверхности накапливаются положительные заряды. Они создают в пленке электрическое поле, достаточное для возникновения полевой эмиссии электронов из металлической подложки электрода в диэлектрик, часть из которых может преодолевать потенциальный барьер на внешней границе пленки и выходить в разрядный объем, улучшая эмиссионные характеристики катода.

Расчеты показали, что при снижении температуры происходит увеличение напряженности электрического поля в разрядном промежутке и напряжения на нем, обусловленное уменьшением концентрации насыщенных паров ртути в смеси, а следовательно, и ее ионизационного коэффициента. Наличие же тонкой диэлектрической пленки на поверхности катода может приводить, вследствие существования полевой эмиссии электронов в пленку, к увеличению эффективного коэффициента электронной эмиссии катода, компенсирующему снижение величины ионизационного коэффициента.

Представленные результаты моделирования характеристик разряда демонстрируют, что в случае катода с диэлектрической пленкой становится возможным возникновение разряда при более низком межэлектродном напряжении. Это обеспечивает зажигание лампы наружного освещения при меньшем напряжении питающей сети и повышает ее надежность в условиях низких температур.

Ключевые слова: ртутная осветительная лампа, слаботочный газовый разряд, диэлектрическая пленка на катоде, полевая электронная эмиссия, напряжение зажигания разряда.

DOI: 10.21122/2220-9506-2019-10-1-7-13

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Introduction

Arc illuminating lamps are nowadays a widespread type of gas discharge devices in which a mixture of argon with fixed density and mercury vapor with the density depending on temperature is often used as the background gas [1–5]. After the lamp turning on the low-current discharge is initiated in it which then transits into the glow and arc modes [6].

A lamp important characteristic is the discharge ignition voltage which equals to the minimum potential difference between the electrodes at which the gas breakdown occurs in the inter-electrode gap. Its reduction results in decrease of the lamp energy consumption and also in increase of its reliability and service time [7]. Value of the ignition voltage is determined by the processes of electron emission from the lamp cathode and ionization of the background gas in the discharge volume. In an argon–mercury mixture, along with the direct ionization of atoms by electrons, also ionization of mercury atoms in collisions with excited argon ones (the Penning reaction) can considerably contribute [4, 8, 9]. This leads to an increase of the ignition voltage of outdoor lamps under a reduction of the ambient temperature due to a decrease of the mercury vapor content in the mixture. As a result, the supply voltage can become insufficient for mixture ignition.

However, when a thin insulating film exists on the cathode surface, positive charges are accumulated on it during the discharge. They generate an electric field in the insulator sufficient for the field emission of electrons from the electrode metal substrate into the film [10, 11]. The emitted electrons are accelerated by the field in the film conduction band and a fraction of them can overcome the potential barrier at the film outer surface and go out into the discharge volume increasing the emission characteristics of the electrode. Therefore a method of reduction of the discharge ignition voltage consists in formation of an insulating layer with thickness 10^1–10^2 nm on the lamp metal electrodes surfaces.

Influence of field electron emission from the metal substrate into the insulating film on the cathode emission properties and the discharge ignition voltage was investigated in [12]. It was shown there that this influence was determined completely by the film emission efficiency δ_f equal to the fraction of electrons emitted from the substrate, which goes out of the film into the discharge. But only fixed δ_f magnitudes of about 0.1 were used in [12] whereas in real lamps its value depends on the film thickness and the electric field strength in it, determined by the discharge conditions [13, 14].

In this work a model of the low-current discharge in an argon-mercury mixture under the presence of a thin insulating film formed at the cathode is developed. An analytical expression for the film emission efficiency δ_f obtained in [14] is used. Dependence of δ_f on the background gas temperature in the low-current discharge in argon-mercury mixture is calculated. It is shown that existence of an insulating film can result in a considerable reduction of the discharge ignition voltage at low temperatures.

Model of the low-current discharge

Let the voltage sufficient for arising of the low-current gas discharge to be applied to the discharge gap (length d) between the flat metal cathode covered with a thin insulating film (thickness H_f) and the metal anode (see Figure 1). The discharge current density j is determined by the equation of the discharge circuit:

$$U_d + U_f + RSj = U_0, \tag{1}$$

where $U_d = E_d d$ and $U_f = E_f H_f$ are the voltage drops in the discharge gap and in the insulating film, respectively, E_d and E_f are the electric field strength values in them, S is the electrode surface area occupied by the discharge, U_0 is the external applied voltage, R is the ballast resistor which value is large enough to ensure a small value of j at which the discharge is low-current one [15].

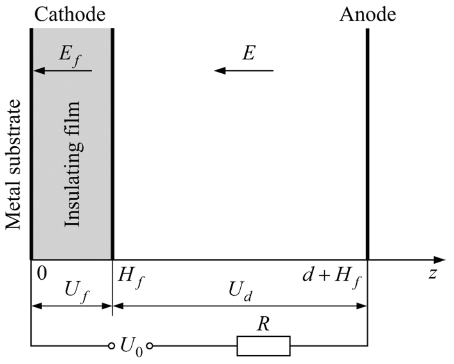

Figure 1 – Schematic of the discharge geometry

Under the current flow in the discharge, bombardment of the cathode by ions proceeds and positive charges are accumulated on the film surface, which generate strong electric field in the film. When its strength E_f reaches value of $\approx 10^8$ V m^{-1} the field electron emission from the cathode metal substrate into the film starts with the macroscopic current density determined by the Fowler-Nordheim equation [14, 16]:

$$j_f\left(E_f\right)=\frac{a s_f E_f^2}{t^2\left(y_0\right)\left(\varphi_m-\chi_d\right)}\exp\left(-\frac{b v\left(y_0\right)\left(m^*/m\right)^{1/2}}{E_f}\left(\varphi_m-\chi_d\right)^{3/2}\right), \quad (2)$$

where $v^2\left(y_0\right)=1-y_0^2+\left(1/3\right)y_0^2\ln y_0$;

$t^2\left(y_0\right)=1+\left(1/9\right)y_0^2\left(1-\ln y_0\right)$; $y_0=c\sqrt{E_f}\Big/\left(\varphi_m-\chi_d\right)$;

$a = 1.541\cdot10^{-6}\,\mathrm{A\cdot eV\cdot V^{-2}}$; $b = 6.831\cdot10^{9}\,\mathrm{V\cdot m^{-1}\cdot eV^{-3/2}}$; $c = 3.795\cdot10^{-5}\,\mathrm{eV\cdot m^{1/2}\cdot V^{-1/2}}$; φ_m is the metal substrate work function; χ_d is the film material electronic affinity; m and m^* are magnitudes of the electron mass in vacuum and in insulator, respectively. Here $E_f = \beta U_f/\varepsilon_f H_f$ [17]; β is the electric field enhancement factor on the relief elements of the metal-insulator boundary; s_f is the fraction of the boundary surface near the relief tops from which the field emission of electrons goes on due to the electric field enhancement on them; ε_f is the high-frequency permittivity of the film material.

Value of E_f can be found from the condition of equality of the macroscopic current density j_f of the field electron emission from the cathode substrate into the film and the discharge current density j, i. e. from equation:

$$j_f\left(E_f\right)=j. \quad (3)$$

Electrons emitted into the film are accelerated by the electric field and decelerated in collisions with phonons [13, 14]. When they reach the film outer boundary a fraction δ_f of them goes out of the film into the discharge increasing the cathode effective secondary electron emission yield γ_{eff}, which equals to the average number of electrons emitted from the surface per an incident ion. Value of the film emission efficiency is determined by an expression [14]:

$$\delta_f=1-\exp\left(-\frac{H_0}{\lambda_e}\right)\sum_{n=0}^{\infty}\frac{H_0^n}{n!\lambda_e^n}\left(1+\frac{\varepsilon_{en}}{\varepsilon_d}\right)\exp\left(-\frac{\varepsilon_{en}}{\varepsilon_d}\right), \quad (4)$$

where $H_0=H_f-H_t$; $\varepsilon_{en}=eE_fH_f-e\varphi_m-n\Delta\varepsilon$;

$\varepsilon_d=\hbar eE_f\Big/2\sqrt{2m^*\left(\varphi_m-\chi_d\right)}\,t\left(y_0\right)$;

H_t is the tunneling length of an electron with the initial energy about the Fermi energy ε_F of the cathode metal substrate; $\Delta\varepsilon$ is the energy lost by an electron in each collision with a phonon in the insulator; λ_e is the average electron path length between collisions with phonons in the direction perpendicular to the cathode surface; $\hbar = h/2\pi$, h is the Planck constant; e is the electron charge magnitude.

In the gas discharge, a substantial fraction of electrons emitted from the cathode surface returns to it due to scattering by the background gas atoms and only their fraction f_{es} goes into the discharge volume. Therefore the real film emission efficiency in the discharge equals to $\delta_{fe} = f_{es}\delta_f$ [15]:

$$f_{es}=1\Big/\left(1+\bar{v}/4w_e\right),$$

where $\bar{v}$ is the average velocity of electrons emitted from the cathode; w_e is the electron drift velocity in the gas.

As a result the cathode effective secondary electron emission yield is determined by expression [14]:

$$\gamma_{eff}=\left(f_{es}\gamma_i+\delta_{fe}\right)\Big/\left(1-\delta_{fe}\right), \quad (5)$$

where γ_i is the cathode ion-electron emission yield.

The condition of low-current discharge self-sustaining in the inter-electrode gap is [6]:

$$\alpha\left(E_d\right)d=\ln\left(1+1/\gamma_{eff}\right), \quad (6)$$

where $\alpha(E_d)$ is the ionization coefficient of the background gas equal to the average number of ionizations of its atoms by an electron per a unit length.

For the argon-mercury mixture $\alpha(E_d)$ value in wide ranges of variation of temperature T and the electric field strength E_d it is determined by expression [18]:

$$\alpha\left(E_d\right)=A\left(N\right)n\exp\left(-B\left(N\right)\sqrt{n/E_d}\right), \quad (7)$$

where $A\left(N\right)=0.18N^3+5.84N^2+54.45N+209.20$;

$B\left(N\right)=0.08N^3+2.50N^2+22.28N+90.40$;

$N=\ln\left(n_{\mathrm{Hg}}\left(T\right)/n\right)$, $n=n_{\mathrm{Ar}}+n_{\mathrm{Hg}}\left(T\right)$;

n_{Ar} and $n_{\mathrm{Hg}}(T)$ are the argon and mercury vapor densities, respectively.

Equations (1)–(7) form a system for the low-current discharge characteristics in the argon-mercury mixture, including its ignition voltage $U_t = U_d + U_f$, when a thin insulating film covers the cathode, as well as under it absence (at $H_f = 0$ and $\delta_f = 0$).

Results of modeling

Calculations were performed for the discharge gap of length $d = 2\cdot10^{-3}$ m filled with the mixture of argon with density $n_{\mathrm{Ar}} = 6.57\cdot10^{23}\ \mathrm{m}^{-3}$ corresponding to its pressure 2660 Pa at temperature 20 °C and saturated mercury vapor, which density grows rapidly with temperature T as it is shown in Figure 2 [18]. The cathode was considered to be aluminum without an insulating film on the surface or with the insulating Al_2O_3 film of thickness $H_f = 16$ nm. The following parameter values were used [13, 14]: $\varphi_m = 4$ eV; $\chi_d = 2$ eV; $\varepsilon_f = 3$; $\beta = 3.8$; $m_e^* = m_e$; $\Delta\varepsilon = 0.125$ eV; $\lambda_e = 0.3$ nm; $s_f = 10^{-3}$; $\gamma_i = 0.03$. The ballast resistor ensured the discharge current density of about 10^{-5} Am^{-2}, i. e. the discharge was low-current [15].

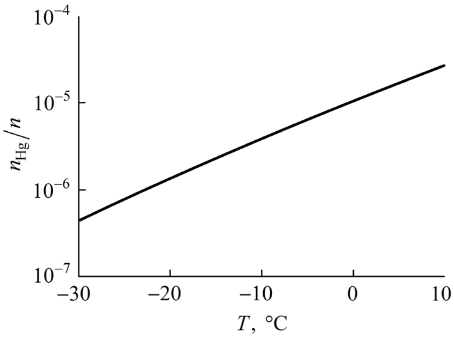

Figure 2 – Relative saturated mercury vapor content in the mixture as a function of temperature [18]

The calculated dependences of discharge characteristics on the mixture temperature are presented in Figures 3–5. It follows from them that in case of cathode without the film temperature reduction from +10 °C to –20 °C (typical for northern countries in winter time), the electric field strength E_d value increases, whereas the discharge current density decreases. This results in discharge ignition voltage U_d increase by value of about 50 V which is in an agreement with experimental data [18]. At a temperature decrease below – 20 °C the discharge characteristics do not change, because the mercury content in the mixture becomes negligibly small and the discharge burns in practically pure argon.

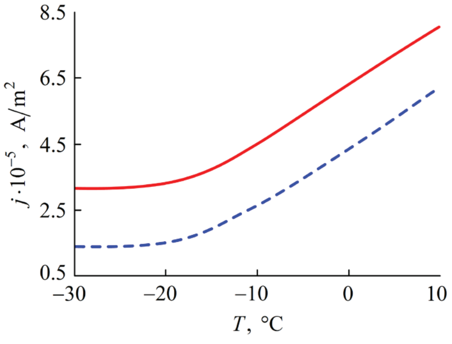

Figure 3 – The discharge current density as a function of temperature for the cathode with insulating film (solid line) and without the film (dashed line)

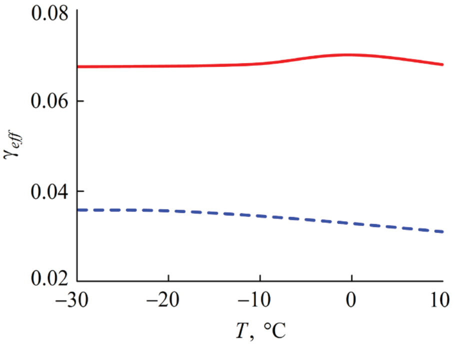

Figure 4 – The cathode effective secondary electron emission yield as a function of temperature. Designations are the same as in Figure 3

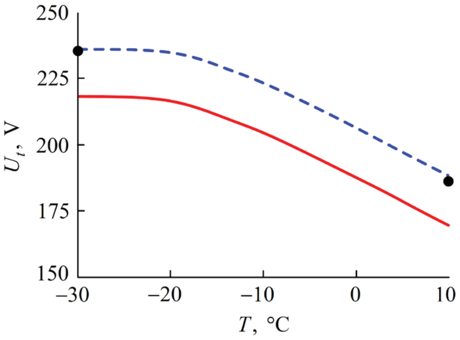

Figure 5 – The discharge ignition voltage as a function of temperature. Designations are the same as in Figure 3. Points are experimental data [19] for the cathode without the film

Under the presence of an insulating film on the cathode a contribution to its effective secondary electron emission yield γ_{eff} along with the ion-electron emission makes the field electron emission from the metal substrate into the insulating film. Therefore, value of γ_{eff} exceeds its value for the cathode without the film considerably. Hence, fulfillment of the discharge sustaining condition (6) becomes possible at a reduced value of the ionization coefficient of the background gas, i. e. at lower electric field strength in it and, consequently, at lower voltage U_t between the electrodes. It can be seen in Figure 5 that the decrease of U_t due to the field electron emission from the cathode substrate is of about 20 V at low temperatures which facilitates the discharge ignition in the arc illuminating lamp under such conditions.

Conclusion

A model of low-current discharge in the argon-mercury mixture when a thin insulating film exists on the cathode surface is developed in the paper. It takes into account the ion-electron emission from the cathode surface, and the field electron emission from the cathode metal substrate into the film, caused by the strong electric field generated in the cathode during the discharge. Using this model dependences of the discharge characteristics on the mixture temperature are calculated.

It is shown that because the saturated mercury vapor density decreases with temperature rapidly, a reduction of the mixture ionization coefficient also takes place. Therefore, the electric field strength in the discharge gap and the voltage drop across it are increased which can prevent the discharge ignition in the lamp at low ambient temperatures. When the cathode surface is covered with a thin insulating film the field electron emission from the cathode metal substrate increases the cathode effective secondary electron emission yield. As a result, the discharge ignition becomes possible at lower value of the ionization coefficient of the background gas and, consequently, at a lower voltage drop between the electrodes. This ensures the discharge ignition under smaller supply voltage values and increases reliability of mercury lamp operation at low ambient temperatures(typical for the northern countries in winter time) and under instability of the supply network voltage.

Acknowledgments

This work was performed in frameworks of the program «Organization of Scientific Researches» of the Russian Federation Ministry of Science and Higher Education in Bauman Moscow State Technical University (project 3.8408.2017/6.7) and was supported financially by the Russian Foundation for Basic Researches and the Kaluga Region Government (project 18-42-400001).

Support from the Basic Research Program of National Research University «Higher School of Economics» is gratefully acknowledged.

Благодарности

Работа выполнена в рамках реализации государственного задания «Организация проведения научных исследований» Минобрнауки РФ в МГТУ имени Н.Э. Баумана (проект 3.8408.2017/6.7) при финансовой поддержке РФФИ и правительства Калужской области (проект 18-42-400001), а также в рамках Программы фундаментальных исследований НИУ ВШЭ.

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