providing the lowest dispersion equal to 128 by 128 pixels or $0.068 \%$ of the total number of pixels.

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## УДК 535.3 <br> QUANTITATIVE RESEARCHES IN THE VISION TECHNICAL SYSTEMS WITH UNLIMITED NUMBER OF ENTRANCE AND OUTPUT MAGNITUDES

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Quantitative researches in the vision technical systems are based on processing of digital images at measurements and modeling of objects properties. In this regard a method of the correlation and regression analysis described in [1] is the convenient instrument of results displaying. The method allows to reveal significant factors of influence and to define interrelations between them (or their absence) [2]. The usage of 3D technologies gives the chance to display triads of magnitudes and to represent the results of multiple parameter researches as a vector columns and sets of covariation matrixes. The review of sources [1-4] has shown that for covariance assessment most often use coefficient of linear correlation of $r_{x, y}$ and a correlation ratio $\eta_{y x}^{\epsilon}$ (for nonlinear dependences) and also a method of the smallest squares and confidential intervals.

1. The Pearson coefficient of linear correlation expresses degree of narrowness of linear communication between two random variables and is calculated according to selective data on a formula:

$$
\begin{equation*}
r_{x, y}=\frac{\sum_{i=1}^{n}\left[x_{i}-\bar{x}\right) \cdot\left(y_{i}-\overline{y)]}\right.}{\sqrt{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2} \cdot \sum_{i=1}^{n}\left(y_{i}-\bar{y}\right)^{2}}} \tag{1}
\end{equation*}
$$

where $x_{i}, y_{i}$ the values of variables $\mathrm{x}, \mathrm{y}$ respectively, n - number of observations.

The coefficient of correlation can accept values from -1 to +1 and the direction of change of characteristics is reflected by the signs «+»» or «-» before it. This coefficient shows communication «force» i. e. synchronism of change of two variables.

According to Fischer the statistical importance of Pearson correlation coefficients in a case of their normal distribution is checked on the basis of Student's $t$-distribution with the set of the probabilistic importance level $\alpha$ and the known factors number in a model [3]. We find the value of $t$-criterion on a formula:

$$
\begin{equation*}
t_{c a l c}=r_{x y} \sqrt{\frac{n-k-1}{1-r_{x y}^{2}}} \tag{2}
\end{equation*}
$$

from here

$$
\begin{equation*}
r_{x y}=\frac{t_{\text {calc }}}{\sqrt{n-k-1+t_{c a l c}^{2}}} \tag{3}
\end{equation*}
$$

where $k$ - number of factors in the model. It is necessary to compare the value of the criterion $t_{\text {calc }}$ with the theoretical value $t_{t a b}$ specified in statistical tables. If $t_{\text {calc }}>t_{\text {tab }}$, the correlation coefficient is considered to be statistically significant, and this coefficient is insignificant when $t_{\text {calc }} \leq t_{t a b}$. If the probability distribution is not normal then the Fisher's Z-criterion is used as a criterion of their significance. For paired correlation coefficients an interval estimate $\left[c_{m i n} ; c_{\max }\right]$ can be constructed using the Fisher Z-transform with a given reliability [4]. Using the Fisher's Z-transform and the selective correlation coefficient $r$ we find the corresponding value of Z which is the hyperbolic arctangent of $r$ [4]:

$$
\mathrm{Z}=\frac{1}{2} \ln \left(\frac{1+r}{1-r}\right)=\operatorname{arctanh}(r)
$$

Further we find the value of $d Z$ corresponding to the confidence level $\mathrm{p}=0,95$ (the value of the Laplace function). We calculate the values of $Z_{\text {min }}$ and $Z_{\max }$ by the following formulas:

$$
\begin{align*}
& Z \min =Z r-d Z  \tag{4}\\
& Z \max =Z r+d Z \tag{5}
\end{align*}
$$

Finally using the inverse Fisher transform we find the lower and upper bounds for the general correlation coefficient $c_{\text {min }}$ and $c_{\text {max }}$ which are hyperbolic tangents of $Z_{\text {min }}$ and $Z_{\max }$ :

$$
r=\frac{\exp (2 z)-1}{\exp (2 z)+1}=\tanh (z)
$$

2. The empirical correlation relation $\eta_{y x}^{\epsilon}$ is used to estimate the tightness of the nonlinear coupling between random variables and calculated using the common $\sigma_{Y}^{2}$ and the intergroup $\sigma_{\bar{Y}_{x}}^{2}$ dispersions the formulas of which are indicated in [3]. After all the additional calculations, we find the empirical correlation relation according to the formula [3]:

$$
\begin{equation*}
\eta_{y x}^{\epsilon}=\sqrt{\frac{\sigma_{\bar{Y}_{X}}^{2}}{\sigma_{\mathrm{Y}}^{2}}} \tag{6}
\end{equation*}
$$

The coefficient $\eta_{y x}^{\epsilon}$ is always in the range from 0 to $1\left(0<\eta_{y x}^{\epsilon} \leq 1\right)$. A functional relationship is observed for $\eta_{y x}^{\epsilon}=1$ between the quantities and the quantities are independent for $\eta_{y x}^{3}=0$. The relationship between the quantities is linear if $\eta_{y x}^{\ni}=\left|r_{x y}\right|$. The condition $\eta_{y x}^{\ni} \geq r_{x y}$ is always satisfied. The verification of the significance of the empirical correlation relation is carried out by the criterion:

$$
\begin{gather*}
F_{H}=\frac{\eta_{y x}^{\epsilon^{2}}(n-m)}{\left(1-\eta_{y x}^{\epsilon_{x}^{2}}\right)(m-1)},  \tag{7}\\
F_{c r}=F(1-\alpha, m-1, n-m), \tag{8}
\end{gather*}
$$

where $F_{H}$ - calculated value of Fisher's coefficient; $F_{c r}$ - theoretical value of Fisher's coefficient; $n-$ the number of observations; $m$ - the number of observations series.

After establishing the significance of the data obtained by means of correlation analysis, we proceed to regression analysis. The tasks of regression analysis are to establish the form of the investigated dependence.

Linear regression reduces to finding an equation of the form:

$$
\begin{equation*}
y_{x}=\alpha_{0}+\alpha_{1} x \tag{9}
\end{equation*}
$$

where $x$ - an individual factor attribute value; $\mathrm{a}_{0}$, $\mathrm{a}_{1}$ - parameters of the straight line equation (regression equation); $\mathrm{y}_{\mathrm{x}}$ - the theoretical value of the resulting factor.

This equation shows the average value of the change in the effective characteristic x by one unit of its measurement and the sign of the parameter is the direction of this change.

By the classical approach the parameters $a_{0}$, $a_{1}$ of the equation are found by the least squares method [4] by which the most suitable regression line is determined minimizing the vertical distance of all points of the correlation field from this line. The distance from each point of the field to the vertical regression line is a random error $e_{i}$. The distance from each point of the field to the vertical regression line is a random error $e_{j}$. Summing up the errors in the square we get the total error minimizing which we can determine the most suitable regression line. To do this, we equate partial derivatives to zero and obtain a system of two linear equations:

$$
\begin{gather*}
\left\{\begin{array}{c}
\sum y=n a_{0}+a_{1} \sum x, \\
\sum x y=a_{0} \sum x+a_{1} \sum x^{2}
\end{array}\right.  \tag{10}\\
a_{0}=\frac{\sum y \sum x^{2}-\sum x y \sum x}{n \sum x^{2}-\left(\sum x\right)^{2}},  \tag{11}\\
a_{1}=\frac{n \sum x y-\sum x \sum y}{n \sum x^{2}-\left(\sum x\right)^{2}} . \tag{12}
\end{gather*}
$$

Parameter $\mathrm{a}_{0}$ shows the averaged effect on the effective sign of the factorial signs unaccounted in the equation, parameter $a_{1}$ is the regression coefficient and shows the change in the effective sign when the factorial one changes by the unit of its own measurement. Often the studied features have different units of measurement, therefore, to assess the influence of the factor characteristic on the effective elasticity coefficient is used.

The coefficient is calculated for each point and on average for the whole population by the formula:

$$
\begin{equation*}
\mathrm{C}=y_{x}^{\prime} \frac{x_{i}}{y_{x}^{\prime}}, \tag{13}
\end{equation*}
$$

where $\mathrm{y}_{\mathrm{x}}^{\prime}$ - the first derivative of the regression equation.

The coefficient of elasticity shows the percentage change in the effective feature when the factor attribute changes by $1 \%$. To test the significance of the regression model we use Fisher's criteria:

$$
\begin{gather*}
F_{H}=\frac{r_{x y}^{2}(n-2)}{\left(1-r_{x y}^{2}\right)},  \tag{14}\\
F_{c r}=F(1-\alpha, 1, n-2) \tag{15}
\end{gather*}
$$

Model significance condition:

$$
\begin{equation*}
F_{H}>F_{c r} \tag{16}
\end{equation*}
$$

As an example of a visual representation of the results of correlation and regression analysis in 3-dimensional space, we use the data of the study of vocal speech as a non-stationary random process described in [5]. The results of data correlation analysis in Matlab can be seen in figure 1.


Figure 1-3D-presentation of the results of correlation analysis in Matlab
Thus with the help of correlation and regression analysis we can judge the reliability of the data with a certain accuracy as well as having some mathematical model (with the possibility of its implementation in 3-dimensional space), with a certain degree of probability we can predict further research results.

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## УДК 535.3

# RECOMMENDATIONS FOR THE DISPLAY AND ANALYSIS OF THE RESULTS OF MULTIPARAMETER RESEARCH IN HARDWARE AND SOFTWARE ENVIRONMENTS Saukova Y., Alkhimovich M., Mirgorod Y. 

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Increasingly, it is necessary to analyze and process the results of multiparameter research of the objects geometric and optical properties at the control and testing of products. The capabilities of modern software and hardware environments allow you to implement these procedures. The purpose of this work is to analyze the capabilities of modern software products from the standpoint of effective 3-D modeling of the results of multiparameter research.

1. Software product of Waterloo Maple Inc. (Canada) Maple is used for analytical mathematical calculations, data visualization, and modeling. Maple has a built-in plot3d function in the kernel for plotting three-dimensional surfaces. There is a possibility to specify the types of coordinate systems using the transformation formulas from a rectangular coordinate system to another, i.e. transformation $(u, v, w) \rightarrow(\mathrm{x}, \mathrm{y}, z)[1]$. To build a graph, we need to set the equation of the function (in explicit or implicit form) and then the program will display its three-dimensional image. The plot3d function allows you to build several shapes that intersect in space at the same time. To do this, instead of describing one surface, we need to specify a list of descriptions of a number of surfaces. An example of such construction for two functions is shown in the figure 1.


Figure 1 - An example of the construction of surfaces in the environment Maple

Also, special three-dimensional structures can be used, such as GRID (specifying the surface over a portion of the coordinate plane bounded by segments [a, b] and [C, d], according to the data specified by the variable-listlist, see figure 2) or MESH (specifying the data of the listlist variable, which contains the full coordinates of all surface points, it is possible to specify the last one in case of uneven grid).


Figure 2 - Example of creating A grid-type graphical structure
2. Software product Mathcad company Parametric Technology Corporation is universal for mathematical modeling and solutions of mass various fields of science and technology. In the Mathcad environment, there are actually no function graphs in the mathematical understanding of the term, but there is a visualization of data located in vectors and matrices (both lines and surfaces are plotted by interpolated points).

Mathcad easily integrates with a huge number of databases and third-party SOFTWARE including Microsoft Excel, MATLAB by MathWorks, etc. The ability to use in the calculation of quantities with dimensions in different systems of units is an advantage of this environment [2]. The results of the calculations also obtain the corresponding dimension. This fact greatly simplifies the tracking of errors in physical and engineering calculations. Two-dimensional (2D) and three-dimensional (3D) graphics are used to display and interpret the calcu-

