МИНИСТЕРСТВО ОБРАЗОВАНИЯ РЕСПУБЛИКИ БЕЛАРУСЬ

Белорусский национальный технический университет

Кафедра «Теоретическая механика»

А. В. Чигарев

МАШИНОСТРОИТЕЛЬНЫЙ ДИЗАЙН ДЛЯ МЕХАТРОННЫХ СИСТЕМ

MECHANICAL ENGINEERING DESIGN FOR MECHATRONIC SYSTEMS

Курс лекций в форме презентаций

Часть 1

Минск
БНТУ
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Рассмотрены методы расчета механических компонентов машин, представляющих собой мехатронные системы. Лекции построены в виде презентаций так, что каждая страница на бумажном носителе соответствует слайду в проекторе. Пособие предназначено для студентов специальностей по направлениям мехатроники, робототехники, информационных систем машиностроения, приборостроения и других технических специальностей. Может быть полезно магистрантам, аспирантам, инженерам соответствующих специальностей.

This course of lectures in the form presentation will introduce the mechanical engineering students to the design methods of mechanical elements of machines using only analytical tools of classical mechanics and theory of solid body accordance with International and National Standards for machine design.

Methods of these courses are used by students for studying disciplines “Mechatronics”, “Design of mechatronic systems”, which offered in Belarusian National Technical University for students which to take the profession “Computer mechatronics” and other technical professions.

This course will be useful for engineers which are practiced in field of machine design.

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Lecture
Theme 1

1.1 Definitions and classifications for machine design

1.1.1 Introduction

Design is to formulate a plan to satisfy a particular need and to create something with a physical reality. Consider for an example, design of a table. A number of factors need be considered first:

(a) The purpose for which the table is to be designed such as whether it is to be used as an easy table, an office table or a dining table.

(b) Whether the table is to be designed for a grownup person or a child.

(c) Material for the table, its strength and cost need to be determined.

(d) Finally, the aesthetics of the designed table.

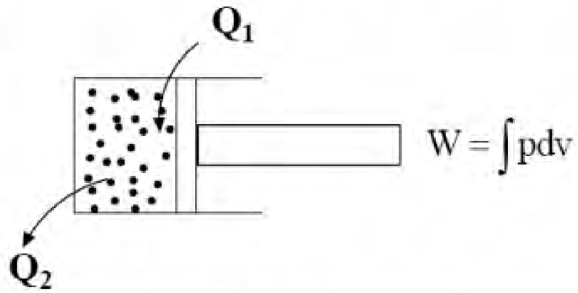
Almost everyone is involved in design, in one way or the other, in our daily lives because problems are posed and they need to be solved.

1.1.2 Basic concept of machine design

Decision making comes in every stage of design. Consider two cars of different makes. They may both be reasonable cars and serve the same purpose but the designs are different. The designers consider different factors and come to certain conclusions leading to an optimum design. Market survey gives an indication of what people want. Existing norms play an important role. Once a critical decision is made, the rest of the design features follow. For example, once we decide the engine capacity, the shape and size, then the subsequent course of the design would follow. A bad decision leads to a bad design and a bad product.

Design may be for different products and with the present specialization and knowledge bank, we have a long list of design disciplines e.g. ship design, plan design, building design, robot design, mechatroniks system and so on.

Here we are concerned with machine design. We now define a machine as a combination of resisting bodies with successfully constrained relative motions which is used to transform other forms of energy into mechanical energy or transmit and modify available energy to do some useful work. If it converts heat into mechanical energy we then call it a heat engine. This is illustrated in figure-1.1.2.1.



1.1.2.1- Conversion of heat to mechanical energy in a piston cylinder arrangement.

In this example for case heat energy $Q_1 > Q_2$, W – energy (work) piston is moved to right, for case heat energy $Q_1 < Q_2$, piston is moved to left.

In many cases however, the machines receive mechanical energy and modify it so that a specific task is carried out, for example a hoist, a bicycle or a hand-winch.

1.1.3 Types of design

There may be several types of design such as

Adaptive design
This is based on existing design, for example, standard products or systems adopted for a new application. Conveyor belts, control system of machines and mechanisms or haulage systems are some of the examples where existing design systems are adapted for a particular use.

Developmental design
Here we start with an existing design but finally a modified design is obtained. A new model of a car is a typical example of a developmental design .

New design
This type of design is an entirely new one but based on existing scientific principles. No scientific invention is involved but requires creative thinking to solve a problem. Examples of this type of design may include designing a small vehicle for transportation of men and material on board a ship or in a desert. Some research activity may be necessary.

1.1.4 Types of design based on methods

Rational design

This is based on determining the stresses and strains of components and thereby deciding their dimensions.

Empirical design

This is based on empirical formulae which in turn is based on experience and experiments. For example, when we tighten a nut on a bolt the force exerted or the stresses induced cannot be determined exactly but experience shows that the tightening force may be given by P=284d where, d is the bolt diameter in mm and P is the applied force in kg. There is no mathematical backing of this equation but it is based on observations and experience.

Industrial design

These are based on industrial considerations and norms viz. market survey, external look, production facilities, low cost, use of existing standard products.

1.1.5 Factors to be considered in machine design

There are many factors to be considered while attacking a design problem. In many cases these are a common sense approach to solving a problem. Some of these factors are as follows:

(a) Device or mechanism which to be used. This would decide the relative arrangement of the constituent elements.

(b) Material which to be used

(c) Forces on the elements

(d) Size, shape weight and space requirements.

(e) The method of manufacturing the components and their assembly.

(f) How will it operate?

(g) Reliability and safety aspects of the machin

(h) Inspectibility of the product

(i) Maintenance, cost and aesthetics of the designed product.

Explanations:

(a) Device or mechanism to be used- This is best judged by understanding the problem thoroughly. Sometimes a particular function can be achieved by a number of means or by using different mechanisms and the designer has to decide which one is most effective under the circumstances. A rough design or layout diagram may be made to crystallize the thoughts regarding the relative arrangement of the elements.

(b) Material- This is a very important aspect of any design. A wrong choice of material may lead to failure, over or undersized product or expensive items. The choice of materials is thus dependent on suitable properties of the material for each component, their suitability of fabrication or manufacture and the cost.

(c) Load- The external loads cause internal stresses in the elements and these stresses must be determined accurately since these will be used in determining the component size.

Loading may be classified:

1) Energy transmission by a machine member.
2) Dead weight.
3) Inertial forces.
4) Thermal effects.
5) Frictional forces.

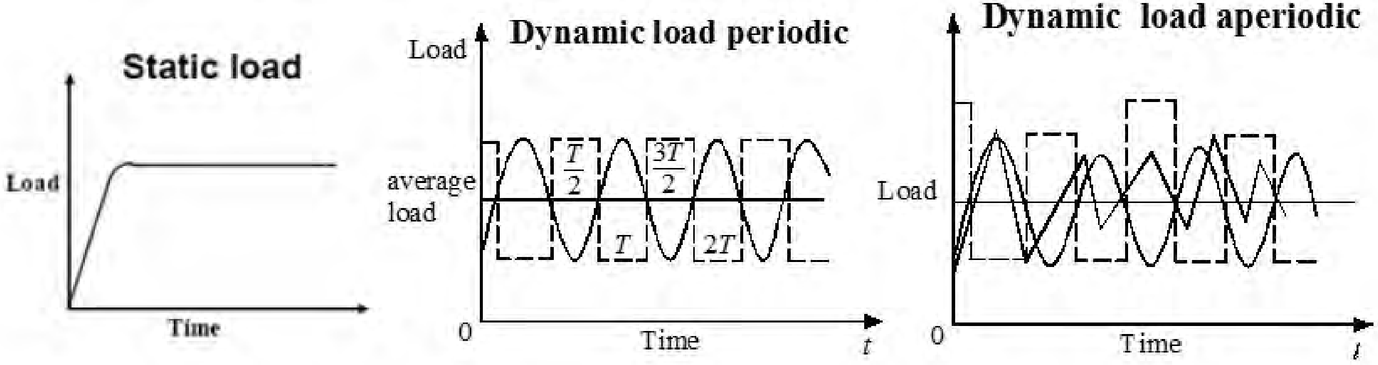
In other ways loads may be classified as:

1) Static load- Does not change in magnitude and direction and normally increases gradually to a steady value.

2) Dynamic load- a) changes in magnitude- for e.g. traffic of varying weight passing a bridge.

b) changes in direction- for e.g. load on piston rod of a double acting cylinder.

The nature of these loads are shown in figure-1.1.3.1.



1.1.5.1 The nature of static and dynamic load

Vibration and shock loading are types of dynamic loading.

(d) Size, shape, space requirements and weight- Preliminary analysis would give an approximate size but if a standard element is to be chosen, the next larger size must be taken. Shapes of standard elements are known but for non-standard element, shapes and space requirements must depend on available space in a particular machine assembly. A scale layout drawing is often useful to arrive at an initial shape and size. Weight is important depending on application. For example, an aircraft must always be made light. This means that the material chosen must have the required strength yet it must be light. Similar arguments apply to choice of material for ships and there too light materials are to be chosen. Portable equipment must be made light.

(e) Manufacture

Care must always be taken to ensure that the designed elements may be manufactured with ease, within the available facilities and at low cost.

(f) How will it operate

In the final stage of the design a designer must ensure that the machine may be operated with ease. In many power operated machines it is simply a matter of pressing a knob or switch to start the machine. However in many other cases, a sequence of operations is to be specified. This sequence must not be complicated and the operations should not require excessive force. Consider the starting, accelerating and stopping a scooter or a car. With time tested design considerations, the sequences have been made user-friendly and as in any other product, these products too go through continuous innovation and development.

(g) Reliability and safety

1. A designed machine should work effectively and reliably. The probability that an element or a machine will not fail in use is called reliability. Reliability lies between $0 \le R < 1$. To ensure this, every detail should be examined. Possible overloading, wear of elements, excessive heat generation and other such detrimental factors must be avoided. There is no single answer for this but an overall safe design approach and care at every stage of design would result in a reliable machine.

2. Safety has become a matter of paramount importance these days in design. Machines must be designed to serve mankind, not to harm it. Industrial regulations ensure that the manufacturer is liable for any damage or harm arising out of a defective product. Use of a factor of safety only in design does not ensure its overall reliability.

(i) Maintenance, cost and aesthetics

Maintenance and safety are often interlinked. Good maintenance ensures good running condition of machinery. Often a regular maintenance schedule is maintained and a thorough check up of moving and loaded parts is carried out to avoid catastrophic failures. Low friction and wear is maintained by proper lubrication. This is a major aspect of design since wherever there are moving parts, friction and wear are inevitable. High friction leads to increased loss of energy. Wear of machine parts leads to loss of material and premature failure.

Cost and aesthetics are essential considerations for product design. Cost is essentially related to the choice of materials which in turn depends on the stresses developed in a given condition. Although in many cases aesthetic considerations are not essential aspects of machine design, ergonomic aspects must be taken into considerations.

1.1.6 Problems with Answers

Q.1: Define machine design.

A.1: A machine is a combination of several machine elements arranged to work together as a whole to accomplish specific purposes. Machine design involves designing the elements and arranging them optimally to obtain some useful work.

Q.2: What is an adaptive design?

A.2: Adaptive design is based on an existing design adapted for a new system or application, for example, design of a new model of passenger car.

Q.3: Suggest briefly the steps to be followed by a designer.

A.3: Machine design requires a thorough knowledge of engineering science in its totality along with a clear decision making capability. Every designer follows his own methodology based on experience and analysis. However, the main steps to be followed in general are :

- Define the problem.
- Make preliminary design decisions.
- Make design sketches.
- Carry out design analysis and optimization.
- Design the elements for strength and durability.
- Prepare documentations to be followed for manufacture.

Q.4: Discuss 'factor of safety ' in view of the reliability in machine design.

A.4: Reliability of a designed machine is concerned with the proper functioning of the elements and the machine as a whole so that the machine does not fail in use within its designed life. There is no single answer to this and an overall safe design approach at every stage of the design is needed. Use of factor of safety in designing the elements is to optimize the design to avoid over-design for reliability.

1.1.7 Summary of this Lecture

The lesson essentially discusses the basic concept of design in general leading to the concept of machine design which involves primarily designing the elements. Different types of design and the factors to be considered have been discussed in detail.
As you well know modern machines are complex systems which consist of mechanical, electrical, electronic, control parts. There **fore dixiplane** machine design in general case is the science about complex systems. Mechatronics is science about modern machins. We will be study only design of mechanical parts of machins.

Lecture
Theme 1

1.2 Classification of materials for machine design

1.2.1 Introduction

Choice of materials for a machine element depends very much on its properties,cost, availability and such other factors. It is therefore important to have some idea of the common engineering materials and their properties before learning the details of design procedure. Common engineering materials are normally classified as **metals** and **nonmetals**. Metals may conveniently be divided into **ferrous** and **non-ferrous** metals. Important ferrous metals for the present purpose are:

(1) cast iron (2) wrought iron (3) steel.

Some of the important **non-ferrous** metals used in engineering design are:

(a) Light metal group such as aluminium and its alloys, magnesium and manganese alloys.

(b) Copper based alloys such as brass (Cu-Zn), bronze (Cu-Sn).

(c) White metal group such as nickel, silver, white bearing metals eg. SnSb7Cu3, Sn60Sb11Pb, zinc etc.

Cast iron, wrought iron and steel will now be discussed under separate headings.

1.2.2 Ferrous materials

1. Cast iron- It is an alloy of iron, carbon and silicon and it is hard and brittle. Carbon content may be within 1.7% to 3% and carbon may be present as free carbon or iron carbide Fe3C. In general the types of cast iron are (a) grey cast iron and (b) white cast iron (c) malleable cast iron (d) spheroidal or nodular cast iron (e) austenitic cast iron (f) abrasion resistant cast iron.

1.a Grey cast iron- Carbon here is mainly in the form of graphite. This type of cast iron is inexpensive and has high compressive strength. Graphite is an excellent solid lubricant and this makes it easily machinable but brittle.

1.b White cast iron- In these cast irons carbon is present in the form of iron carbide (Fe_3C) which is hard and brittle. The presence of iron carbide increases hardness and makes it difficult to machine. Consequently these cast irons are abrasion resistant.

1.c Malleable cast iron- These are white cast irons rendered malleable by annealing. These are tougher than grey cast iron and they can be twisted or bent without fracture. They have excellent machining properties and are inexpensive.

1.d Spheroidal or nodular graphite cast iron- In these cast irons graphite is present in the form of spheres or nodules. They have high tensile strength and good elongation properties. They are designated as, for example, SG50/7, SG80/2 etc where the first number gives the tensile strength in MPa and the second number indicates percentage elongation.

1.e Austenitic cast iron- Depending on the form of graphite present these cast iron can be classified broadly under two headings

Austenitic spheroidal or nodular graphite iron designated, for example, ASGNi20Cr2. These are alloy cast irons and they contain small percentages of silicon, manganese, sulphur, phosphorus etc. They may be produced by adding alloying elements viz. nickel, chromium, molybdenum, copper and manganese in sufficient quantities. These elements give more strength and improved properties. They are used for making automobile parts such as cylinders, pistons, piston rings, brake drums etc.

1.f Abrasion resistant cast iron- These are alloy cast iron and the alloying elements render abrasion resistance.

2. Wrought iron- This is a very pure iron where the iron content is of the order of 99.5%. It is produced by re-melting pig iron and some small amount of silicon, sulphur, or phosphorus may be present. It is tough, malleable and ductile and can easily be forged or welded. It cannot however take sudden shock. Chains, crane hooks, railway couplings and such other components may be made of this
iron.

3. Steel- This is by far the most important engineering material and there is an enormous variety of steel to meet the wide variety of engineering requirements. The present note is an introductory discussion of a vast topic.

3.a Plain carbon steel- The properties of plain carbon steel depend mainly on the carbon percentages and other alloying elements are not usually present in more than 0.5 to 1% such as 0.5% Si or 1% Mn etc.

Following categorization of these steels is sometimes made for convenience.

Detailed properties of these steels may be found in any standard handbook but in general higher carbon percentage indicates higher strength.

3.b Alloy steel- these are steels in which elements other than carbon are added in sufficient quantities to impart desired properties, such as wear resistance, corrosion resistance, electric or magnetic properties. Chief alloying elements added are usually nickel for strength and toughness, chromium for hardness and strength, tungsten for hardness at elevated temperature, vanadium for tensile strength, manganese for high strength in hot rolled and heat treated condition, silicon for high elastic limit, cobalt for hardness and molybdenum for extra tensile strength.

1.2.3 Specifications

A number of systems for grading steel exist in different countries.

The American system is usually termed as SAE (Society of Automobile Engineers) or AISI (American Iron and Steel Industries) systems. For an example, a steel denoted as SAE 1020 indicates 0.2% carbon and 13% tungsten. In this system the first digit indicates the chief alloying material. Digits 1,2,3,4 and 7 refer to carbon, nickel, nickel/chromium, molybdenum and tungsten respectively. More details may be seen in the standards. The second digit or second and third digits give the percentage of the main alloying element and the last two digits indicate the carbon percentage.

1.2.4 Non-ferrous metals

Metals containing elements other than iron as their chief constituents are usually referred to as non-ferrous metals. There is a wide variety of non-metals in practice. However, only a few exemplary ones are discussed below:

4.a Aluminium- This is the white metal produced from Alumina. In its pure state it is weak and soft but addition of small amounts of Cu, Mn, Si and Magnesium makes it hard and strong. It is also corrosion resistant, low weight and non-toxic.

4.b Duralumin- This is an alloy of 4% Cu, 0.5% Mn, 0.5% Mg and aluminium. It is widely used in automobile and aircraft components.

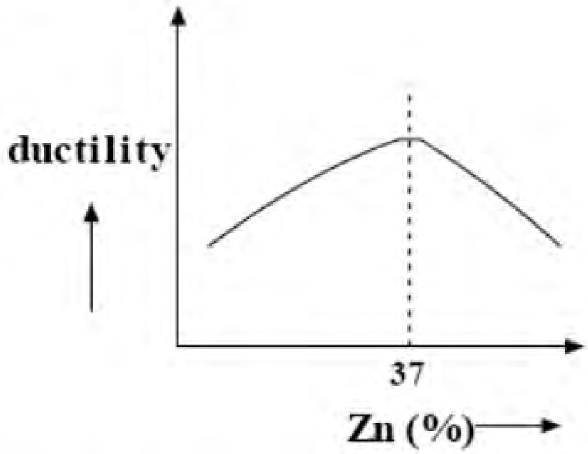
4.c Y-alloy- This is an alloy of 4% Cu, 1.5% Mn, 2% Ni, 6% Si, Mg, Fe and the rest is Al. It gives large strength at high temperature. It is used for aircraft engine parts such as cylinder heads, piston etc.

4.d Magnalium- This is an aluminium alloy with 2 to 10 % magnesium. It also contains 1.75% Cu. Due to its light weight and good strength it is used for aircraft and automobile components.

4.e Copper alloys

Copper is one of the most widely used non-ferrous metals in industry. It is soft, malleable and ductile and is a good conductor of heat and electricity. The following two important copper alloys are widely used in practice:

4.f Brass (Cu-Zn alloy)- It is fundamentally a binary alloy with Zn upto 50% . As Zn percentage increases, ductility increases upto ~37% of Zn beyond which the ductility falls. This is shown in figure-1.2.4.1. Small amount of other elements viz. lead or tin imparts other properties to brass. Lead gives good machining quality and tin imparts strength. Brass is highly corrosion resistant, easily machinable and therefore a good bearing material.



***1.2.4.1** - Variation of ductility of brass with percentage of zinc.*

4.g Bronze (Cu-Sn alloy)-This is mainly a copper-tin alloy where tin percentage may vary between 5 to 25. It provides hardness but tin content also oxidizes resulting in brittleness. Deoxidizers such as Zn may be added. Gun metal is one such alloy where 2% Zn is added as deoxidizing agent and typical compositions are 88% Cu, 10% Sn, 2% Zn. This is suitable for working in cold state. It was originally made for casting guns but used now for boiler fittings, bushes, glands and other such uses.

1.2.5 Non-metals

Non-metallic materials are also used in engineering practice due to principally their low cost, flexibility and resistance to heat and electricity. Though there are many suitable non-metals, the following are important few from design point of view:

5.a Timber- This is a relatively low cost material and a bad conductor of heat and electricity. It has also good elastic and frictional properties and is widely used in foundry patterns and as water lubricated bearings.

5.b Leather- This is widely used in engineering for its flexibility and wear resistance. It is widely used for belt drives, washers and such other applications.

5.c Rubber- It has high bulk modulus and is used for drive elements, sealing, vibration isolation and similar applications.

5.d Plastics

These are synthetic materials which can be moulded into desired shapes under pressure with or without application of heat. These are now extensively used in various industrial applications for their corrosion resistance, dimensional stability and relatively low cost.

There are two main types of plastics:

5.d.1 Thermosetting plastics- Thermosetting plastics are formed under heat and pressure. It initially softens and with increasing heat and pressure, polymerisation takes place. This results in hardening of the material. These plastics cannot be deformed or remoulded again under heat and pressure. Some examples of thermosetting plastics are phenol formaldehyde (Bakelite), phenol-furfural (Durite), epoxy resins, phenolic resins etc.

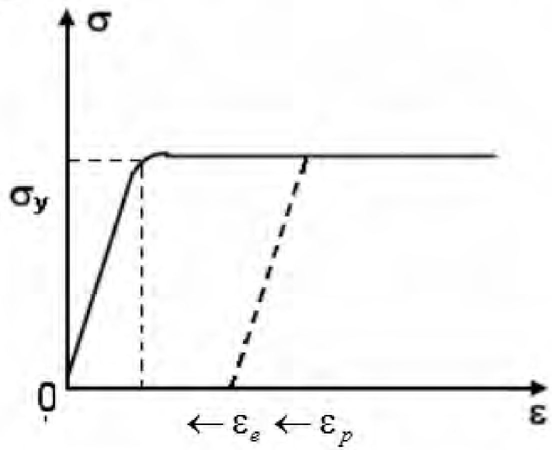
5.d.2 Thermoplastics- Thermoplastics do not become hard with the application of heat and pressure and no chemical change takes place. They remain soft at elevated temperatures until they are hardened by cooling. These can be re-melted and remoulded by application of heat and pressure. Some examples of thermoplastics are cellulose nitrate (celluloid), polythene, polyvinyl acetate, polyvinyl chloride (PVC) etc.

1.2.6 Mechanical properties of common engineering materials

The important properties from design point of view are:

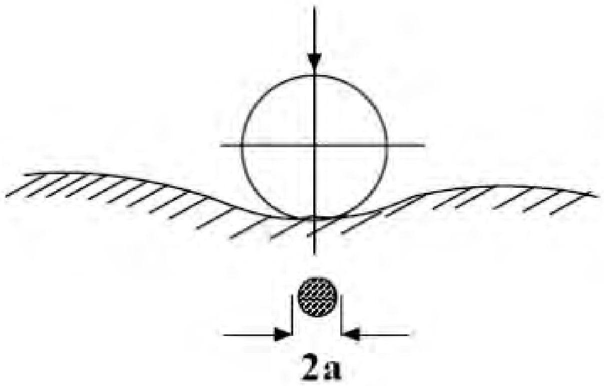
(a) Elasticity- This is the property of a material to regain its original shape after deformation when the external forces are removed. All materials are plastic to some extent but the degree varies, for example, both mild steel and rubber are elastic materials but steel is more elastic than rubber.

(b) Plasticity- This is associated with the permanent deformation of material when the stress level exceeds the yield point. Under plastic conditions materials ideally deform without any increase in stress. A typical stressstrain diagram for an elastic-perfectly plastic material is shown in the figure-1.2.6.1. Mises-Henky criterion gives a good starting point for plasticity analysis. The criterion is given as $\sigma = \sigma_y$ where the stress at the tensile yield point. For $\sigma < \sigma_y$ deformation ε_e of material is elastic, for $\sigma \geq \sigma_y$ deformation ε_p of material is plastic



1.2.6.1 a) - *Stress-strain diagram of an elastic-perfectly plastic material*

A typical example of plastic flow is the indentation test where a spherical ball is pressed in a semi-infinite body where 2a is the contact diameter. In a simplified model we may write that if $\frac{P}{\pi a^2} > p_m$ plastic flow occurs where, p_m is the flow pressure. This is also shown in figure 1.2.6.1 b.



1.2.6.1 b) *- The plastic indentation.*

(c) Hardness- Property of the material that enables it to resist permanent deformation, penetration, indentation etc. Size of indentations by various types of indenters are the measure of hardness e.g. Brinnel hardness test, Rockwell hardness test, Vickers hardness (diamond pyramid) test. These tests give hardness numbers which are related to yield pressure (MPa).

(d) Ductility- This is the property of the material that enables it to be drawn out or elongated to an appreciable extent before rupture occurs. The percentage elongation or percentage reduction in area before rupture of a test specimen is the measure of ductility. Normally if percentage elongation exceeds 15% the material is ductile and if it is less than 5% the material is brittle. Lead, copper, aluminium, mild steel are typical ductile materials.

(e) Malleability- It is a special case of ductility where it can be rolled into thin sheets but it is not necessary to be so strong. Lead, soft steel, wrought iron, copper and aluminium are some materials in order of diminishing malleability.

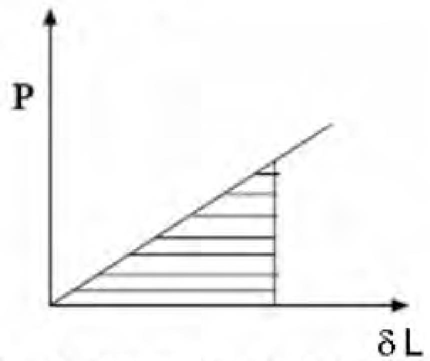
(f) Brittleness- This is opposite to ductility. Brittle materials show little deformation before fracture and failure occur suddenly without any warning. Normally if the elongation is less than 5% the material is considered to be brittle. E.g. cast iron, glass, ceramics are typical brittle materials.

(g) Resilience- This is the property of the material that enables it to resist shock and impact by storing energy. The measure of resilience is the strain energy absorbed per unit volume. For a rod of length L subjected to tensile load P, a linear load-deflection plot is shown in figure-1.2.6.2.

Strain energy (energy stored)

$W = \frac{1}{2} P\delta\delta = \frac{1}{2}\frac{P}{A}\frac{\delta L}{L} AL = \frac{1}{2}\sigma\varepsilon V$, $\sigma = \frac{P}{A}$, $\varepsilon = \frac{\delta L}{L}$, $V = AL$ where σ is stress, ε strain, V is volume, A is area, L is length.

Strain energy/unit volume $W = \frac{1}{2}\sigma\varepsilon$



1.2.6.2 *- A linear load-deflection plot.*

(h) Toughness- This is the property which enables a material to be twisted, bent or stretched under impact load or high stress before rupture. It may be considered to be the ability of the material to absorb energy in the plastic zone. The measure of toughness is the amount of energy absorbed after being stressed upto the point of fracture.

(i) Creep- When a member is subjected to a constant load over a long period of time it undergoes a slow permanent deformation and this is termed as "creep". This is dependent on temperature. Usually at elevated temperatures creep is high.

1.2.7 Problems with Answers

Q.1: Classify common engineering materials.
A.1: Common engineering materials can be broadly classified into metals and non-metals. Metals include ferrous and non-ferrous metal and the nonmetals include timber, leather, rubber and a large variety of polymers. Among the ferrous metals different varieties of cast iron, wrought iron and alloy steels are extensively used in industry. There are also a large variety of timber, leather and polymers that are used in industry.
Q.2: What are the advantages of malleable cast iron over white or grey cast iron?
A.2: Malleable cast iron are tougher than grey or white cast iron and can be twisted or bent without fracture. They also have excellent machining properties and are relatively inexpensive.
Q.3: A standard alloy steel used for making engineering components is 20Cr18 Ni2. State the composition of the steel.

Q.5: Name two important copper alloys and give their typical compositions.

A.5: Two most important copper alloys are bronze and brass. Bronze is a Cu-Sn alloy with the typical composition of 88% Cu, 10% Sn and 2% Zn. Brass is a Cu-Zn alloy with the typical composition of red brass of 85% Cu , 15% Zn.

Q.6: List at least five important non-metals commonly used in machine design.

A.6: Some important non-metals for industrial uses are: Timber, leather, rubber, bakelite, nylon, polythene, polytetraflutoethylene (PTFE).

Q.7: State atleast 5 important mechanical properties of materials to be considered in machine design.

A.7: Some important properties of materials to be considered in design are: Elastic limit, yield and ultimate strength, hardness and toughness.

Q.8: Define resilience and discuss its implication in the choice of materials in machine design.

A.8: Resilience is defined as the property of a material that enables it to resist shock and impact. The property is important in choosing materials for machine parts subjected to shock loading, such as, fasteners, springs etc.

1.2.8 Summary of this Lecture

In this lecture the properties and uses of different types of metals and nonmetals, generally used in machine design, are discussed. Primarily ferrous and non-ferrous metals and some non-metals are discussed. Mechanical properties of some common engineering materials are also discussed briefly.

Lecture
Theme 1

1.3. Limit and Tolerance, Fit system and standard limit

1.3.1 Design and Manufacturing

A machine element, after design, requires to be manufactured to give it a shape of a product. Therefore, in addition to standard design practices like, selection of proper material, ensuring proper strength and dimension to guard against failure, a designer should have knowledge of basic manufacturing aspects.

1.3.2 Limits

Fig. 1.3.1 explains the terminologies used in defining **tolerance** and **limit**. The zero line, shown in the figure, is the basic size or the nominal size. The definition of the terminologies is given below. For the convenience, shaft and hole are chosen to be two mating components.

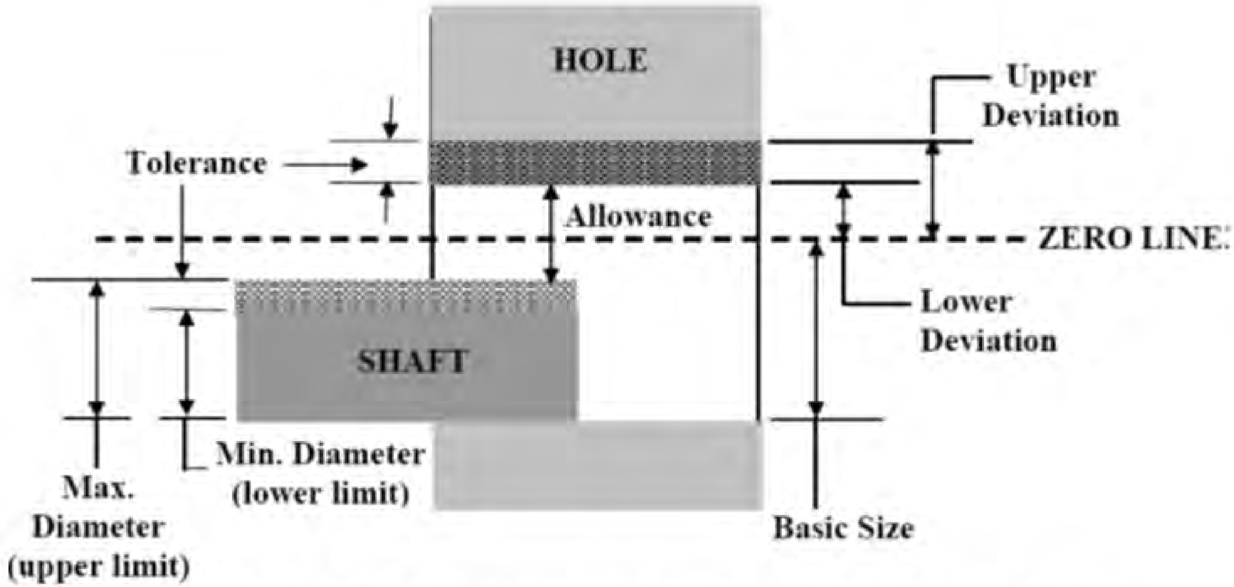


Fig. 1.3.1 Interrelationship between tolerances and limits

Tolerance

Tolerance is the difference between maximum and minimum dimensions of a component, ie, between upper limit and lower limit. Depending on the type of application, the permissible variation of dimension is set as per available standard grades (see Fig. 1.3.1).

Tolerance is of two types, **bilateral and unilateral**. When tolerance is present on both sides of nominal size, it is termed as bilateral; unilateral has tolerance only on one side. The Fig.1.3.2 shows the types of tolerance. 50^{0}_{-y}, 50^{+y}_{0} and 50^{+y}_{-y} is a typical example of specifying tolerance for a shaft of nominal diameter of 50mm. First two values denote unilateral tolerance and the third value denotes bilateral tolerance. Values of the tolerance are given as x and y respectively.

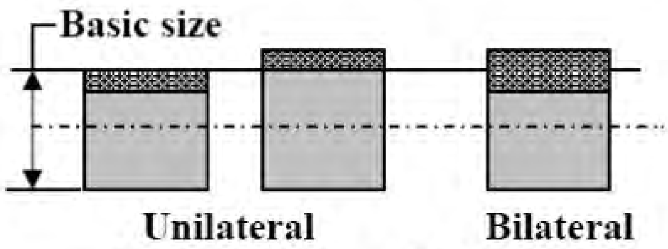


Fig. 1.3.2 Types of tolerance

Allowance

It is the difference of dimension between two mating parts.

Upper deviation

It is the difference of dimension between the maximum possible size of the component and its nominal size.

Lower deviation

Similarly, it is the difference of dimension between the minimum possible size of the component and its nominal size.

Fundamental deviation

It defines the location of the tolerance zone with respect to the nominal size. For that matter, either of the deviations may be considered.

1.3.3 Fit System

We have learnt above that a machine part when manufactured has a specified tolerance. Therefore, when two mating parts fit with each other, the nature of fit is dependent on the limits of tolerances and fundamental deviations of the mating parts. The nature of assembly of two mating parts is defined by three types of fit system, Clearance Fit, Transition Fit and Interference Fit. The fit system is shown schematically in Fig.1.3.3.

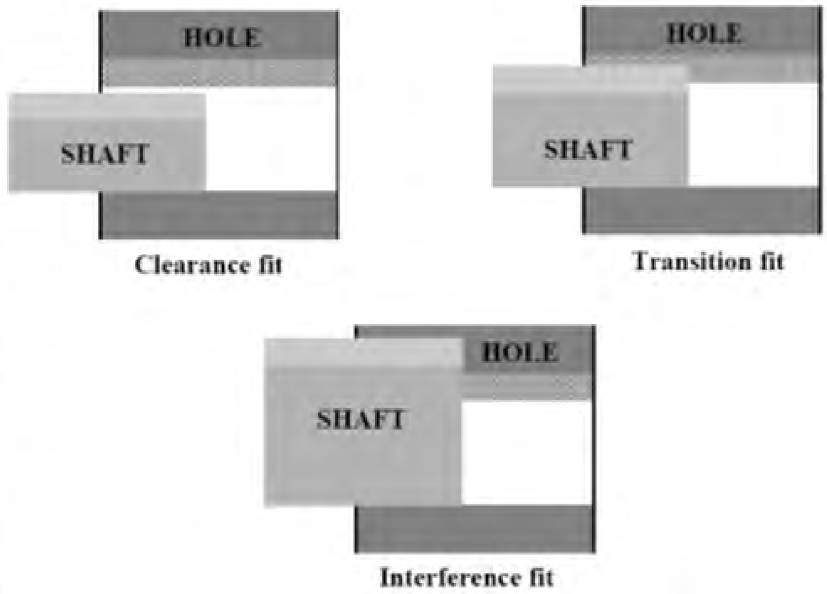


Fig. 1.3.3 Schematic view of Fit system

There are two ways of representing a system. One is the hole basis and the other is the shaft basis. In the hole basis system the dimension of the hole is considered to be the datum, whereas, in the shaft basis system dimension of the shaft is considered to be the datum. The holes are normally made by drilling, followed by reaming. Therefore, the dimension of a hole is fixed due to the nature of the tool used. On the contrary, the dimension of a shaft is easily controllable by standard manufacturing processes. For this reason, the hole basis system is much more popular than the shaft basis system. Here, we shall discuss fit system on hole basis.

Clearance Fit

In this type of fit, the shaft of largest possible diameter can also be fitted easily even in the hole of smallest possible diameter.

Transition Fit

In this case, there will be a clearance between the minimum dimension of the shaft and the minimum dimension of the hole. If we look at the figure carefully, then it is observed that if the shaft dimension is maximum and the hole dimension is minimum then an overlap will result and this creates a certain amount of tightness in the fitting of the shaft inside the hole. Hence, transition fit may have either clearance or overlap in the fit.

Interference Fit

In this case, no matter whatever may be the tolerance level in shaft and the hole, there is always a overlapping of the matting parts. This is known as interference fit. Interference fit is a form of a tight fit.

1.3.4 Standard limit and fit system

Fig. 1.3.4 shows the schematic view of a **standard limit and fit system**. In this figure tolerance is denoted as IT and it has 18 grades; greater the number, more is the tolerance limit. The fundamental deviations for the hole are denoted by capital letters from A and ZC, having altogether 25 divisions. Similarly, the fundamental deviations for the shaft is denoted by small letters from a to zc.

Here H or h is a typical case, where the fundamental deviation is zero having an unilateral tolerance of a specified IT grade.

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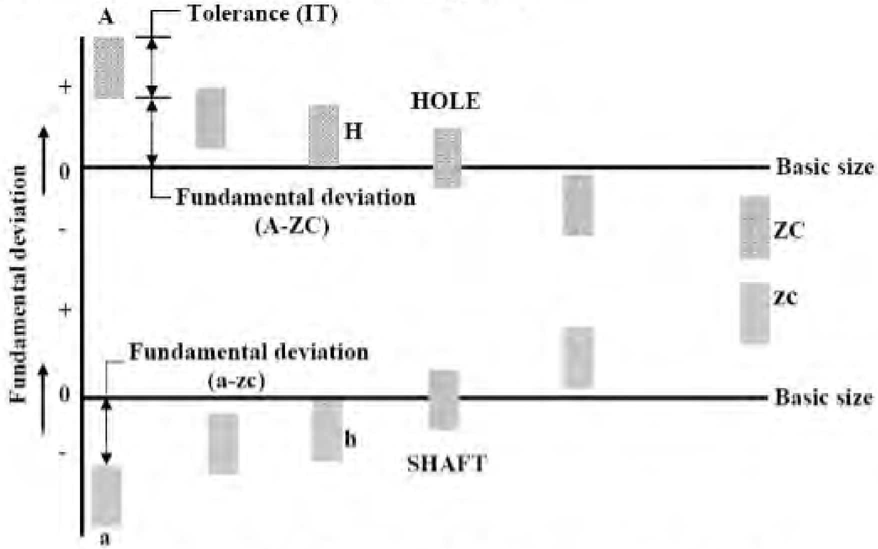


Fig. 1.3.4 Schematic view of standard limit and fit system

Therefore in standard limits and fit system we find that,

Standard tolerances
18 grades: IT01 ,IT0 and IT1-1T16

Fundamental deviations
25 types: A- ZC (For holes)
a- zc (For shafts)

The values of standard tolerances and fundamental deviations can be obtained by consulting design **hand book**. It is to be noted that the choice of tolerance grade is related to the type of manufacturing process; for example, attainable tolerance grade for lapping process is lower compared to plain milling. Similarly, choice of fundamental deviation largely depends on the nature of fit, running fit or tight fit etc. Manufacturing processes involving lower tolerance grade are generally costly. Hence the designer has to keep in view the manufacturing processes to make the design effective and inexpensive.

Sample designation of limit and fit, 50H6/g5.

The designation means that the nominal size of the hole and the shaft is 50 mm. H is the nature of fit for the hole basis system and its fundamental deviation is zero. The tolerance grade for making the hole is IT6. Similarly, the shaft has the fit type g, for which the fundamental deviation is negative, that is, its dimension is lower than the nominal size, and tolerance grade is IT5. The approximate zones for fit are shown in Fig. 1.3.5.

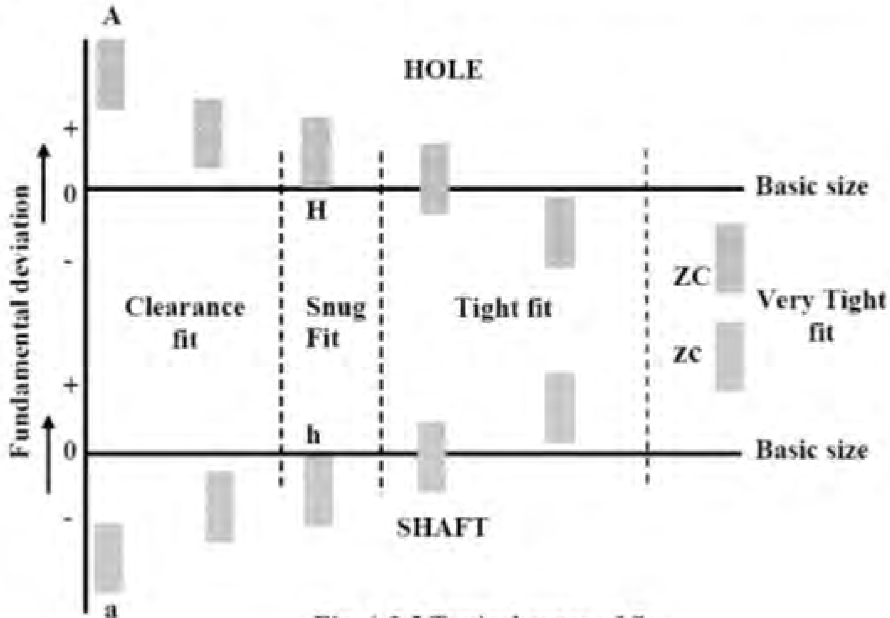


Fig. 1.3.5 Typical zones of fit

Lecture
Theme 1

1.3. The types of common manufacturing processes

1.3.5 Preferred numbers

A designed product needs standardization. It means that some of its important specified parameter should be common in nature. For example, the sizes of the ingots available in the market have standard sizes. A manufacturer does not produce ingots of sizes of his wish, he follows a definite pattern and for that matter designer can choose the dimensions from those standard available sizes. Motor speed, engine power of a tractor, machine tool speed and feed, all follow a definite pattern or series. This also helps in interchangeability of products. It has been observed that if the sizes are put in the form of geometric progression, then wide ranges are covered with a definite sequence.

These numbers are called preferred numbers having common ratios as,

$$\sqrt[5]{10} \approx 1.58,\ \sqrt[10]{10} \approx 1.26,\ \sqrt[20]{10} \approx 1.12 \ \text{ and } \ \sqrt[40]{10} \approx 1.06$$

Depending on the common ratio, four basic series are formed; these are R5, R10 , R20 and R40 . These are named as Renard series. Many other derived series are formed by multiplying or dividing the basic series by 10, 100 etc.

Typical values of the common ratio for four basic G.P. series are given below.

Preferred Numbers

R5:	$\sqrt[5]{10}$	***1.58***: 1.0, 1.6, 2.5, 4.0,…
R10:	$\sqrt[10]{10}$	***1.26***: 1.0, 1.25, 1.6, 2.0,…
R20:	$\sqrt[20]{10}$	***1.12***: 1.0, 1.12, 1.25, 1.4,…
R40:	$\sqrt[40]{10}$	***1.06***: 1.0, 1.06, 1.12, 1.18,..

Few examples

R_{10} , R_{20} and R_{40} : Thickness of sheet metals, wire diameter

R_5 , R_{10} , R_{20} : Speed layout in a machine tool (R_{10} : 1000, 1250,1600, 2000)

R_{20} or R_{40} : Machine tool feed

R_5 : Capacities of hydraulic cylinder

1.3.6 Common manufacturing processes

The types of common manufacturing processes are given below in the Fig.1.3.6.

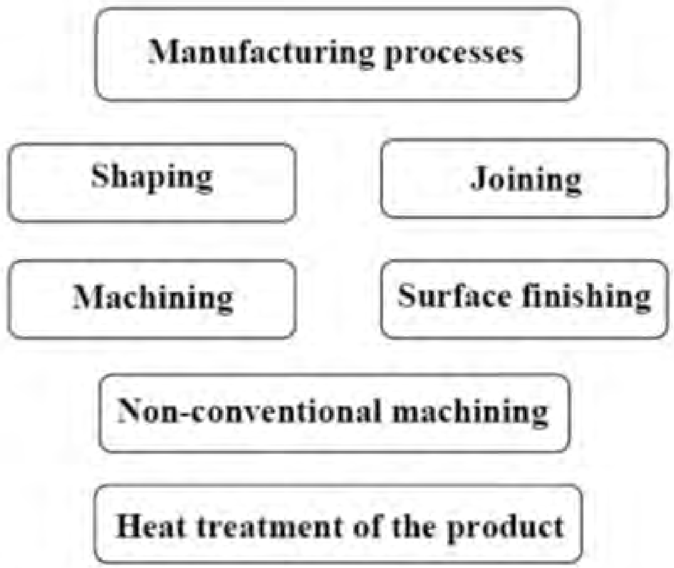


Fig. 1.3.6 Common manufacturing processes

The types of shaping processes are given below in the Fig.1.3.7.

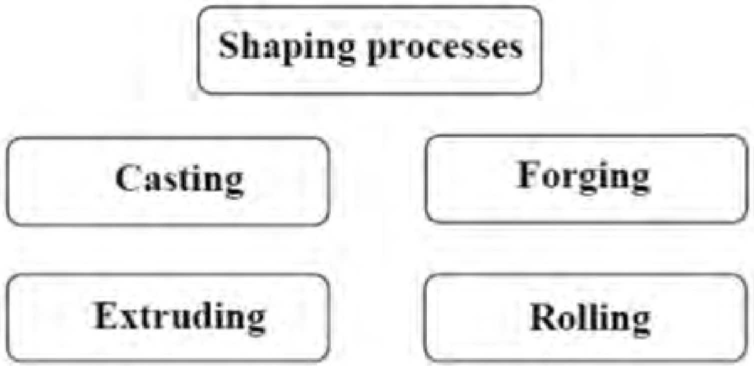


Fig. 1.3.7 Shaping processes

Following are the type of machining processes, shown in Fig.1.3.8.

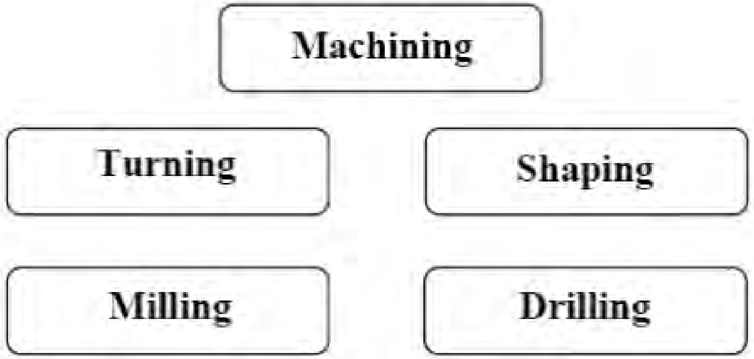


Fig. 1.3.8 Machining processes

Various joining processes are shown in Fig.1.3.9.

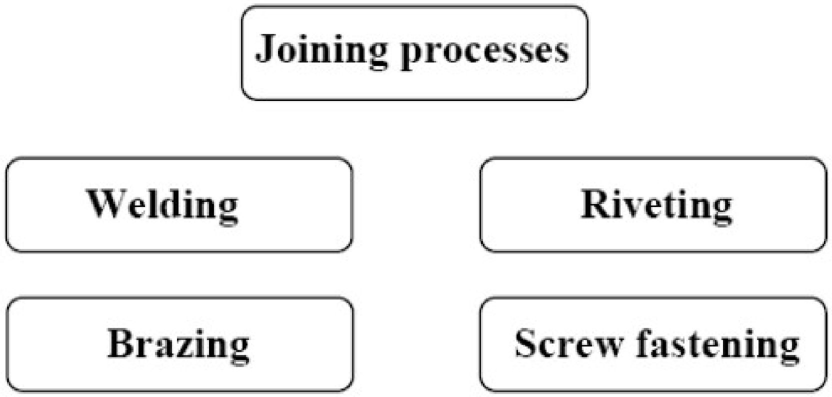


Fig. 1.3.9 Joining processes

The surface finishing processes are given below (Fig.1.3.10),

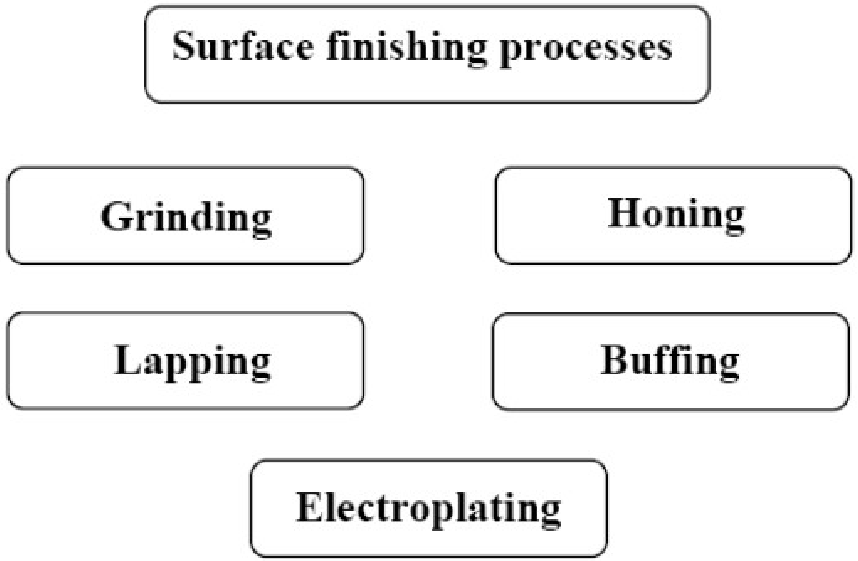


Fig. 1.3.10 Surface finishing processes

The non-conventional machining processes are as follows (Fig.1.3.11),

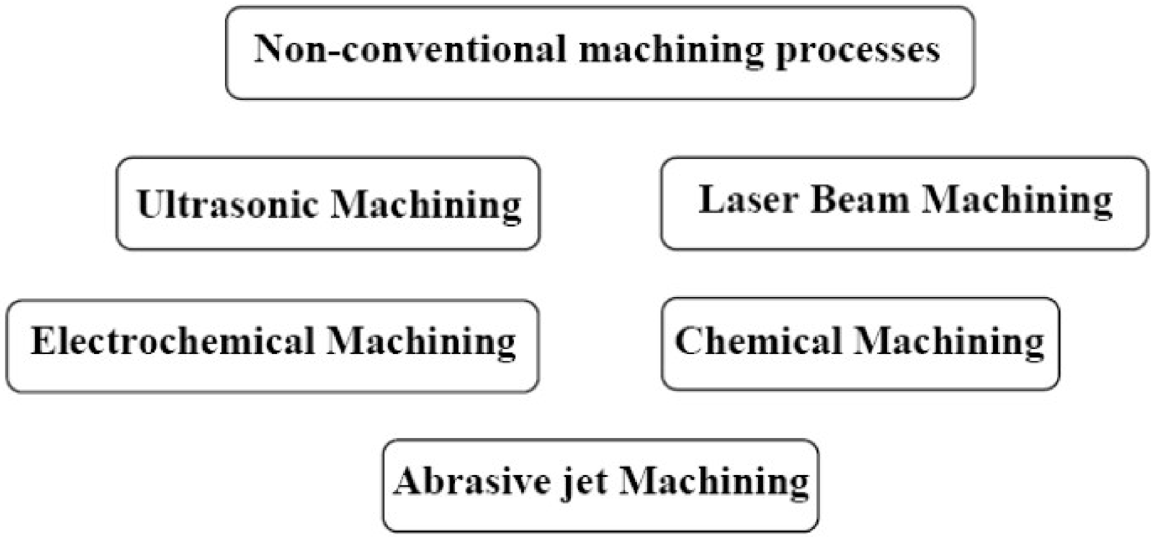


Fig. 1.3.11 Non conventional machining processes

Problems and answers

Q1. What is meant by tolerance? How many types of tolerance is there?

A1. Tolerance is the difference between maximum and minimum dimensions of a component, ie, between upper limit and lower limit. Depending on the type of application, the permissible variation of dimension is set as per available standard grades. Tolerance is of two types, bilateral and unilateral. When tolerance is present on both sides of nominal size, it is termed as bilateral; unilateral has tolerance only on one side.

Q2. What are the types fit? Describe the differences.

A2. The nature of assembly of two mating parts is defined by three types of fit system, Clearance Fit, Transition Fit and Interference Fit.

Clearance Fit: In this type of fit, the shaft of largest possible diameter can be fitted easily in the hole of smallest possible diameter.

Interference Fit : In this type of fit, irrespective of tolerance grade there is always a overlapping of the matting parts.

Transition Fit: In this case, a clearance is present between the minimum dimension of the shaft and the minimum dimension of the hole. However, the fit is tight, if the shaft dimension is maximum and the hole dimension is minimum. Hence, transition fit have both the characteristics of clearance fit and interference fit.

Q3. What are preferred numbers?

A3. Preferred numbers are the numbers belonging to four categories of geometric progression series, called basic series, having common ratio of,

$$\sqrt[5]{10} \approx 1.58,\ \sqrt[10]{10} \approx 1.26,\ \sqrt[20]{10} \approx 1.12 \text{ and } \sqrt[40]{10} \approx 1.06$$

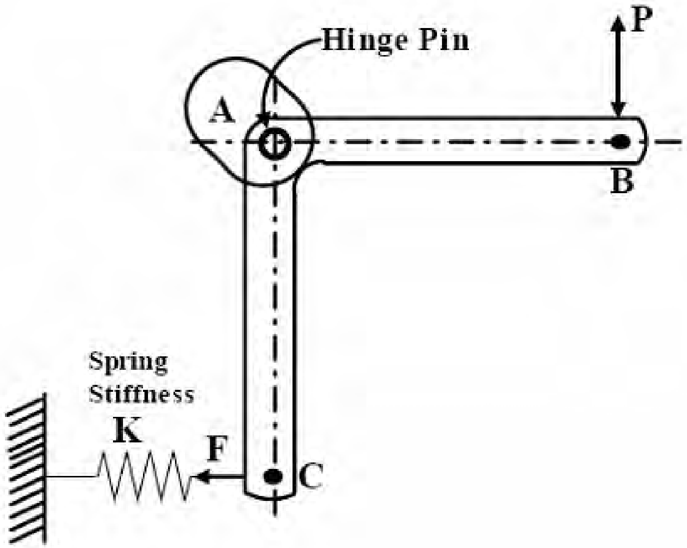
Preferred numbers of derived series are formed by multiplying or dividing the basic series by 10, 100 etc. These numbers are used to build-up or manufacture a product range. The range of operational speeds of a machine or the range of powers of a typical machine may be also as per a series of preferred numbers.

Lecture
Theme 2
Stresses in machine elements

2.1. Simple stresses

2.1.1 Introduction

Stresses are developed in machine elements due to applied load and machine design involves ensuring that the elements can sustain the induced stresses without yielding. Consider a simple lever as shown in figure-2.1.1.1:



2.1.1.1 *- A simple lever subjected to forces at the ends.*

A proper design of the spring would ensure the necessary force P at the lever end B. The stresses developed in sections AB and AC would decide the optimum cross-section of the lever provided that the material has been chosen correctly.

The design of the hinge depends on the stresses developed due to the reaction forces at A. A closer look at the arrangement would reveal that the following types of stresses are developed in different elements:

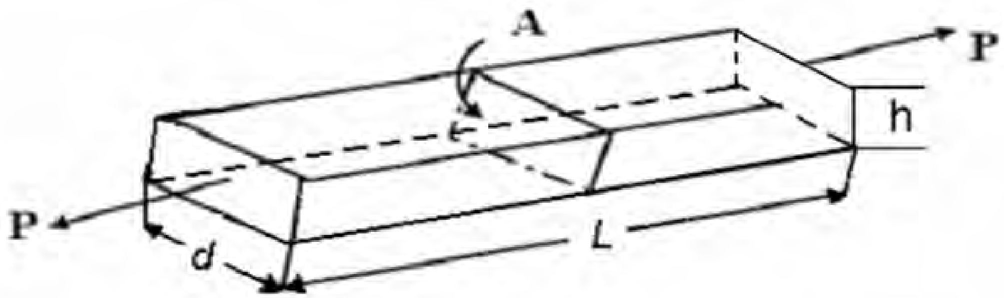
Lever arms AB and AC	-	Bending stresses
Hinge pin	-	Shear and bearing stresses.
Spring	-	Shear stress.

It is therefore important to understand the implications of these and other simple stresses. Although it is more fundamental to consider the state of stress at a point and stress distribution, in elementary design analysis simple average stresses at critical cross-sections are considered to be sufficient. More fundamental issues of stress distribution in design analysis will be discussed later in this lecture.

2.1.2 Some basic issues of simple stresses

Tensile stress

The stress developed in the bar (figure-2.1.2.1) subjected to tensile loading is given by $\sigma_t = \frac{P}{A}$, P is force, A is area of section $A = h \cdot d$

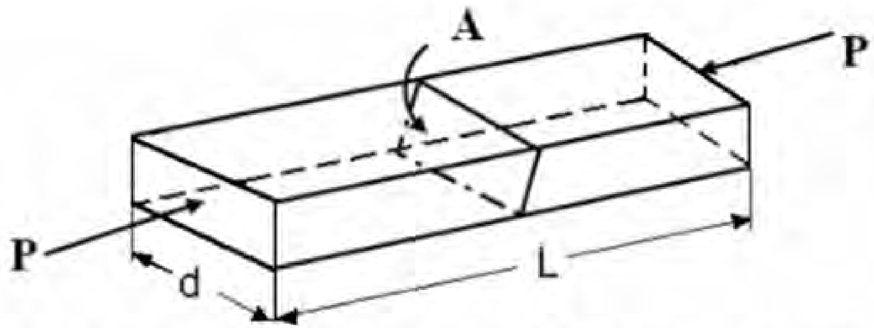


***2.1.2.1** - A prismatic bar subjected to tensile loading.*

Compressive stress

The stress developed in the bar (figure-2.1.2.2) subjected to compressive loading is given by

$$\sigma_c = \frac{P}{A}$$



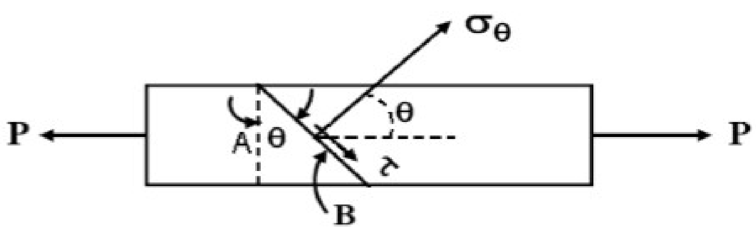
***2.1.2.2** - A prismatic bar subjected to compressive loading.*

However, if we consider the stresses on an inclined cross-section B (figure-2.1.2.3) then the normal stress perpendicular to the section is

$$\sigma_\theta = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{B}\cos\theta, \;\; B = A/\cos\theta$$

and shear stress parallel to the section

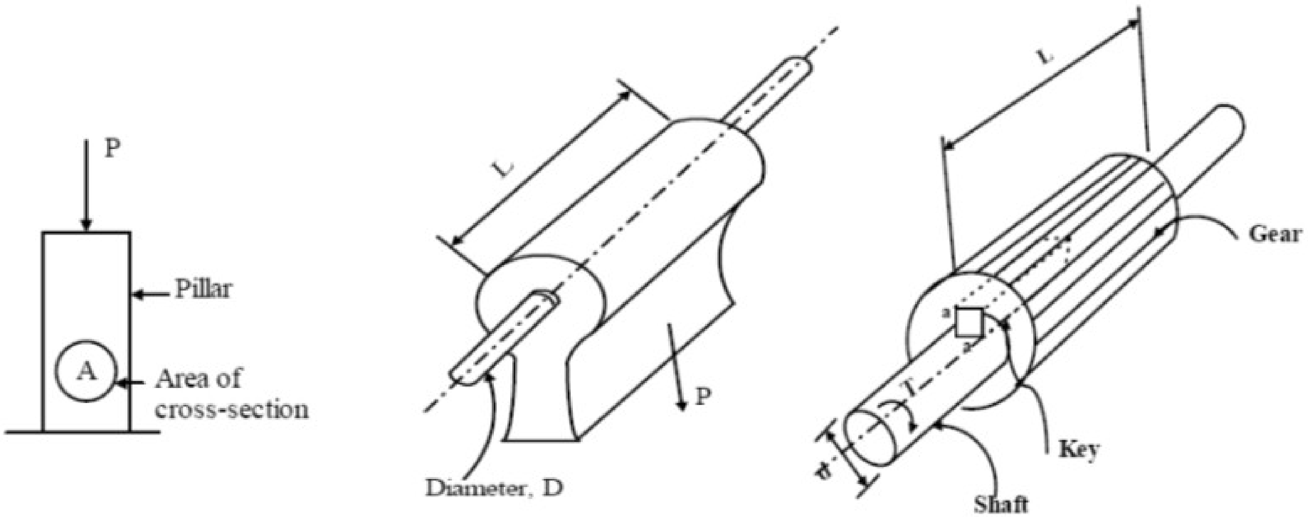
$$\tau = \frac{P\sin\theta}{A/\cos\theta}$$



2.1.2.3 *- Stresses developed at an inclined section of a bar subjected to tensile loading.*

Bearing stress

When a body is pressed against another, the compressive stress developed is termed bearing stress. For example, bearing stress developed at the contact between a pillar and ground (figure- 2.1.2.4a) is $\sigma_{br} = \frac{P}{A}$, at the contact surface between a pin and a member with a circular hole (figure- 2.1.2.4b) is $\sigma_{br} = \frac{P}{Ld}$ and at the faces of a rectangular key fixing a gear hub on a shaft (figure- 2.1.2.4c) is $\sigma_{br} = \frac{4T}{aLd}$.



(a) (b) (c)

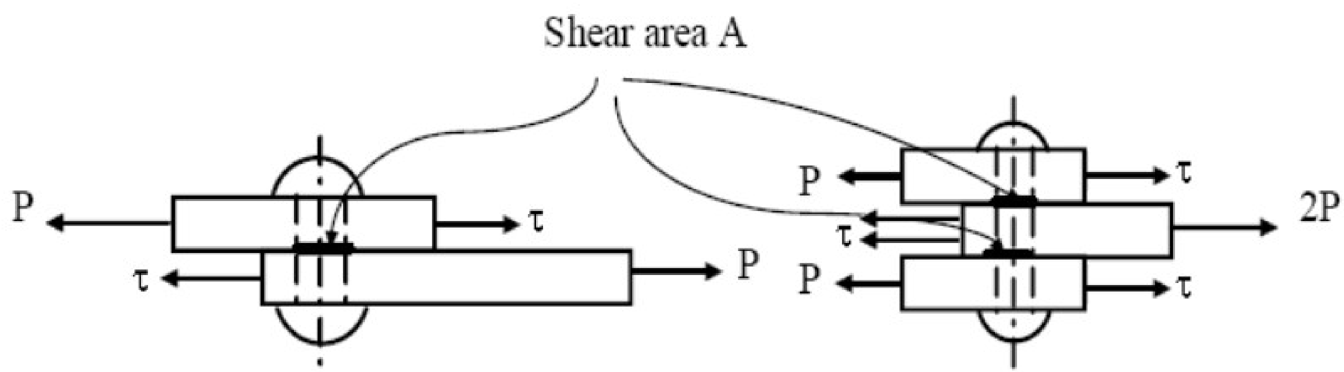
2.1.2.4 - The bearing stresses developed in pillar and machine parts.

The pressure developed may be irregular in the above examples but the expressions give the average values of the stresses.

Shear stress

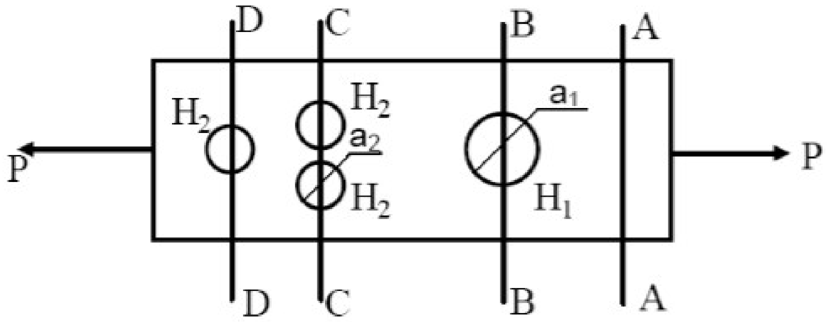
When forces are transmitted from one part of a body to other, the stresses developed in a plane parallel to the applied force are the shear stresses (figure-2.1.2.5) and the average values of the shear stresses are given by

$\tau = \frac{P}{A}$ in single shear $\qquad$ $\tau = \frac{P}{2A}$ in double shear



2.1.2.5 *- Stresses developed in single and double shear modes*

In design problems, critical sections must be considered to find normal or shear stresses. We consider a plate with holes under a tensile load (figure-2.1.2.6) to explain the concept of critical sections.



2.1.2.6 *- The concept of critical sections explained with the help of a loaded plate with holes at selected locations.*

Let the cross-sectional area of the plate, the larger hole H_1 and the smaller holes H_2 be A, a_1, a_2 respectively. If $2a_2 > a_1$ the critical section in the above example is CC and the average normal stress at the critical section is

$$\sigma_2 = \frac{P}{A - 2a_2} > \sigma_1 = \frac{P}{A - a_1}; \text{ as such } A - 2a_2 < A - a_1$$

2.1.3 Bending of beams

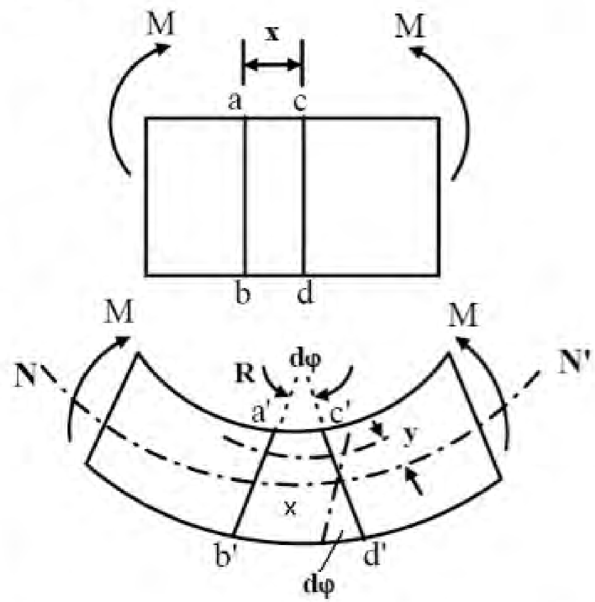
2.1.3.1 Bending stresses

Consider two sections ab and cd in a beam subjected to a pure bending. Due to bending the top layer is under compression and the bottom layer is under tension. This is shown in figure-2.1.3.1.1. This means that in between the two extreme layers there must be a layer which remains un-stretched and this layer is known as neutral layer. Let this be denoted by NN′.

We consider that a plane section remains plane after bending- a basic assumption in pure bending theory. If the rotation of cd with respect to ab is dφ the contraction of a layer y distance away from the neutral axis is given by ds=y dφ and original length of the layer is x=R dφ, R being the radius of curvature of the beam. This gives the strain ε in the layer as $\varepsilon = \frac{y}{R}$

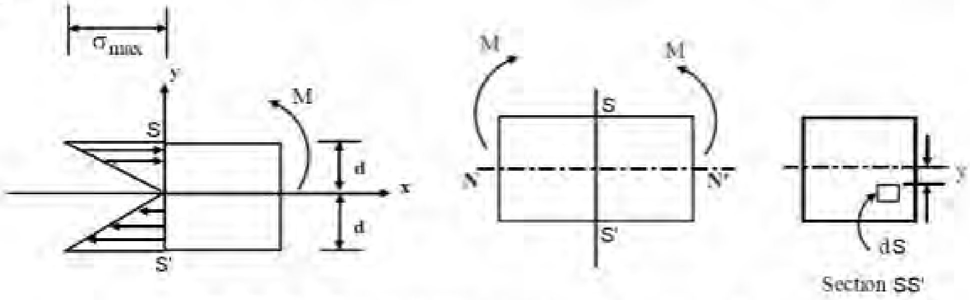
We also consider that the material obeys Hooke's law σ = Eε. This is another basic assumption in pure bending theory and substituting the expression for ε we have

$$\frac{\sigma}{y} = \frac{E}{R}$$



2.1.3.1.1- Pure bending of beams

Consider now a small element dS which distance y away from the neutral axis. This is shown in the figure 2.1.3.1.2



***2.1.3.1.2** - Bending stress developed at any cross-section*

Axial force on the element $dF_x = \sigma_x dS$ and considering the linearity in stress variation across the section $\frac{\sigma_x}{\sigma_{max}} = \frac{y}{d}$ we have where σ_x and σ_{max} are the stresses at distances y and d respectively from the neutral axis.

The axial force on the element is thus given by $dF_x = \frac{\sigma_{max} y}{d} dS$. **Where dS is area at section SS'.**

For static equilibrium total force at any cross-section $F_x = \int_{SS'} \frac{\sigma_{max}}{d} y dS = 0$

This gives $\int_{SS'} y dS = \bar{y} S = 0$ and since $S \neq 0$, $\bar{y} = 0$. This means that the neutral axis passes through the centroid.

Again for static equilibrium total moment about NA must the applied moment M. This is given by

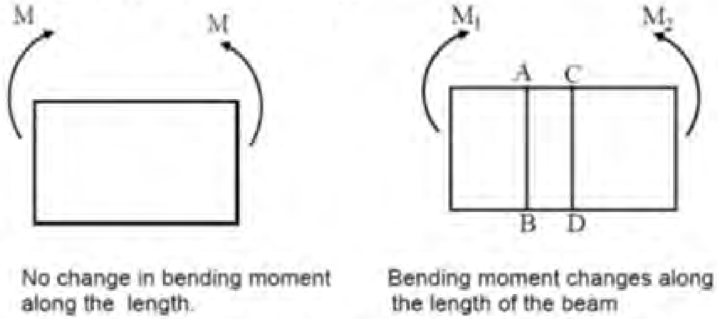
$$dM = \frac{\sigma_{max} y^2 dS}{d}, \quad \int_{SS'} \frac{\sigma_{max} y}{d} y dS = M \quad \text{and this gives} \quad \sigma_{max} = \frac{Md}{I}, \quad I = \int y^2 dS.$$

For any fibre at a distance of y from the centre line we may therefore write $\sigma = \frac{My}{I}$

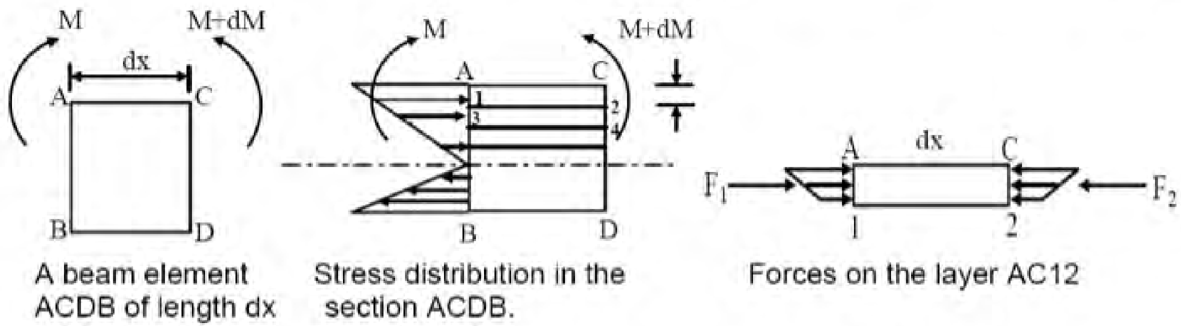
We therefore have the general equation for pure bending as $\boxed{\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}}$

2.1.3.2 Shear stress in bending

In an idealized situation of pure bending of beams, no shear stress occurs across the section. However, in most realistic conditions shear stresses do occur in beams under bending. This can be visualized if we consider the arguments depicted in figure-2.1.3.2.1 and 2.1.3.2.2.



2.1.3.2.1- Bending of beams with a steady and varying moment along its length.



2.1.3.2.2 - Shear stress developed in a beam subjected to a moment varying along the length

When bending moment changes along the beam length, layer AC12 for example, would tend to slide against section 1243 and this is repeated in subsequent layers. This would cause interplanar shear forces F_1 and F_2 at the faces A1 and C2 and since the $F = \int_A \sigma_x dS$ force at any cross-section is given by , we may write

$$F_1 = \frac{M}{I} Q \quad \text{and} \quad F_2 = \frac{(M + dM)}{I} Q$$

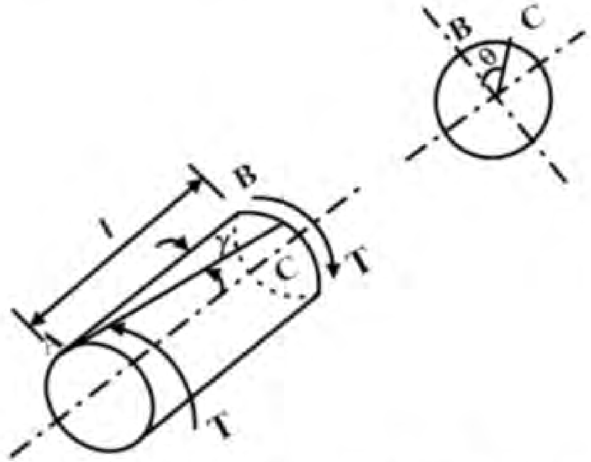
Here M and dM are the bending moment and its increment over the length dx and Q is the 1st moment of area about the neutral axis. Since shear stress across the layers can be given by $\frac{dM}{dx}$ and $\tau = \frac{VQ}{It}$ shear force is given by V = $\tau = \frac{dF}{tdx}$

2.1.4 Torsion of circular members

A torque applied to a member causes shear stress. In order to establish a relation between the torque and shear stress developed in a circular member, the following assumptions are needed:

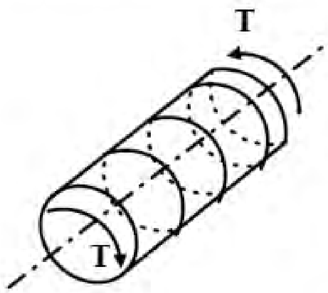
1. Material is homogeneous and isotropic
2. A plane section perpendicular to the axis of the circular member remains plane even after twisting i.e. no warping.
3. Materials obey Hooke's law.

Consider now a circular member subjected to a torque T as shown in figure 2.1.4.1

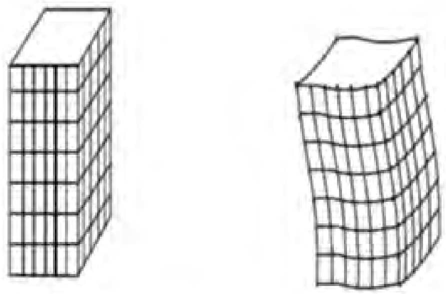


2.1.4.1 - A circular member of radius r and length L subjected to torque T.

The assumption of plane section remaining plane assumes no warping in a circular member as shown in figure- 2.1.4.2



2.1.4.2 - Plane section remains plane- No warping.



2.1.4.3 -Warping during torsion of a non-circular member.

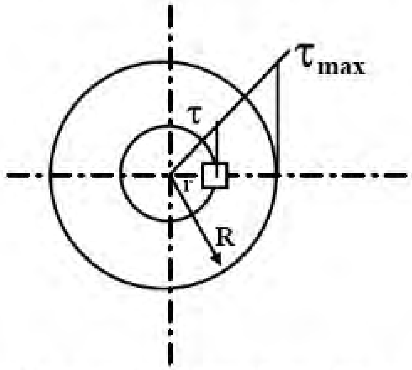
Let the point B on the circumference of the member move to point C during twisting and let the angle of twist be θ. We may also assume that strain γ varies linearly from the central axis. This gives

$$\gamma l = r\theta \text{ and from Hooke's law } \gamma = \frac{\tau}{G}$$

where τ is the shear stress developed and G is the modulus of rigidity. This gives

$$\frac{\tau}{r} = \frac{G\theta}{l}$$

Consider now, an element of area dA at a radius r as shown in figure-2.1.4.4. The torque on the element is given by $T = \int \tau r dA$



2.1.4.4 - Shear stress variation in a circular cross-section during torsion.

For linear variation of shear stress we have $\frac{\tau}{\tau_{max}} = \frac{r}{R}$

Combining this with the torque equation we may write $T = \frac{\tau_{max}}{R} \int r^2 dA$.

Now $\int r^2 dA$ may be identified as the polar moment of inertia J .

And this gives $T = \frac{\tau_{max}}{R} J$.

Therefore for any radius r we may write in general $T/J=\tau/r$.

We have thus the general torsion equation for circular shafts as

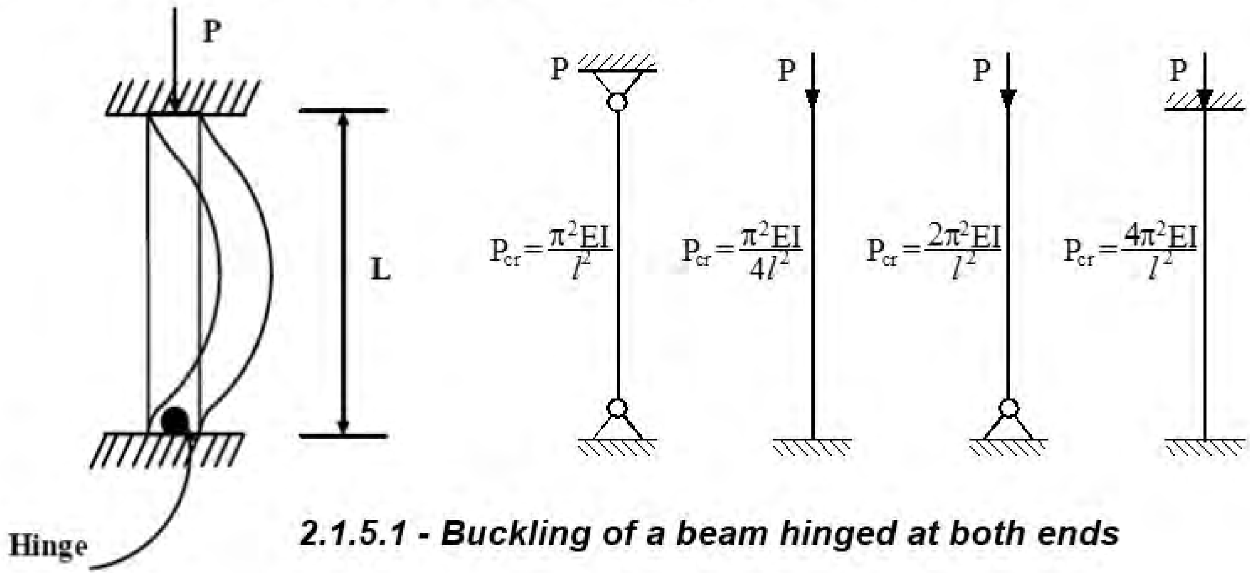
$$\boxed{\frac{\mathbf{T}}{\mathbf{J}} = \frac{\boldsymbol{\tau}}{\mathbf{r}} = \frac{\mathbf{G\theta}}{\mathbf{l}}}$$

2.1.5 Buckling

The compressive stress of P/A is applicable only to short members but for long compression members there may be buckling, which is due to elastic instability. The critical load for buckling of a column with different end fixing conditions is given by Euler's formula (figure-2.1.5.1)

$$P_{cr} = n\frac{\pi^2 EI}{l^2}$$

where E is the elastic modulus, I the second moment of area, l the column length and n is a constant that depends on the end condition. For columns with both ends hinged n=1, columns with one end free and other end fixed n=0.25, columns with one end fixed and other end hinged n=2, and for columns with both ends fixed n=4.

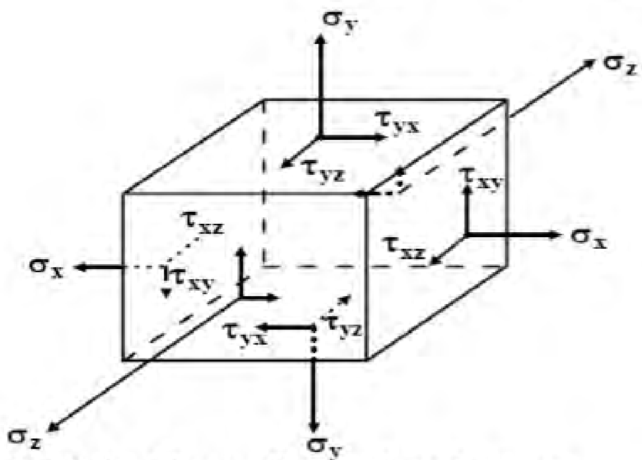


2.1.5.1 - Buckling of a beam hinged at both ends

2.1.6 Stress at a point—its implication in design

The state of stress at a point is given by nine stress components as shown in figure 2.1.6.1 and this is represented by the general matrix as shown below σ_{ij} is tensor or matrix of stress.

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \qquad \begin{matrix} i = 1,2,3, \quad j = 1,2,3, \\ 1 = x, \;\; 2 = y, \;\; 3 = z. \end{matrix}$$



2.1.6.1 - Three dimensional stress field on an infinitesimal element.

$$\left.\begin{matrix} \sigma_x = \sigma_{11} = \sigma_{xx} \\ \sigma_y = \sigma_{22} = \sigma_{yy} \\ \sigma_z = \sigma_{33} = \sigma_{zz} \end{matrix}\right\} \text{tensile stresses.}$$

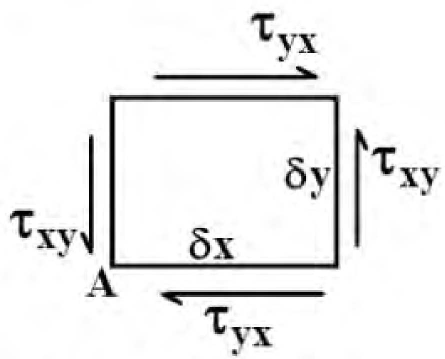
$$\tau_{yx}, \;\; \tau_{zx}, \;\; \tau_{zy} - \text{shear stresses.}$$

On crossection perpendiecu larity to axis y, z, x.

Consider now a two dimensional stress element subjected only to shear stresses.

For equilibrium of a 2-D element we take moment of all the forces about point A (figure-2.1.6.2) and equate to zero as follows:

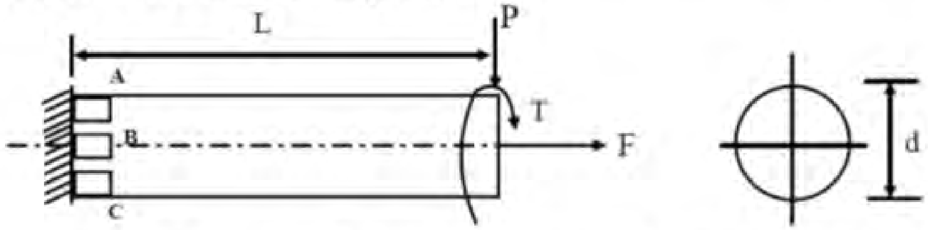
$$\left(\tau_{xy}\delta y\delta z\right)\delta x-\left(\tau_{yx}\delta x\delta z\right)\delta y=0$$



2.1.6.2 - Complimentary shear stresses on a 2-D element.

This gives $\tau_{xy}=\tau_{yx}$ indicating that τ_{xy} and τ_{yx} are complimentary. On similar arguments we may write $\tau_{yz}=\tau_{zy}$ and $\tau_{zx}=\tau_{xz}$. This means that the state of stress at a point can be given by six stress components only. It is important to understand the implication of this state of stress at a point in the design of machine elements where all or some of the stresses discussed above may act.

For example, let us consider a cantilever beam of circular cross-section subjected to a vertical loading P at the free end and an axial loading F in addition to a torque T as shown in figure 2.1.6.3. Let the diameter of cross-section and the length of the beam be d and L respectively.



2.1.6.3 - A cantilever beam subjected to bending, torsion and an axial loading.

The maximum stresses developed in the beam are :

Bending stress, $\sigma_A = \dfrac{32PL}{\pi d^3}$

Axial stress, $\sigma_B = \dfrac{4F}{\pi d^2}$

Torsional shear stress $\tau = \dfrac{16T}{\pi d^3}$

The formulae were obtained before, here F – is force along axial, T – is moment.

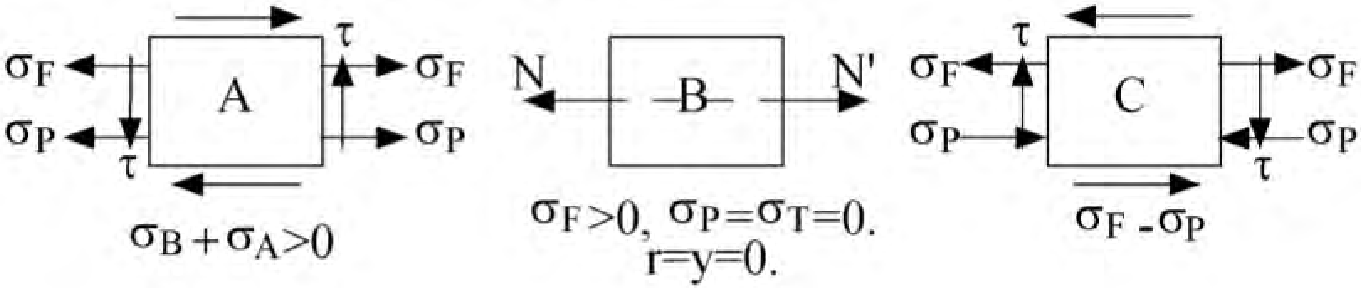
It is now necessary to consider the most vulnerable section and element. Since the axial and torsional shear stresses are constant through out the length, the most vulnerable section is the built-up end.

We now consider the three elements A, B and C. There is no bending stress on the element B and the bending and axial stresses on the element C act in the opposite direction. Therefore, for the safe design of the beam we consider the stresses on the element A which is shown in figure 2.1.6.4.

On element B located on neutral axis NN'.

On element C bending stresses is contract, axial stresses ore tensile.

On element A bending stresses are tensile and axial stress are tensile.

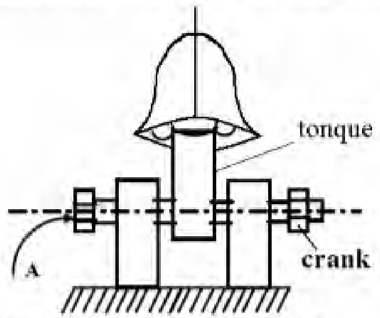


2.1.6.4 - Stresses developed on elements A, B, C in figure-2.1.6.3

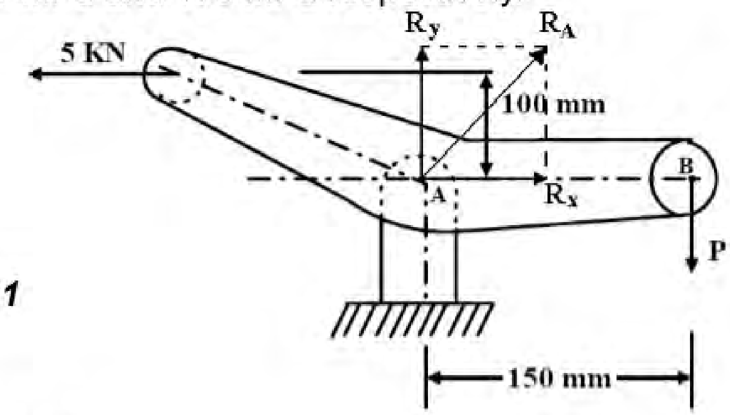
Principal stresses and maximum shear stresses can now be obtained and using a suitable failure theory a suitable diameter of the bar may be obtained. In this case in cantilever beam we have combination of simple stresses: tensile, bending, torsinal stresses.

2.1.7 Problems with Answers

Q.1: What stresses are developed in the pin A for the bell crank mechanism shown in the figure-2.1.7.1? Find the safe diameter of the pin if the allowable tensile and shear stresses for the pin material are 350 MPa and 170 MPa respectively.



2.1.7.1



Equation of force equilibium

A.1: y: $R_y + P_B = 0$,

x: $R_x - P_C = 0$. Hene R – constrain face.

Force at B = $\frac{5x0.1}{0.15} = 3.33 KN$

Resultant force at A= $\sqrt{5^2 + 3.33^2}$ kN = 6 kN.

Stresses developed in pin A: (a) shear stress (b) bearing stress

Considering double shear at A, pin diameter d = $\sqrt{\frac{2x6x10^3}{\pi x170x10^6}}$ m = 4.7 mm

Considering bearing stress at A, pin diameter d = $\frac{6x10^3}{0.01x7.5x10^6}$ m = 8mm

A safe pin diameter is 10 mm.

Q.2: What are the basic assumptions in deriving the bending equation?

A.2: The basic assumptions in deriving bending equation are:

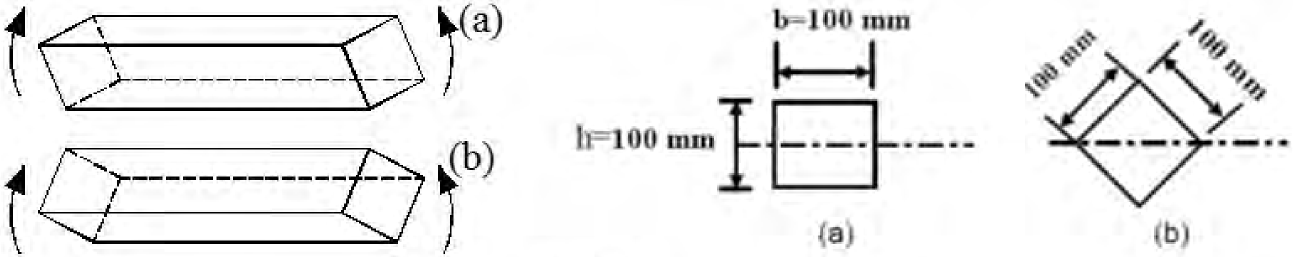
a) The beam is straight with a constant area of cross-section and is symmetrical about the plane of bending.

b) Material is homogeneous and isotropic.

c) Plane sections normal to the beam axis remain plane even after bending.

d) Material obeys Hooke's law

Q.3: Two cast iron machine parts of cross-sections shown in figure-2.1.7.2 are subjected to bending moments. Which of the two sections can carry a higher moment and determine the magnitude of the applied moments?



A.3:

2.1.7.2

Assuming that bending takes place about the horizontal axis, the 2nd moment of areas of the two sections are:

$$I_a = \frac{b.b^3}{12} \qquad I_b = 2\frac{\left(\sqrt{2}b\right)\left(\frac{b}{\sqrt{2}}\right)^3}{36} + 2\frac{\left(\sqrt{2}b\right)\left(\frac{b}{\sqrt{2}}\right)}{2}\left(\frac{b/\sqrt{2}}{3}\right)^2 = \frac{b^4}{12}$$

$$\therefore I_a = I_b$$

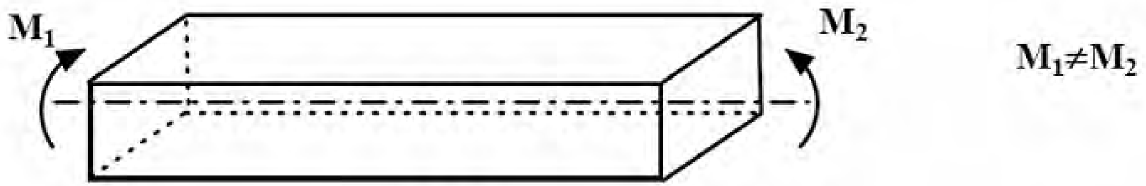
Considering that the bending stress σ_B is same for both the beams and moments applied M_a and M_b, we have

$$\sigma_B = \frac{M_a y_a}{I_a} = \frac{M_b y_b}{I_b}$$

Here, $y_a = 0.5b$, $y_b = b/\sqrt{2}$. Then $M_a = \sqrt{2}M_b$, $M_a > M_b$.

Q.4: Under what condition transverse shear stresses are developed in a beam subjected to a bending moment?

A.4: Pure bending of beams is an idealized condition and in the most realistic situation,bending moment would vary along the bending axis (figure-2.1.7.3).

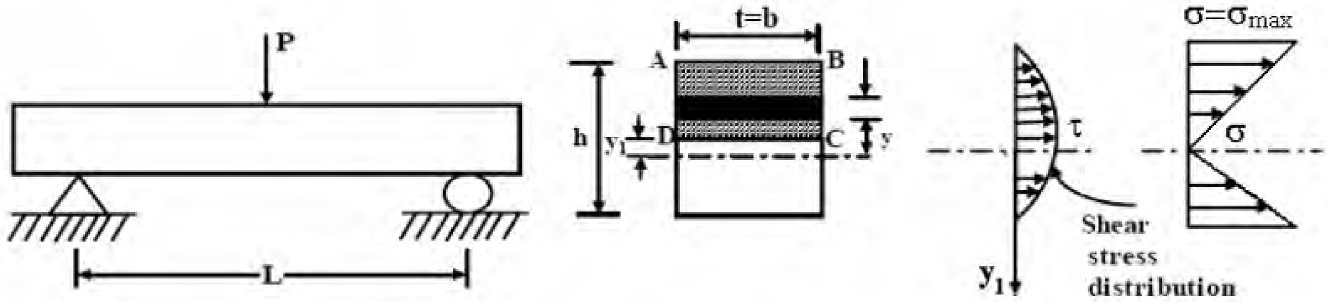


2.1.7.3

Under this condition transverse shear stresses would be developed in a beam.

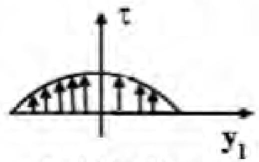
Q.5: Show how the transverse shear stress is distributed in a beam of solid rectangular cross-section transmitting a vertical shear force.

A.5: Consider a beam with a rectangular cross-section (figure-2.1.7.4). Consider now a longitudinal cut through the beam at a distance of y1 from the neutral axis isolating an area ABCD. An infinitesimal area within the isolated area at a distance y from the neutral axis is then considered to find the first moment of area Q.



2.1.7.4

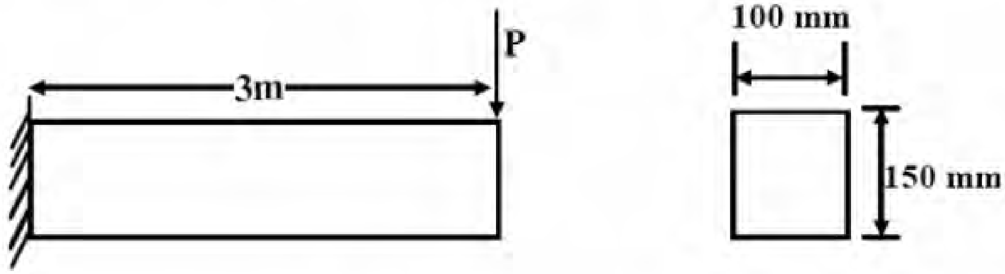
A simply supported beam with a Enlarged view of the rectangular cross-section Concentrated load at the centre.



Horizontal shear stress at y, $\tau = \frac{VQ}{It} = \frac{V}{It}\int_{y_1}^{h} bydy$

This gives $\tau = \frac{V}{2I}\left[\frac{h^2}{4} - y_1^2\right]$ indicating a parabolic distribution of shear stress across the cross-section. Here, V is shear force, I is the second moment of area of the beam cross-section, t is the beam width which is b in this case.

Q.6: A 3m long cantilever beam of solid rectangular cross-section of 100mm width and 150mm depth is subjected to an end loading P as shown in the figure-2.1.7.5. If the allowable shear stress in the beam is 150 MPa, find the safe value of P based on shear alone.



2.1.7.5

A.6: Maximum shear stress in a rectangular cross-section $\tau_{max} = \frac{3}{2}\frac{V}{A}$

where, A is the cross-section area of the beam.

Substituting values we have τmax= 100P and for an allowable shear stress of 150 MPa the safe value of P works out to be 1.5 MN.

Q.7: What are the basic assumptions in deriving the torsion equation for a circular member?

A.7: Basic assumptions in deriving the torsion formula are:

a) Material is homogenous and isotropic.

b) A plane section perpendicular to the axis remains plane even after the torque is applied. This means there is no warpage.

c) In a circular member subjected to a torque, shear strain varies linearly from the central axis.

d) Material obeys Hooke's law.

Q.8: In a design problem it is necessary to replace a 2m long aluminium shaft of 100mm diameter by a tubular steel shaft of the same outside diameter transmitting the same torque and having the same angle of twist. Find the inner radius of the steel bar if G_{Al} = 28GPa and G_{St} = 84GPa.

A.8:

Since the torque transmitted and angle of twist are the same for both the solid and hollow shafts, we may write from torsion formula

$$\tau_{Al} J_{Al} = \tau_{St} J_{St} \quad \text{and} \quad \frac{\tau_{Al}}{\tau_{St}} = \frac{G_{Al}}{G_{St}}$$

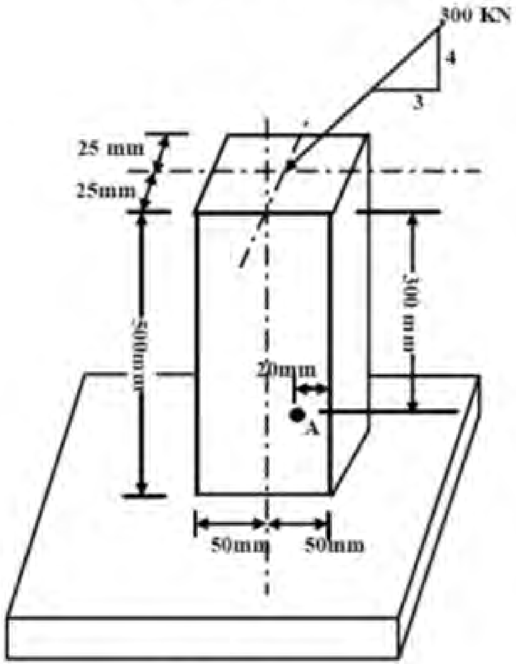
where τ, J and G are shear stress, polar moment of inertia and modulus of rigidity respectively. This gives

$$\frac{d_0^4 - d_i^4}{d_0^4} = \frac{28}{84} \quad \text{and with } d_0 = 100\,\text{mm} \;\; d_i = 90.36\,\text{mm}$$

$$1 - \frac{d_1^4}{d_0^4} = \frac{29}{84}: \quad \frac{d_1^4}{d_0^4} = 1 - \frac{28}{84} = \frac{56}{84} = \frac{14}{21} = \frac{2}{3};$$

$$d_{Al} = 100 = d_0: \quad d_{St} = d_1 = 90{,}36.$$

Q.10: Show the stresses on the element at A in figure-2.1.7.6.



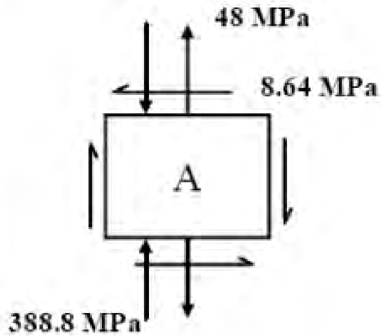
2.1.7.6

A.10: The element A is subjected to a compressive stress due to the vertical component 240 KN and a bending stress due to a moment caused by the horizontalcomponent 180 KN.

Compressive stress, $\sigma_c = \dfrac{240}{0.05x0.1} = 48\,\text{MPa}$

Bending (tensile) stress, $\sigma_B = \dfrac{(180x0.3)x0.03}{\left(\dfrac{0.05x0.1^3}{12}\right)} = 388.8\,\text{MPa}$

Shear stress due to bending = $\dfrac{VQ}{It} = 8.64\,\text{MPa}$



2.1.8 Summary of this Lecture

It is important to analyse the stresses developed in machine parts and design the components accordingly. In this lecture simple stresses such as tensile, compressive, bearing, shear, bending and torsional shear stress and buckling of beams have been discussed along with necessary formulations. Methods of combining normal and shear stresses are also discussed.

Lecture
Theme 2
Stresses in machine elements

2.2. Compound stresses in machine parts

2.2.1 Introduction

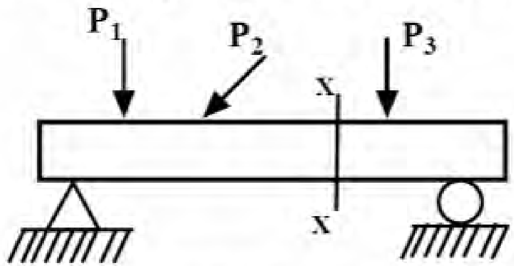
The elements of a force system acting at a section of a member are axial force, shear force and bending moment and the formulae for these force systems were derived based on the assumption that only a single force element is acting at the section. Figure-2.2.1.1 shows a simply supported beam while figure-2.2.1.2 shows the forces and the moment acting at any cross-section X-X of the beam. The force system can be given as:

Axial force : $\sigma = \frac{P}{A}$

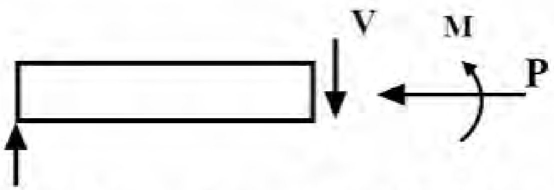
Bending moment : $\sigma = \frac{My}{I}$

Shearforce : $\tau = \frac{VQ}{It}$

Torque $T = \frac{\tau J}{r}$



2.2.1.1 *- A simply supported beam with concentrated loads*

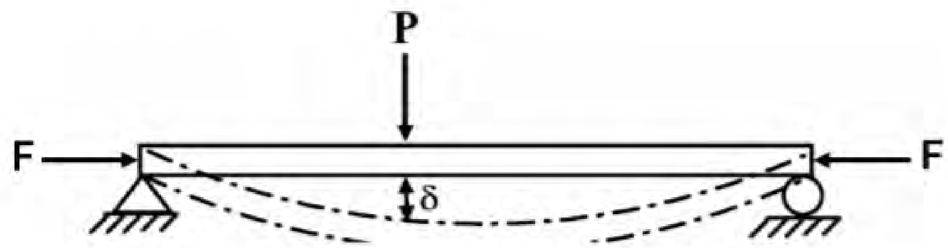


2.2.1.2 *- Force systems on section XX of figure-2.2.1.1*

where, σ is the normal stress, τ the shear stress, P the normal load, A is the crosssectional area, M is the moment acting at section X-X, V the shear stress acting at section X-X, Q the first moment of area, I is the moment of inertia, t the width at which transverse shear is calculated, J is the polar moment of inertia and r is the radius of the circular cross-section.

Combined effect of these elements at a section may be obtained by the method of superposition provided that the following limitations are tolerated:

(a) Deformation is small (figure-2.2.1.3)



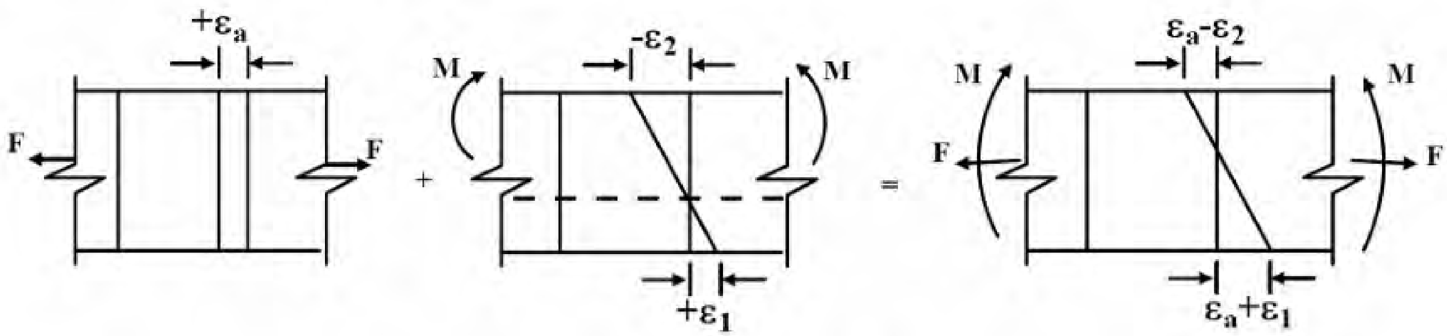
2.2.1.3 - Small deflection of a simply supported beam with a concentrated load

If the deflection is large, another additional moment of Pδ would be developed.

(b) Superposition of strains are more fundamental than stress superposition and the principle applies to both elastic and inelastic cases.

2.2.2 Strain superposition due to combined effect of axial force P and bending moment M.

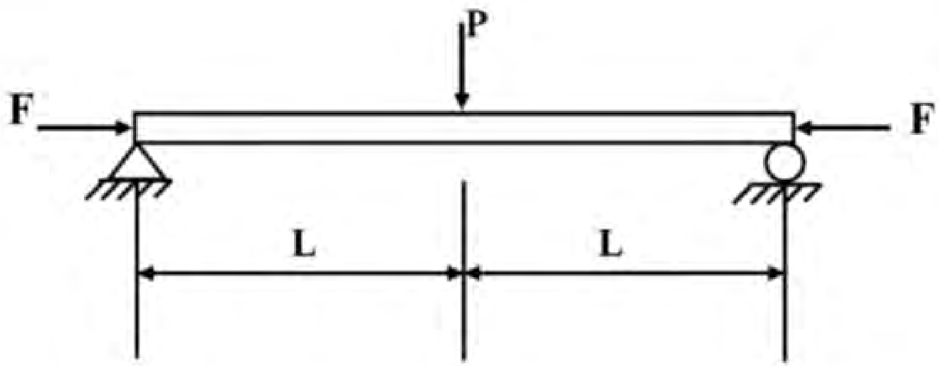
Figure-2.2.2.1 shows the combined action of a tensile axial force and bending moment on a beam with a circular cross-section. At any cross-section of the beam, the axial force produces an axial strain ε_a while the moment M causes a bending strain. If the applied moment causes upward bending such that the strain at the upper most layer is compressive ($-\varepsilon_2$) and that at the lower most layer is tensile ($+\varepsilon_1$), consequently the strains at the lowermost fibre are additive ($\varepsilon_a+\varepsilon_1$) and the strains at the uppermost fibre are subtractive ($\varepsilon_a-\varepsilon_2$). This is demonstrated in figure-2.2.2.1.



2.2.2.1 - Superposition of strain due to axial loading and bending moment.

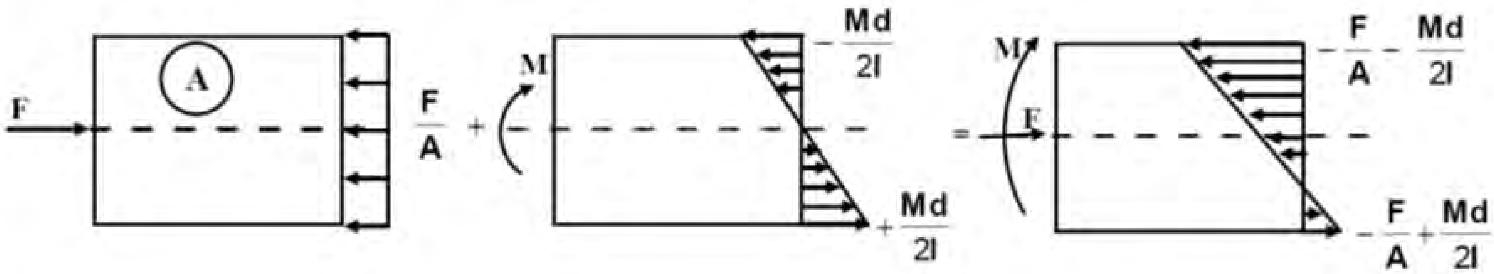
2.2.3 Superposition of stresses due to axial force and bending moment

In linear elasticity, stresses of same kind may be superposed in homogeneous and isotropic materials. One such example (figure-2.2.3.1) is a simply supported beam with a central vertical load P and an axial compressive load F.



2.2.3.1 - A simply supported beam with an axial and transverse loading.

At any section a compressive stress of $4F/\pi d^2$ and a bending stress of My/I are produced. Here d is the diameter of the circular bar, I is the second moment of area and the moment is PL/2 where the beam length is 2L. Total stresses at the upper and lower most fibres in any beam cross-section are $-\left(\frac{32M}{2\pi d^3}+\frac{4F}{\pi d^2}\right)$ and $\left(\frac{32M}{2\pi d^3}-\frac{4F}{\pi d^2}\right)$ respectively. This is illustrated in figure-2.2.3.2



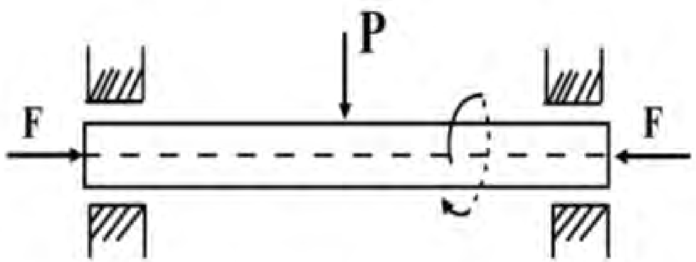
2.2.3.2 - Combined stresses due to axial loading and bending moment.

2.2.4 Superposition of stresses due to axial force, bending moment and torsion

Until now, we have been discussing the methods of compounding stresses of same kind for example, axial and bending stresses both of which are normal stresses. However, in many cases members on machine elements are subjected to both normal and shear stresses, for example, a shaft subjected to torsion, bending and axial force. This is shown in figure-2.2.4.1.

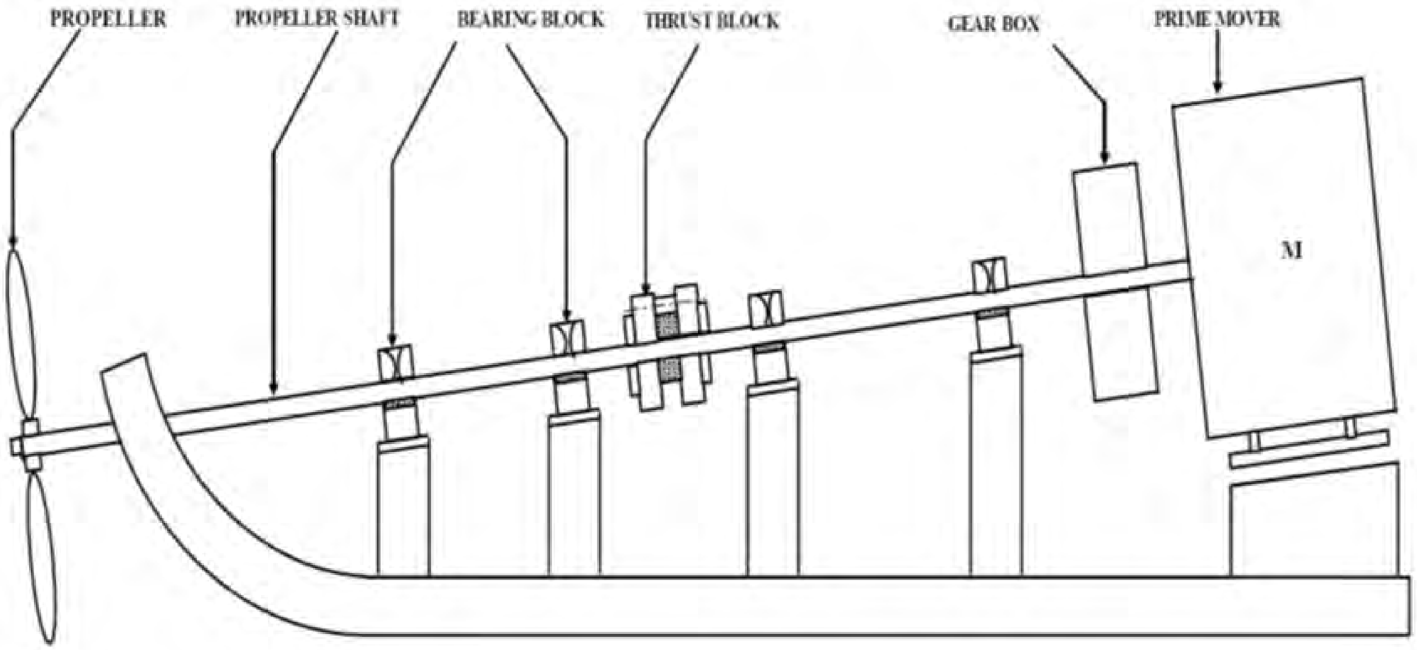
In this case we have

$$\sigma_F = \frac{F}{A};\ \sigma_P = \frac{M_y}{I};\ T = \frac{\tau J}{r}$$



2.2.4.1 - A simply supported shaft subjected to axial force bending moment and torsion.

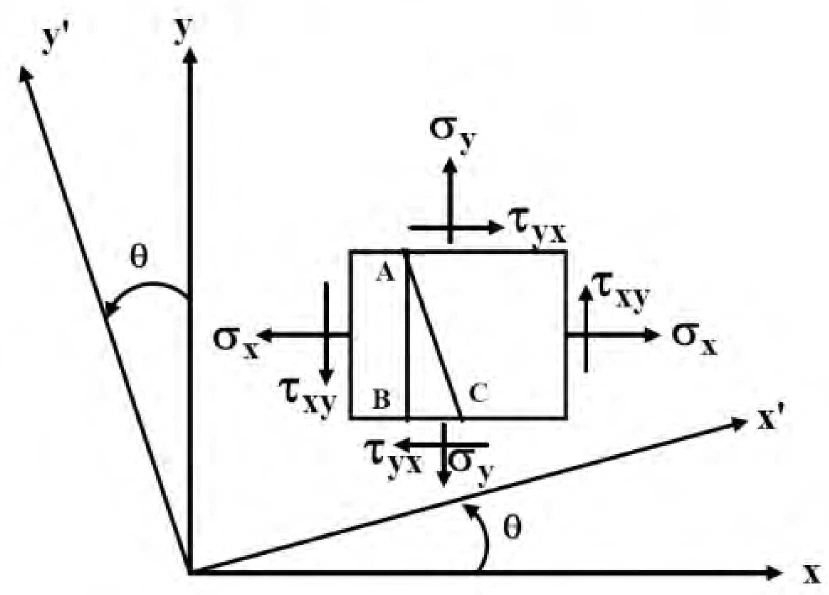
A typical example of this type of loading is seen in a ship's propeller shafts. Figure-2.2.4.2 gives a schematic view of a propulsion system. In such cases normal and shearing stresses need to be compounded.



2.2.4.2 - A schematic diagram of a typical marine propulsion shafting

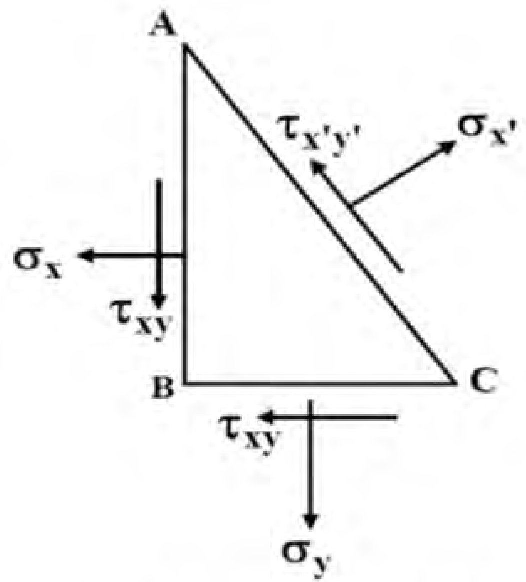
2.2.5 Transformation of plane stresses

Consider a state of general plane stress in x-y co-ordinate system. We now wish to transform this to another stress system in, say, x′- y′ co-ordinates, which is inclined at an angle θ. This is shown in figure-2.2.5.1.



2.2.5.1 - Transformation of stresses from x-y to x'-y' co-ordinate system.

A two dimensional stress field acting on the faces of a cubic element is shown in figure-2.2.5.2. In plane stress assumptions, the non-zero stresses are σx, σy and $\tau_{xy}=\tau_{yx}$.We may now isolate an element ABC such that the plane AC is inclined at an angle θ and the stresses on the inclined face are σ'_x and τ'_{xy} .



2.2.5.2 - Stresses on an isolated triangular element

Considering the force equilibrium in x-direction we may write

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

This may be reduced to

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (1)$$

Similarly, force equilibrium in y-direction gives

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (2)$$

Since plane AC can assume any arbitrary inclination, a stationary value of σx′ is given by

$$\frac{d\sigma_{x'}}{d\theta} = 0$$

This gives

$$\tan 2\theta = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} \qquad (3)$$

This equation has two roots and let the two values of θ be θ_1 and $(\theta_1 + 90^o)$. Therefore these two planes are the planes of maximum and minimum normal stresses. Now if we set $\tau_{x'y'}=0$ we get the values of θ corresponding to planes of zero shear stress.

This also gives

$$\tan 2\theta = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2}$$

And this is same as equation (3) indicating that at the planes of maximum and minimum stresses no shearing stress occurs. These planes are known as **Principal planes** and stresses acting on these planes are known as **Principal stresses.** From equation (1) and (3) the principal stresses are given as

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (4)$$

In the same way, condition for maximum shear stress is obtained from

$$\frac{d}{d\theta}(\tau_{x'y'}) = 0$$

$$\tan 2\theta = -\frac{\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} \qquad (5)$$

This also gives two values of θ say θ_2 and (θ_2+90°), at which shear stress is maximum or minimum. Combining equations (2) and (5) the two values of maximum shear stresses are given by

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (6)$$

One important thing to note here is that values of tan2θ2 is negative reciprocal of $\tan 2\theta_1$ and thus θ_1 and θ_2 are 45° apart. This means that principal planes and planes of maximum shear stresses are 45° apart. It also follows that although no shear stress exists at the principal planes, normal stresses may act at the planes of maximum shear stresses.

2.2.6 An example

Consider an element with the following stress system (figure-2.2.6.1)

σ_x=-10 MPa, σ_y = +20 MPa, τ = -20 MPa.

We need to find the principal stresses and show their senses on a properly oriented element.

Solution:

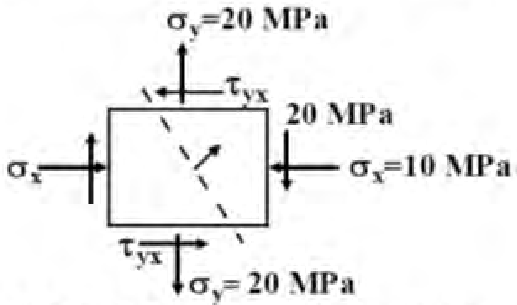
The principal stresses are

$$\sigma_{1,2} = \frac{-10+20}{2} \pm \sqrt{\left(\frac{-10-20}{2}\right)^2 + (-20)^2}$$

This gives -20 MPa and 30 MPa

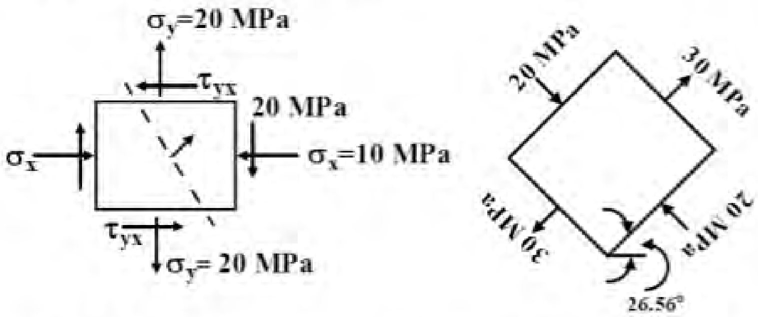
The principal planes are given by

$$\tan 2\theta_1 = \frac{-20}{(-10-20)/2} = 1.33$$



2.2.6.1 - A 2-D element with normal and shear stresses.

The two values are 26.56 and 116.56°. The oriented element to show the principal stresses is shown in figure-2.2.6.2.



2.2.6.2 - Orientation of the loaded element in the left to show the principal stresses.

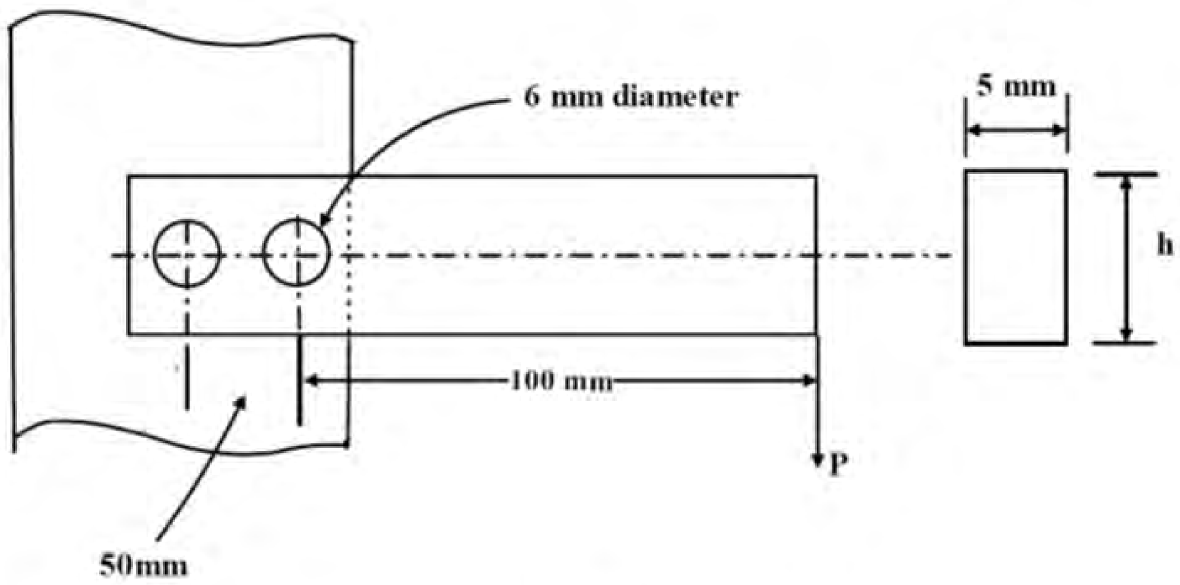
In this case tensor of stress has simple form $\sigma_{ij} = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix}$, $\tau_{xy} = 0$. If we turn the element on angle 26.56° then we have simple state of element in principal axises.

2.2.7 Problems with Answers

Q.1: A 5mm thick steel bar is fastened to a ground plate by two 6 mm diameter pins as shown in figure-2.2.7.1. If the load P at the free end of the steel bar is 5 KN, find

(a) The shear stress in each pin

(b) The direct bearing stress in each pin.



2.2.7.1

A.1:

Due to the application of force P the bar will tend to rotate about point 'O' causing shear and bearing stresses in the pins A and B. This is shown in figure-2.2.7.2F. Let the forces at pins A and B be F_A and F_B and equating moments about 'O',

$$5x10^3x0.125 = (F_A+F_B)x\ 0.025 \qquad (1)$$

Also, from force balance, $F_A+P = F_B$ (2)

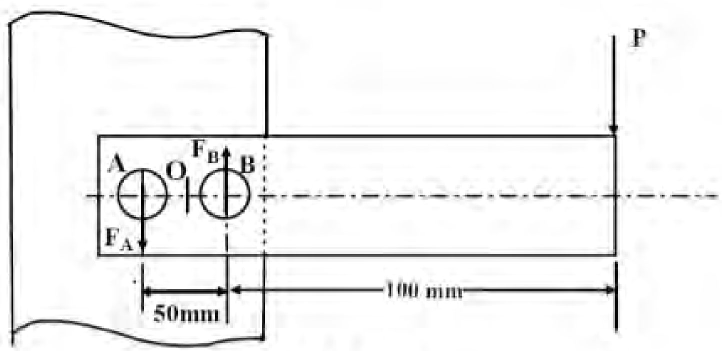
Solving equations-1 and 2 we have, F_A =10 KN and F_B = 15 KN.

(a) Shear stress in pin A = $\frac{10x10^3}{\left(\frac{\pi x0.006^2}{4}\right)} = 354\,\text{MPa}$

Shear stress in pin B = $\frac{15x10^3}{\left(\frac{\pi x0.006^2}{4}\right)} = 530.5\,\text{MPa}$

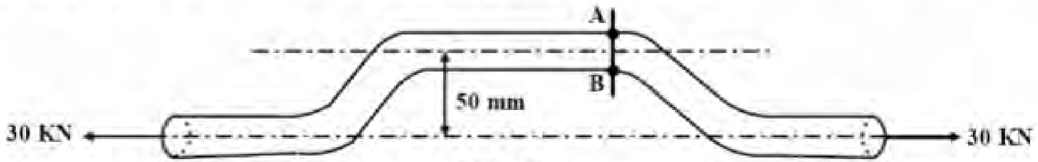
(b) Bearing stress in pin A = $\frac{10x10^3}{(0.006x0.005)} = 333\,\text{MPa}$

Bearing stress in pin B = $\frac{15x10^3}{(0.006x0.005)} = 500\,\text{MPa}$



2.2.7.2

Q.2: A 100 mm diameter off-set link is transmitting an axial pull of 30 KN as shown in the figure- 2.2.7.3. Find the stresses at points A and B.



2.2.7.3

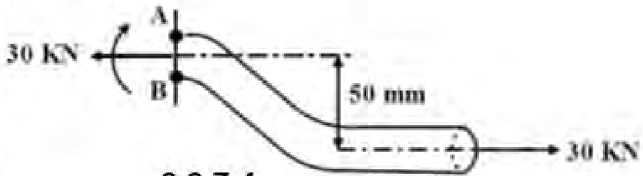
A.2: The force system at section AB is shown in figure-2.2.7.4.

$$\sigma_A = \sigma_A^{(b)} + \sigma_A^{(t)} = -\frac{30\times10^3\times0.05\times0.05}{\frac{\pi}{64}(0.1)^4} + \frac{30\times10^3}{\frac{\pi}{4}(0.1)^2} = -11.46\text{MPa}$$

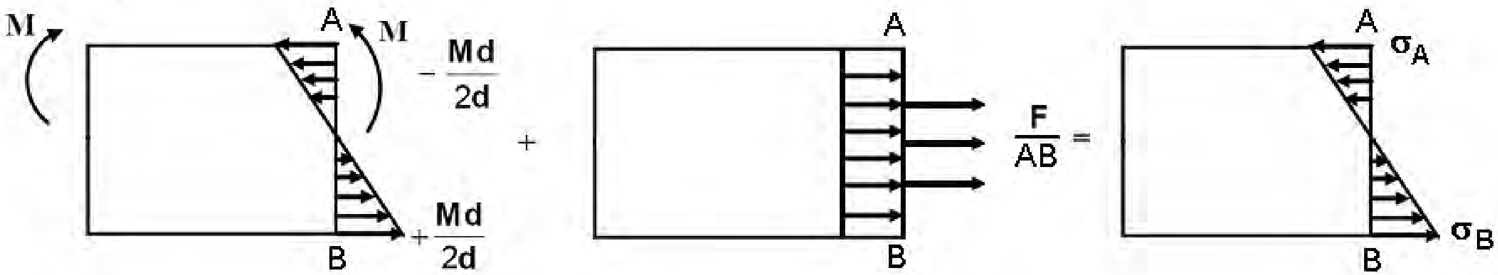
$$\sigma_B = \sigma_B^{(b)} + \sigma_B^{(t)} = \frac{30\times10^3\times0.05\times0.05}{\frac{\pi}{64}(0.1)^4} + \frac{30\times10^3}{\frac{\pi}{4}(0.1)^2} = 19.1\text{MPa}$$

At section AB in this case we have pure bending plus stretch at paint B or minus carpesscs at paint A.

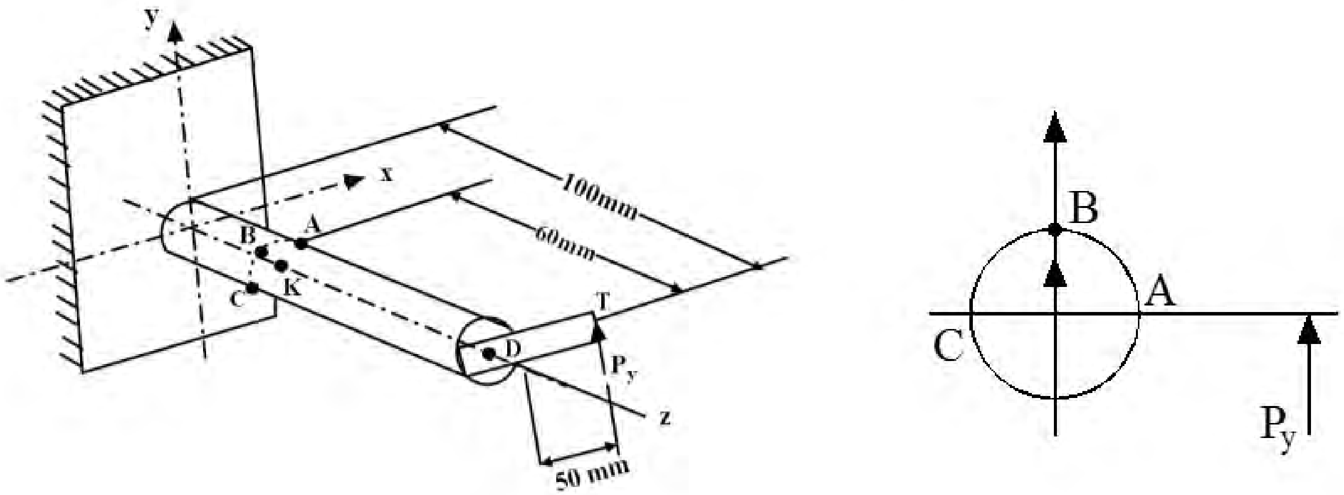
We applicate princinl of superposition for solving.



2.2.7.4



Q.3: A vertical load P_y = 20 KN is applied at the free end of a cylindrical bar of radius 50 mm as shown in figure-2.2.7.5. Determine the principal and maximum shear stresses at the points A, B and C.



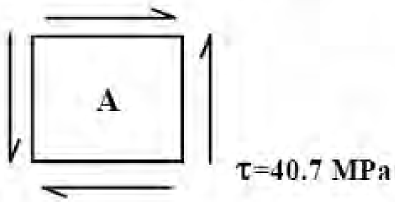
2.2.7.5

A.3: At section ABC a bending moment of 1.2 KN-m and a torque of 1KN-m act. On elements A and C there is no bending stress because they are in plane perpendicularity to force P_y. Only torsional shear stress acts and

$$M^{(b)} = P_y DK = 1.2KN,$$

$$M^{(T)} = P_y DT = 1KN$$

$$\tau = \frac{16T}{\pi d^3} = 40.7 \text{ MPa}$$



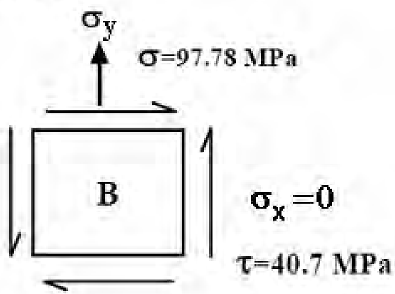
$$\tau_A = \tau_C$$

On element B both bending (compressive) and torsional shear stress act.

$$\sigma_B = \frac{32M}{\pi d^3} = 97.78\,\text{MPa} = \frac{Mr}{I}, \ r = 50$$

$$\tau = 40.7\,\text{MPa}$$

Principal stresses at B = $\left(\frac{97.78}{2} \pm \sqrt{\left(\frac{97.78}{2}\right)^2 + (40.7)^2}\right)$



$$\sigma_{B1} = 112.5\text{MPa}; \quad \sigma_{B2} = -14.72\text{MPa}$$

Q.4: A propeller shaft for a launch transmits 75 KW at 150 rpm and is subjected to a maximum bending moment of 1KN-m and an axial thrust of 70 KN. Find the shaft diameter based on maximum principal stress if the shear strength of the shaft material is limited to 100 MPa.

A.4:

$$\text{Torque, T} = \frac{75\text{x}10^3}{\left(\frac{2\pi\text{x}150}{60}\right)} = 4775\,\text{Nm; then, } \tau = \frac{24.3}{d^3}\,\text{KPa}$$

$$\text{Maximum bending moment} = 1\,\text{KNm; then, } \sigma_b = \frac{10.19}{d^3}\,\text{KPa}$$

$$\text{Axial force} = 70\,\text{KN; then, } \sigma = \frac{70}{\frac{\pi d^2}{4}}\,\text{KPa} = \frac{89.12}{d^2}\,\text{KPa}$$

$$\text{Maximum shear stress} = \sqrt{\left(\frac{89.12}{2d^2} - \frac{10.19}{2d^3}\right)^2 + \left(\frac{24.3}{d^3}\right)^2} = 100\text{x}10^3$$

Solving we get the value of shaft diameter d = 63.4 mm.

2.2.8 Summary of this Lecture

The stresses developed at a section within a loaded body and methods of superposing similar stresses have been discussed. Methods of combining normal and shear stresses using transformation of plane stresses have been illustrated. Formulations for principal stresses and maximum shear stresses have been derived and typical examples are solved.

Lecture

Theme 2

Stresses in machine elements

2.3. Strain analysis

2.3.1 Introduction

No matter what stresses are imposed on an elastic body, provided the material does not rupture, displacement at any point can have only one value. Therefore the displacement at any point can be completely given by the three single valued components u, v and w along the three co-ordinate axes x, y and z respectively. The normal and shear strains may be derived in terms of these displacements.

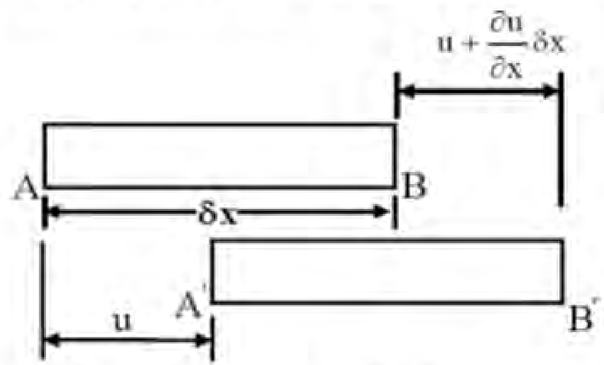
2.3.2 Normal strais

Consider an element AB of length δx (figure-2.3.2.1). If displacement of end A is u, that of end B is $u+\frac{\partial u}{\partial x}\delta x$. This gives an increase in length of ($u+\frac{\partial u}{\partial x}\delta x$ -u) and therefore the strain in x-direction is $\frac{\partial u}{\partial x}$.Similarly, strains in y and z directions are $\frac{\partial v}{\partial y}$ and $\frac{\partial w}{\partial z}$.Therefore, we may write the three normal strain components as

$$\varepsilon_x = \frac{\partial u}{\partial x},\ \varepsilon_y = \frac{\partial v}{\partial y} \text{ and } \varepsilon_z = \frac{\partial w}{\partial z}.$$

$$u_A = u(x),\ u_B = u(x+\delta x) = u(x) + \frac{\partial u}{\partial x}\delta x + \ldots$$

AB is initial length till deformation, A'B' is length after deforme.



2.3.2.1 - Change in length of an infinitesimal element.

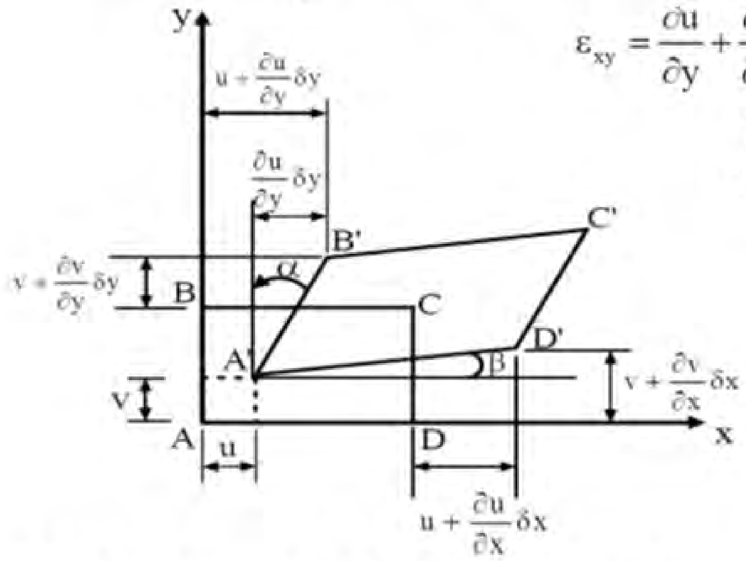
2.3.3 Shear strain

In the same way we may define the shear strains. For this purpose consider an element ABCD in x-y plane and let the displaced position of the element be A′B′C′D′ (Figure-2.3.3.1). This gives shear strain in xy plane as $\varepsilon_{xy}=\alpha+\beta$ where α is the angle made by the displaced line B′C′ with the vertical and β is the angle made by the displaced line A′D′ with the horizontal. This gives

$$\mathrm{tg}\,\alpha \approx \alpha = \frac{\frac{\partial u}{\partial y}\delta y}{\delta y} = \frac{\partial u}{\partial y} \quad \text{and} \quad \mathrm{tg}\,\beta \approx \beta = \frac{\frac{\partial v}{\partial x}\delta x}{\delta x} = \frac{\partial v}{\partial x}$$

We may therefore write the three shear strain components as

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \; \varepsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \; \text{and} \; \varepsilon_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$



2.3.3.1 - Shear strain associated with the distortion of an infinitesimal element.

Therefore, the complete strain matrix can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$$

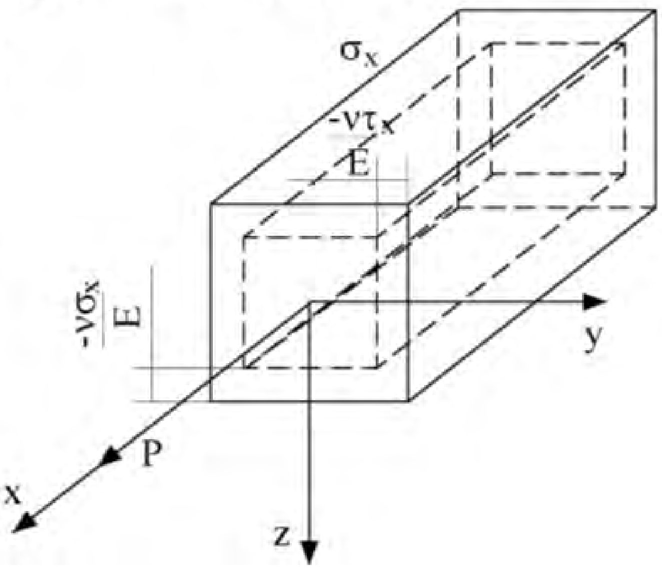
2.3.4 Constitutive equation

The state of strain at a point can be completely described by the six strain components and the strain components in their turns can be completely defined by the displacement components u, v, and w. **The constitutive equations relate stresses and strains and in linear elasticity we simply have $\sigma = E\varepsilon$ where E is modulus of elasticity.** It is also known that σx produces a strain of σ_x/E where E is direction, $-\nu\sigma_x/E$ in y-direction and $-\nu\sigma x/E$ in z-direction. The bar is stretched in direction x and is contracted in directions y and z. Therefore we may write the generalized **Hooke's law** as

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right], \quad \varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\varepsilon_{xy} = \gamma_{xy} = \frac{\tau_{xy}}{G}, \quad \varepsilon_{yz} = \gamma_{yz} = \frac{\tau_{yz}}{G} \text{ and } \varepsilon_{zx} = \gamma_{zx} = \frac{\tau_{zx}}{G}$$



In general each strain is dependent on each stress and we may write. Hooke's law in the form

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

For isotropic material mechanical and physical properties are indepent on direction. In this case we get

$$K_{11} = K_{22} = K_{33} = \frac{1}{E}$$

$$K_{12} = K_{13} = K_{21} = K_{23} = K_{31} = K_{32} = -\frac{\nu}{E}$$

$$K_{44} = K_{55} = K_{66} = \frac{1}{G}$$

Rest of the elements in K matrix are zero.

On substitution, this reduces the general constitutive equation to equations for isotropic materials as given by the generalized Hooke's law. Since the principal stress and strains axes coincide, we may write the principal strains in terms of principal stresses as

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right]$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \nu(\sigma_3 + \sigma_1)\right]$$

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu(\sigma_1 + \sigma_2)\right]$$

From the point of view of volume change or dilatation resulting from hydrostatic pressure we also have

$$\bar{\sigma} = K\Delta$$

where $\bar{\sigma} = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right) = \frac{1}{3}\left(\sigma_1 + \sigma_2 + \sigma_3\right)$ and $\Delta = (\varepsilon_x + \varepsilon_y + \varepsilon_z) = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

These equations allow the principal strain components to be defined in terms of principal stresses. For isotropic and homogeneous materials only two constants viz. E and ν are sufficient to relate the stresses and strains.

The strain transformation follows the same set of rules as those used in stress transformation except that the shear strains are halved wherever they appear.

2.3.5 Relations between E, G and K

The largest maximum shear strain and shear stress can be given by

$\gamma_{max} = \varepsilon_2 - \varepsilon_3$ and $\tau_{max} = \frac{\sigma_2 - \sigma_3}{2}$ and since $\gamma_{max} = \frac{\tau_{max}}{G}$ we have

$\frac{1}{E}\left[\sigma_2 - \nu(\sigma_1 + \sigma_3)\right] - \frac{1}{E}\left[\sigma_3 - \nu(\sigma_1 + \sigma_2)\right] = \frac{1}{G}\left(\frac{\sigma_2 - \sigma_3}{2}\right)$ and this gives

$$\boxed{G = E/2(1+\nu)}$$

Where G is modulus of rigidity or shear modulus, $0 \le \nu \le 0.5$ is Puasson's rafio.

Considering now the hydrostatic state of stress and strain we may write

$\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = K(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$. Substituting ε_1, ε_2 and ε_3 in terms of σ_1 , σ_2 and σ_3

we may write

$\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = K\left[(\sigma_1 + \sigma_2 + \sigma_3) - 2\nu(\sigma_1 + \sigma_2 + \sigma_3)\right]$ and this gives

$$\boxed{K = E/3(1-2\nu)}.$$

Where K is modulus or modulus of compession.

2.3.6 Elementary thermoelasticity

So far the state of strain at a point was considered entirely due to applied forces. Changes in temperature may also cause stresses if a thermal gradient or some external constraints exist. Provided that the materials remain linearly elastic, stress pattern due to thermal effect may be superimposed upon that due to applied forces and we may write

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha T, \qquad \varepsilon_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right] + \alpha T, \qquad \varepsilon_{yz} = \frac{\tau_{yz}}{G}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] + \alpha T, \qquad \varepsilon_{zx} = \frac{\tau_{zx}}{G}$$

Where T is temperature, α is cocfficient of thermal expension.

It is important to note that the shear strains are not affected directly by temperature changes.

It is sometimes convenient to express stresses in terms of strains. This may be done using the relation $\Delta(\varepsilon)=\varepsilon_x+\varepsilon_y+\varepsilon_z$. Substituting the above expressions for ε_x, ε_y and ε_z we have,

$$\Delta(\varepsilon)=\frac{1}{E}\left[(1-2\nu)\left(\sigma_x+\sigma_y+\sigma_z\right)\right]+3\alpha T$$

and substituting $K = E/3(1-2\nu)$ we have

$$\Delta(\sigma)=\frac{1}{3K}\left(\sigma_x+\sigma_y+\sigma_z\right)+3\alpha T\ .$$

Combining this with $\varepsilon_x=\frac{1}{E}\left[\sigma_x-\nu(\sigma_y+\sigma_z)\right]+\alpha T$ we have

$$\sigma_x=\frac{E\varepsilon_x}{1+\nu}+\frac{3\nu K(\Delta(\varepsilon)-3\alpha T)}{1+\nu}-\frac{E\alpha T}{1+\nu}$$

Substituting $G=E/2(1+\nu)$ and $\lambda=\frac{3\nu K}{1+\nu}$ we may write the normal and shear stresses as

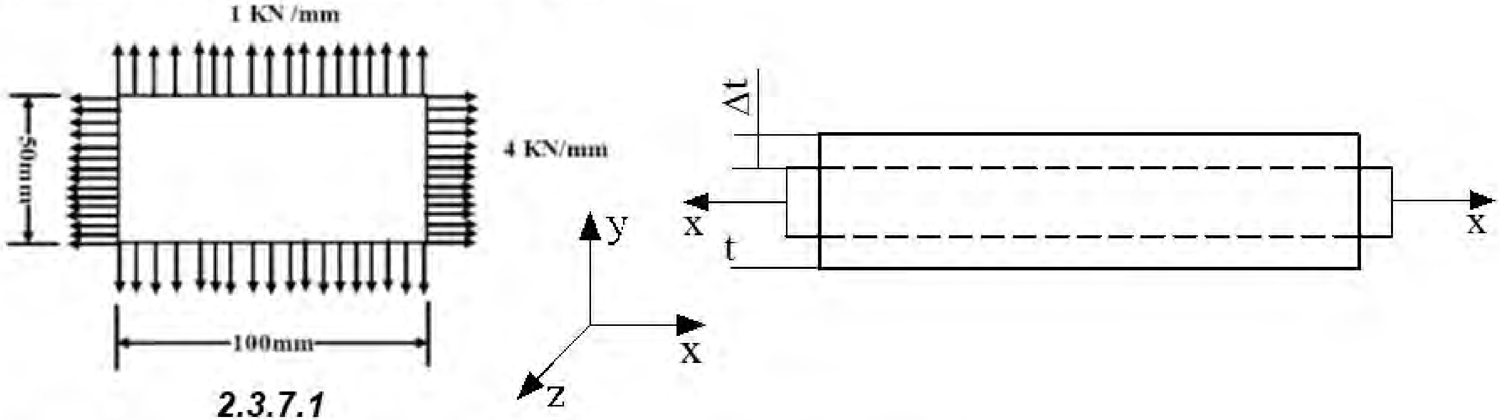
$$\sigma_x=2G\varepsilon_x+\lambda\Delta(\varepsilon)-3K\alpha T \qquad \tau_{xy}=G\varepsilon_{xy}$$
$$\sigma_y=2G\varepsilon_y+\lambda\Delta(\varepsilon)-3K\alpha T \qquad \tau_{yz}=G\varepsilon_{yz}$$
$$\sigma_z=2G\varepsilon_z+\lambda\Delta(\varepsilon)-3K\alpha T \qquad \tau_{zx}=G\varepsilon_{zx}$$

Where λ Lames coefficient, for T=0 we get Hooke's law.

These equations are considered to be suitable in thermoelastic situations.

2.3.7 Problems with Answers

Q.1: A rectangular plate of 10mm thickness is subjected to uniformly distributed load along its edges as shown in figure-2.3.7.1. Find the change in thickness due to the loading. E=200 GPa, v = 0.3



2.3.7.1

A.1: Here $\sigma_x = 400$ MPa, $\sigma_y = 100$ MPa and $\sigma_z = 0$

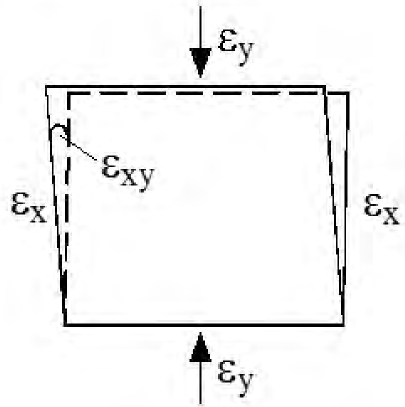
This gives $\varepsilon_z = -\frac{\nu}{E}\left(\sigma_x + \sigma_y\right) = -7.5\text{x}10^{-4}$

Now, $\varepsilon_z = \frac{\Delta t}{t}$ where, t is the thickness and Δt is the change in thickness.

Therefore, the change in thickness = 7.5 μm.

Q.2: At a point in a loaded member, a state of plane stress exists and the strains are ε_x=-90x10^{-6}, ε_y=-30x10^{-6} and ε_{xy}=120x10^{-6}. If the elastic constants E , v and G are 200 GPa , 0.3 and 84 GPa respectively, determine the normal stresses σ_x and σ_y and the shear stress τ_{xy} at the point.

A.2:



$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\sigma_x\right]$$

$$\varepsilon_{xy} = \frac{\tau_{xy}}{G}$$

This gives $\sigma_x = \frac{E}{1-\nu^2}\left[\varepsilon_x + \nu\varepsilon_y\right]$

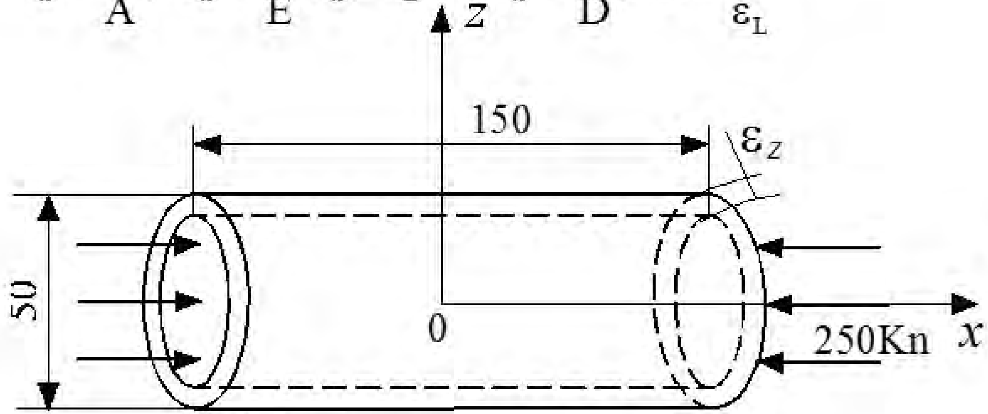
$$\sigma_y = \frac{E}{1-\nu^2}\left[\varepsilon_y + \nu\varepsilon_x\right]$$

Substituting values, we get

σ_x = -21.75 MPa, σ_y = -12.53 MPa and τ_{xy} = 9.23 MPa.

Q.3: A rod 50 mm in diameter and 150 mm long is compressed axially by an uniformly distributed load of 250 KN. Find the change in diameter of the rod if E = 200 GPa and v=0.3.

A.3: $\tau_x = \frac{F}{A}$, $\varepsilon_x = \frac{\lambda}{E}\sigma_x$, $\varepsilon_L = \nu\varepsilon_x = \frac{\Delta}{D}$, $D = \frac{\Delta}{\varepsilon_L}$



Axial stress $\sigma_x = \frac{250}{\frac{\pi}{4}(0.05)^2} = 127.3\,\text{MPa}$

Axial strain, $\varepsilon_x = 0.636\text{x}10^{-3}$

Lateral strain = $\nu\varepsilon_x = 1.9\text{x}10^{-4}$

Now, lateral strain, $\varepsilon_L = \frac{\Delta}{D}$ and this gives $\Delta(\varepsilon) = 9.5\mu\text{m}.$

Q.4: If a steel rod of 50 mm diameter and 1m long is constrained at the ends and heated to 200°C from an initial temperature of 20°C, what would be the axial load developed? Will the rod buckle? Take the coefficient of thermal expansion, α=12x10^{-6} per °C and E=200 GPa.

A.4: Thermal strain, $\varepsilon_t = \alpha \Delta T = 2.16 \text{x} 10^{-3}$, $\Delta T = T_1 - T_0$

In the absence of any applied load, the force developed due to thermal expansion, $F = E\varepsilon_t A = 848 KN$

For buckling to occur the critical load is given by

$$F_{cr} = \frac{\pi^2 EI}{l^2} = 605.59\, KN.$$

So F>F_{cr} Therefore, the rod will buckle when heated to 200°C.

2.3.8 Summary of this Lecture

Normal and shear strains along with the 3-D strain matrix have been defined. Generalized Hooke's law and elementary thermo-elasticity are discussed.

Lecture

Theme 3

Design for Strength

3.1. Design for static loading

3.1.1 Introduction

Machine parts fail when the stresses induced by external forces exceed their strength. The external loads cause internal stresses in the elements and the component size depends on the stresses developed. Stresses developed in a link subjected to uniaxial loading is shown in **figure-3.1.1.1**. Loading may be due to:

a) The energy transmitted by a machine element.

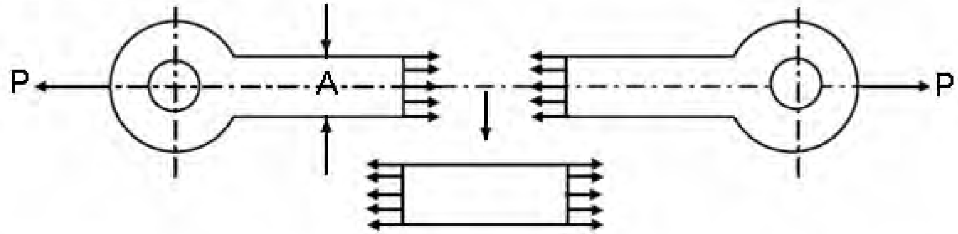
b) Dead weight.

c) Inertial forces.

d) Thermal loading.

e) Frictional forces.

$$\sigma = \frac{P}{A}$$



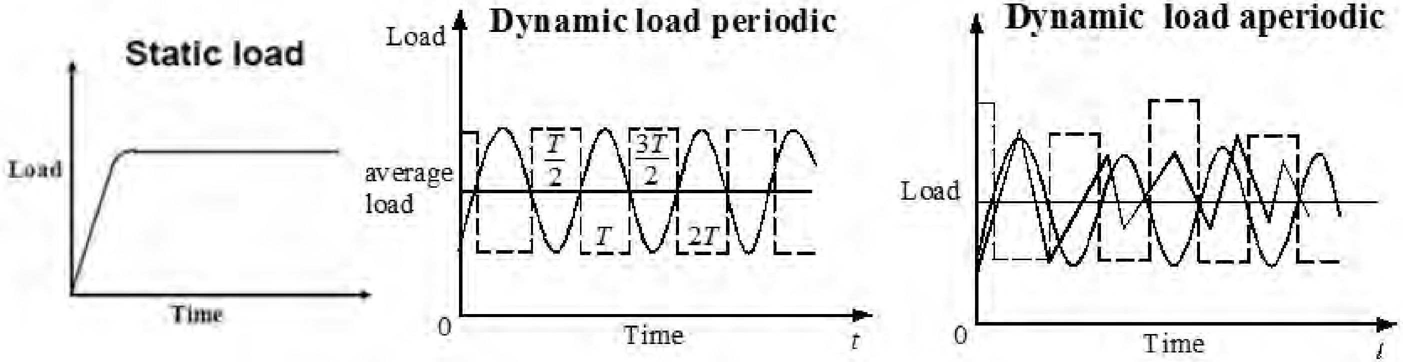
3.1.1.1 - Stresses developed in a link subjected to uniaxial loading

In another way, load may be classified as:

a) Static load- Load does not change in magnitude and direction and normally increases gradually to a steady value.

b) Dynamic load- Load may change in magnitude for example, traffic of varying weight passing a bridge.Load may change in direction, for example, load on piston rod of a double acting cylinder.

Vibration and shock are types of dynamic loading. **Figure-3.1.1.2** shows load vs time characteristics for both static and dynamic loading of machine elements.



3.1.1.2 - Types of loading on machine elements.

3.1.2 Allowable Stresses: Factor of Safety

Determination of stresses in structural or machine components would be meaningless unless they are compared with the material strength. If the induced stress is less than or equal to the limiting material strength then the designed component may be considered to be safe and an indication about the size of the component is obtained. The strength of various materials for engineering applications is determined in the laboratory with standard specimens. For example, for tension and compression tests a round rod of specified dimension is used in a tensile test machine where load is applied until fracture occurs. This test is usually carried out in a **Universal testing machine**. The load at which the specimen finally ruptures is known as **Ultimate load P_U and the ratio of load to original cross-sectional area A is the Ultimate stress σ_U.**

$$\frac{P_U}{A} = \sigma_U$$

Similar tests are carried out for bending, shear and torsion and the results for different materials are available in handbooks. For design purpose an allowable stress is used in place of the critical stress to take into account the uncertainties including the following:

1) Uncertainty in loading.
2) Inhomogeneity of materials.
3) Various material behaviors. e.g. corrosion, plastic flow, creep.
4) Residual stresses due to different manufacturing process.
5) Fluctuating load (fatigue loading): Experimental results and plot-ultimate strength depends on number of cycles.
6) Safety and reliability.

For ductile materials, the yield strength and for brittle materials the ultimate strength are taken as the critical stress.

An allowable stress is set considerably lower than the ultimate strength. The ratio of ultimate to allowable load or stress is known as factor of safety i.e.

$$\frac{\textbf{Ultimate Stress}}{\textbf{Allowable Stress}} = \textbf{F.S.} \qquad \frac{\sigma_U}{\sigma_a} = \text{F.S.}$$

The ratio must always be greater than unity. It is easier to refer to the ratio of stresses since this applies to material properties.

3.1.3 Theories of failure

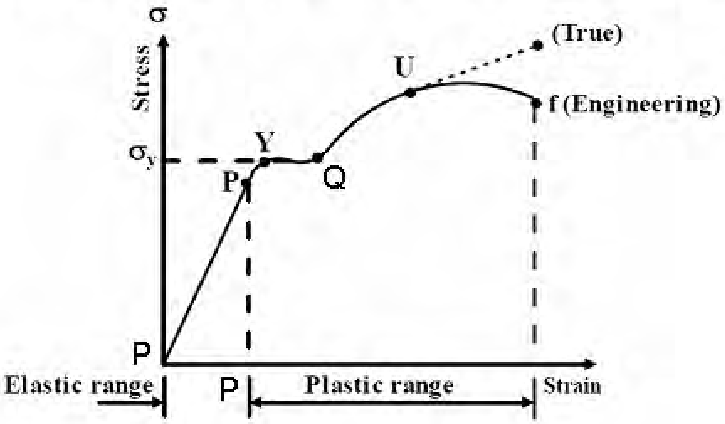
When a machine element is subjected to a system of complex stress system, it is important to predict the mode of failure so that the design methodology may be based on a particular failure criterion. Theories of failure are essentially a set of failure criteria developed for the ease of design. In machine design an element is said to have failed if it ceases to perform its function. There are basically two types of mechanical failure:

(a) **Yielding**- This is due to excessive inelastic deformation rendering the machine part unsuitable to perform its function. This mostly occurs in ductile materials.

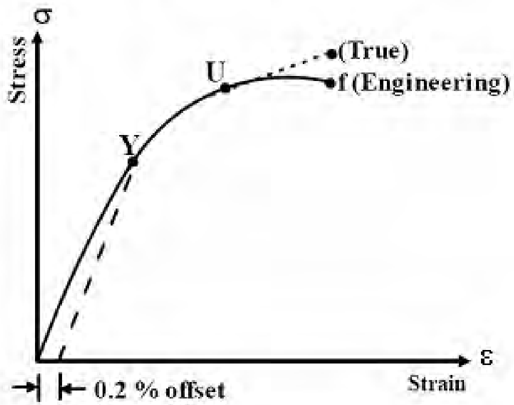
(b) **Fracture**- in this case the component tears apart in two or more parts. This mostly occurs in brittle materials. There is no sharp line of demarcation between ductile and brittle materials. However a rough guideline is that if percentage elongation is less than 5% then the material may be treated as brittle and if it is more than 15% then the material is ductile. However, there are many instances when a ductile material may fail by fracture. This may occur if a material is subjected to

(a) Cyclic loading.
(b) Long term static loading at elevated temperature.
(c) Impact loading.
(d) Work hardening.
(e) Severe quenching.

Yielding and fracture can be visualized in a typical tensile test as shown in the clipping- Typical engineering stress-strain relationship from simple tension ests for same engineering materials are shown in **figure- 3.1.3.1**.

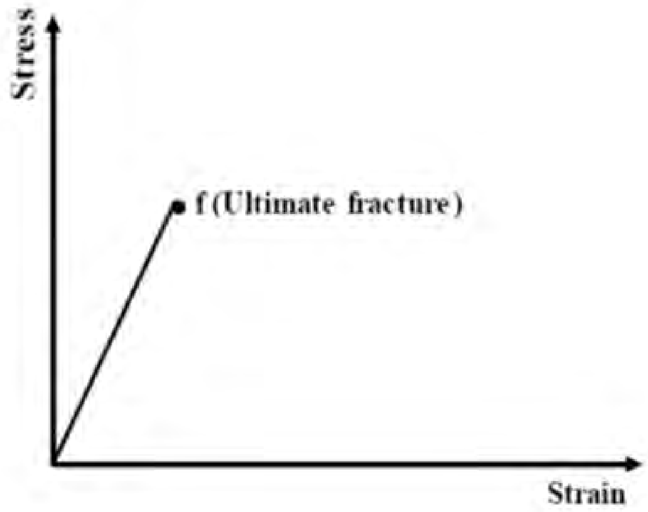


3.1.3.1 - (a) Stress-strain diagram for a ductile material e.g. low carbon steel.

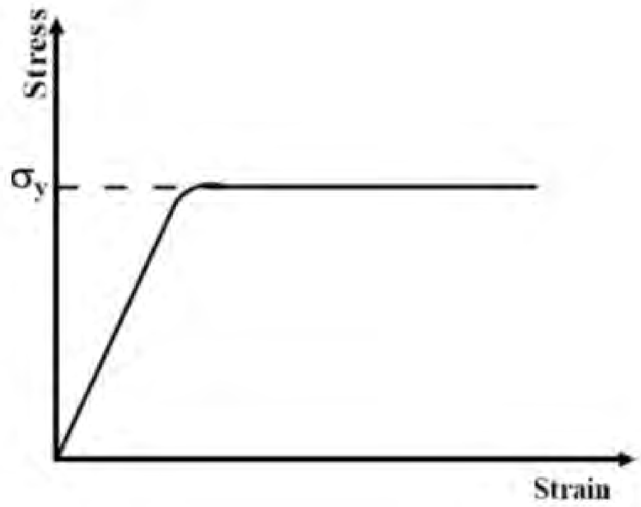


3.1.3.1 - (b) Stress-strain diagram for low ductility.

YQ on diagram is area of yeld, QU is area of plastic hardening, Uf is of plastic dishardening or fracture.



3.1.3.1 - (c) Stress-strain diagram for a brittle material.



3.1.3.1 - (d) Stress-strain diagram for an elastic – perfectly plastic material.

For a typical ductile material as shown in **figure-3.1.3.1 (a)** there is a definite yield point where material begins to yield more rapidly without any change in stress level. Corresponding stress is σ_y . Close to yield point is the proportional limit which marks the transition from elastic to plastic range. Beyond elastic limit for an elastic- perfectly plastic material yielding would continue without further rise in stress i.e. stress-strain diagram would be parallel to parallel to strain axis beyond the yield point. However, for most ductile materials, such as, low-carbon steel beyond yield point the stress in the specimens rises upto a peak value known as ultimate tensile stress σ_o . Beyond this point the specimen starts to neck-down i.e. the reduction in cross-sectional area. However, the stress-strain curve falls till a point where fracture occurs. The drop in stress is apparent since original cross-sectional area is used to calculate the stress. If instantaneous cross-sectional area is used the curve would rise as shown in **figure- 3.1.3.1 (a)** . For a material with low ductility there is no definite yield point and usually off-set yield points are defined for convenience. This is shown in **figure-3.1.3.1**. For a brittle material stress increases linearly with strain till fracture occurs. These are demonstrated in the **clipping- 3.1.3.2** .

3.1.4 Yield criteria

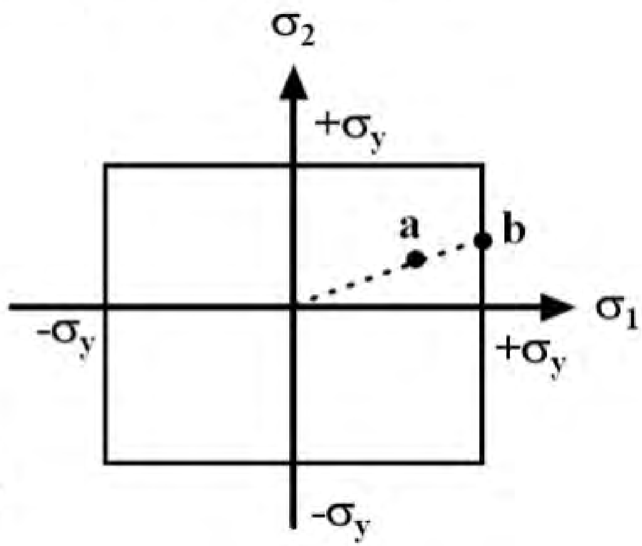
There are numerous yield criteria, going as far back as Coulomb (1773). Many of these were originally developed for brittle materials but were later applied to ductile materials. Some of the more common ones will be discussed briefly here.

3.1.4.1 Maximum principal stress theory (Rankine theory)

According to this, if one of the principal stresses σ_1 (maximum principal stress), σ_2 (minimum principal stress) or σ_3 exceeds the yield stress, yielding would occur. In a two dimensional loading situation for a ductile material where tensile and compressive yield stress are nearly of same magnitude

$$\sigma_1 = \pm \sigma_y \qquad \sigma_2 = \pm\sigma_y$$

Using this, a yield surface may be drawn, as shown in **figure- 3.1.4.1.1**. Yielding occurs when the state of stress is at the boundary of the rectangle. Consider, for example, the state of stress of a thin walled pressure vessel. Here $\sigma_1 = 2\sigma_2$, σ_1 being the circumferential or hoop stress and σ2 the axial stress. As the pressure in the vessel increases the stress follows the dotted line. At a point (say) a, the stresses are still within the elastic limit but at b, σ_1 reaches σy although σ_2 is still less than σy. Yielding will then begin at point b. This theory of yielding has very poor agreement with experiment. **However, the theory has been used successfully for brittle materials**



3.1.4.1.1 - Yield surface orresponding to maximum principal stress theory

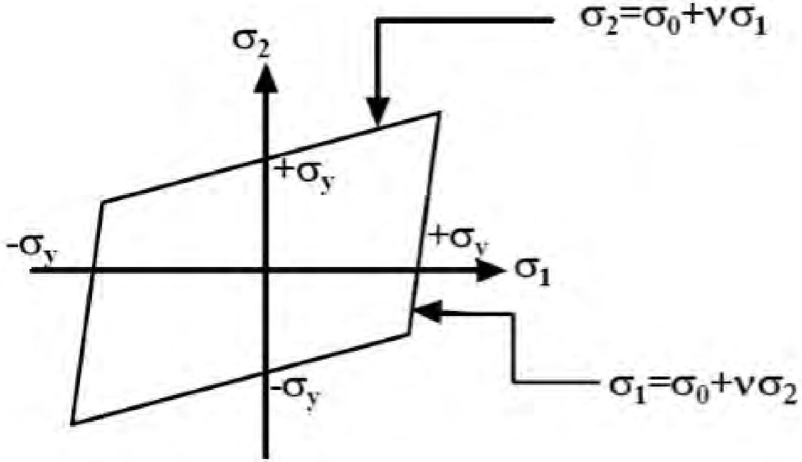
3.1.4.2 Maximum principal strain theory (St. Venant's theory)

According to this theory, yielding will occur when the maximum principal strain just exceeds the strain at the tensile yield point in either simple tension or compression. If ε_1 and ε_2 are maximum and minimum principal strains corresponding to σ_1 and σ_2, in the limiting case

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad |\sigma_1| \geq |\sigma_2| \qquad\qquad \varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad |\sigma_2| \geq |\sigma_1|$$

This gives, $E\varepsilon_1 = \sigma_1 - \nu\sigma_2 = \pm\sigma_0 \qquad E\varepsilon_2 = \sigma_2 - \nu\sigma_1 = \pm\sigma_0$

The boundary of a yield surface in this case is thus given as shown in **figure-3.1.4.2.1**



3.1.4.2.1- Yield surface corresponding to maximum principal strain theory

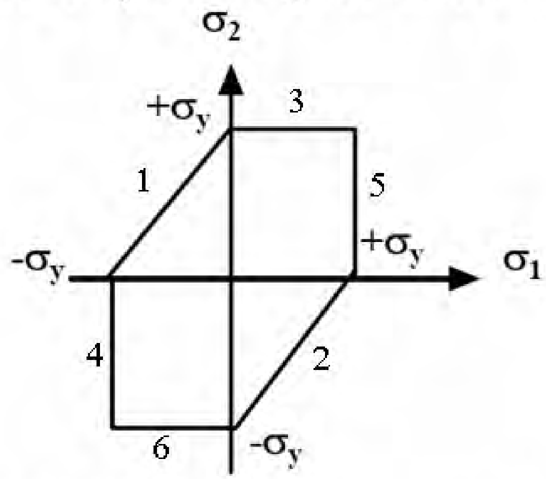
3.1.4.3 Maximum shear stress theory (Tresca theory)

According to this theory, yielding would occur when the maximum shear stress just exceeds the shear stress at the tensile yield point. At the tensile yield point $\sigma_2 = \sigma_3 = 0$ and thus maximum shear stress is $\sigma_y/2$. This gives us six conditions for a three-dimensional stress situation:

$$\sigma_1 - \sigma_2 = \pm\sigma_y$$

$$\sigma_2 - \sigma_3 = \pm\sigma_y$$

$$\sigma_3 - \sigma_1 = \pm\sigma_y$$



3.1.4.3.1 - Yield surface corresponding to maximum shear stress theory

In a biaxial stress situation (**figure-3.1.4.3.1**) case, $\sigma_3 = 0$ and this gives

$1.\ \sigma_1 - \sigma_2 = \sigma_y \quad \text{if } \sigma_1 > 0, \sigma_2 < 0$

$2.\ \sigma_1 - \sigma_2 = -\sigma_y \quad \text{if } \sigma_1 < 0, \sigma_2 > 0$

$3.\ \sigma_2 = \sigma_y \quad \text{if } \sigma_2 > \sigma_1 > 0$

$4.\ \sigma_1 = -\sigma_y \quad \text{if } \sigma_1 < \sigma_2 < 0$

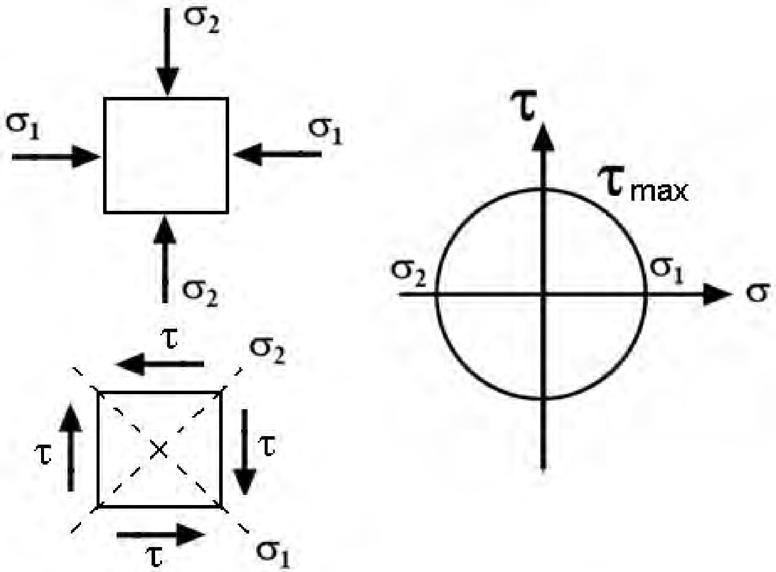
$5.\ \sigma_1 = -\sigma_y \quad \text{if } \sigma_1 > \sigma_2 > 0$

$6.\ \sigma_2 = -\sigma_y \quad \text{if } \sigma_2 < \sigma_1 < 0$

This criterion agrees well with experiment.

In the case of pure shear, σ_1=-σ_2=k (say), σ_3 = 0 and this gives σ_1-σ_2=2k=σ_y

This indicates that yield stress in pure shear is half the tensile yield stress and this is also seen in the Mohr's circle (figure- 3.1.4.3.2) for pure shear.



$$k = \frac{\sigma_y}{2} = \frac{\sigma_1 - \sigma_2}{2}$$

3.1.4.3.2 - Mohr's circle for pure shear

3.1.4.4 Maximum strain energy theory (Beltrami's theory)

According to this theory failure would occur when the total strain energy absorbed at a point per unit volume exceeds the strain energy absorbed per unit volume at the tensile yield point. This may be given

$$\frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3) = \frac{1}{2}\sigma_y\varepsilon_y \quad \text{by} \quad \frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3) = \frac{1}{2}\sigma_y\varepsilon_y$$

Substituting, ε_1, ε_2 , ε_3 and ε_y in terms of stresses we have

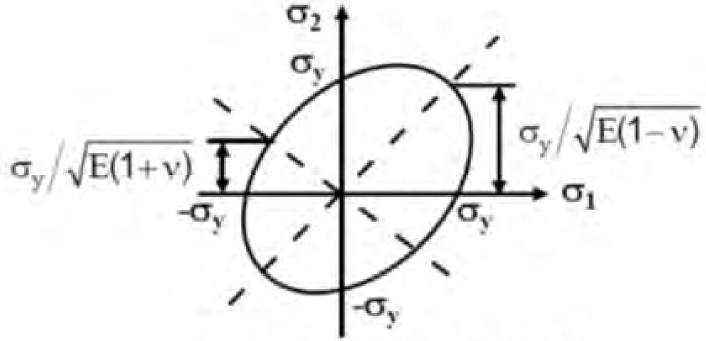
$$\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 - 2\upsilon(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = \sigma_y{}^2$$

$$\begin{cases} \varepsilon_1 = \dfrac{1}{E}[\sigma_1 - U(\sigma_2 + \sigma_1)] \\ \varepsilon_2 = \dfrac{1}{E}[\sigma_2 - U(\sigma_3 + \sigma_1)] \\ \varepsilon_3 = \dfrac{1}{E}[\sigma_3 - U(\sigma_1 + \sigma_2)] \end{cases}$$

For plan state of stresses, σ_3=0. This may be written as

For 2D: $$\left(\frac{\sigma_1}{\sigma_y}\right)^2+\left(\frac{\sigma_2}{\sigma_y}\right)^2-2\nu\left(\frac{\sigma_1\sigma_2}{\sigma_y{}^2}\right)=1\ ,\ \ \varepsilon_2-\varepsilon_1=\frac{1}{E}\left[\sigma_2-\sigma_1\right]$$

This is the equation of an ellipse and the yield surface is shown in **figure-3.1.4.4.1** .



3.1.4.4.1 - Yield surface corresponding to Maximum strain energy theory.

It has been shown earlier that only distortion energy can cause yielding but in the above expression at sufficiently high hydrostatic pressure $\sigma_1=\sigma_2=\sigma_3=\sigma$ (say), yielding may also occur.

3.1.4.5 Distortion energy theory(von Mises yield criterion)

According to this theory yielding would occur when total distortion energy absorbed per unit volume due to applied loads exceeds the distortion energy absorbed per unit volume at the tensile yield point. Total strain energy ET and strain energy for volume change E_V can be given as

$$E_T = \frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3) \text{ and } E_V = \frac{3}{2}\sigma_{av}\varepsilon_{av}$$

Substituting strains in terms of stresses the distortion energy can be given as

$$E_d = E_T - E_V = \frac{2(1+\nu)}{6E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1\right)$$

At the tensile yield point, $\sigma_1 = \sigma_y$, for 1-D $\sigma_2 = \sigma_3 = 0$ which gives

$$E_{dy} = \frac{2(1+\nu)}{3E}\sigma_y^2$$

The failure criterion is thus obtained by equating E_d and E_{dy} , which gives

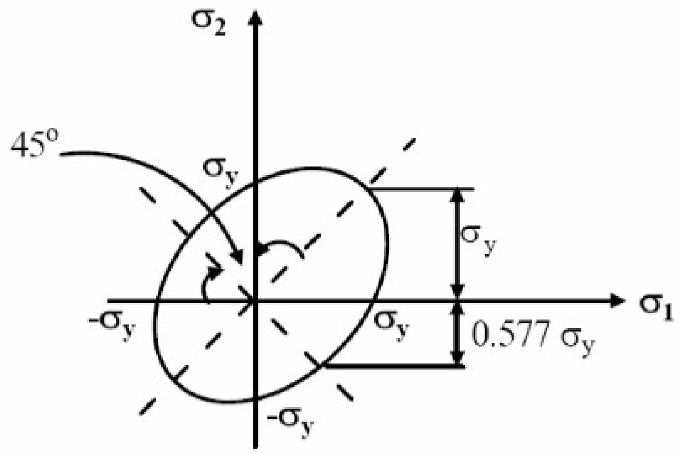
$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$$

In a 2-D situation if $\sigma_3 = 0$, the criterion reduces to

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2$$

$$\text{i.e.}\quad \left(\frac{\sigma_1}{\sigma_y}\right)^2 + \left(\frac{\sigma_2}{\sigma_y}\right)^2 - \left(\frac{\sigma_1}{\sigma_y}\right)\left(\frac{\sigma_2}{\sigma_y}\right) = 1$$

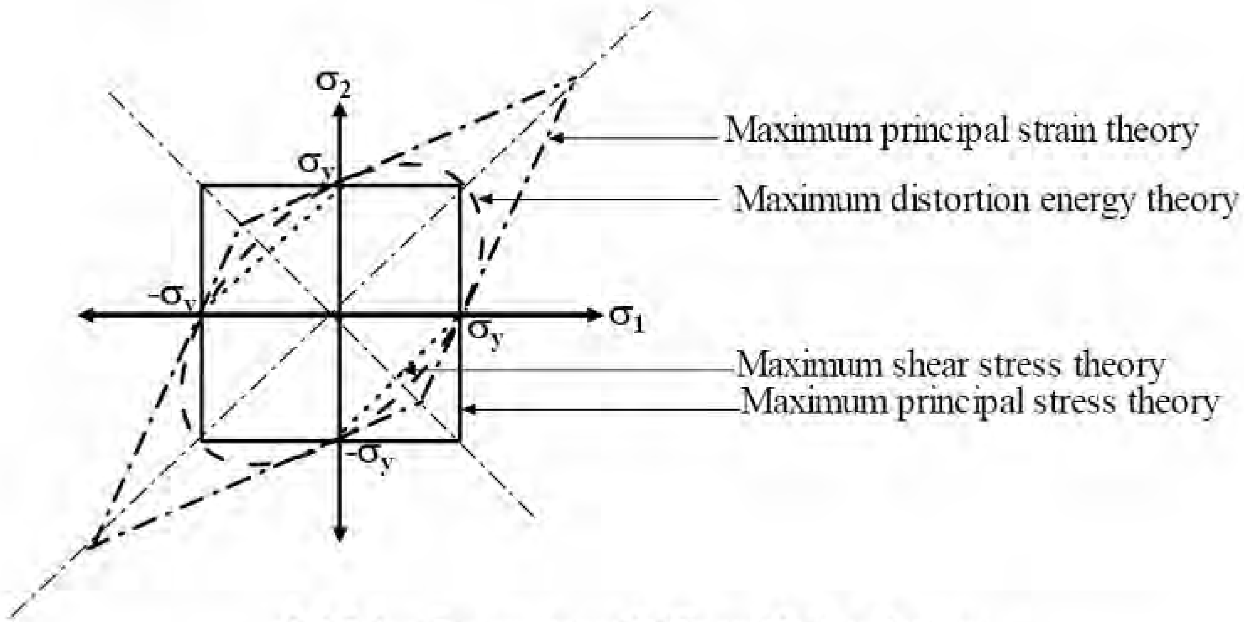
This is an equation of ellipse and the yield surface is shown in **figure-3.1.4.5.1** . This theory agrees very well with experimental results and is widely used for ductile materials.



3.1.4.5.1 *- Yield surface corresponding to von Mises yield criterion.*

3.1.5 Superposition of yield surface

A comparison among the different failure theories can be made by superposing the yield surfaces as shown in **figure- 3.1.5.1**.



3.1.5.1 *- Comparison of different failure theories.*

It is clear that an immediate assessment of failure probability can be made just by plotting any experimental in the combined yield surface. Failure of ductile materials is most accurately governed by the distortion energy theory where as the maximum principal strain theory is used for brittle materials.

3.1.6 Problems with Answers

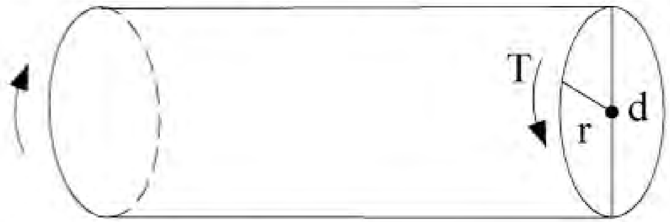
Q.1: A shaft is loaded by a torque of 5 KN-m. The material has a yield point of 350 MPa. Find the required diameter using

(a) Maximum shear stress theory

(b) Maximum distortion energy theory

Take a factor of safety of 2.5.

$$\tau = \frac{TJ}{r}$$



τ is shear stress, T is torque, r is radius, J is polar moment of inertia.

A.1: Torsional shear stress induced in the shaft due to 5 KN-m torque is

$$\tau = \frac{16x(5x10^3)}{\pi d^3}$$ where d is the shaft diameter in m.

(b) Maximum shear stress theory,

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$ Since $\sigma_x = \sigma_y = 0$, $\tau_{max}=25.46x10^3/d^3 = \frac{\sigma_Y}{2xF.S.} = \frac{350x10^6}{2x2.5}$

This gives d=71.3 mm.

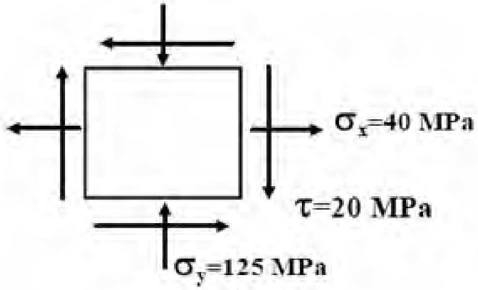
(b) Maximum distortion energy theory

In this case $\sigma_1 = 25.46x10^3/d^3$ $\sigma_2 = -25.46x10^3/d^3$

According to this theory, $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 2(\sigma_Y/F.S)^2$

Since $\sigma_3 = 0$, substituting values of σ_1, σ_2 and σ_Y we get d=68 mm.

Q.2: The state of stress at a point for a material is shown in the **figure-3.1.6.1**. Find the factor of safety using (a) Maximum shear stress theory (b) Maximum distortion energy theory. Take the tensile yield strength of the material as 400 MPa.



3.1.6.1

A.2:

From the Mohr's circle, shown in **figure-3.1.6.2**

$$\sigma_1 = 42.38 \text{ MPa}$$

$$\sigma_2 = -127.38 \text{ MPa}$$

(a) Maximum shear stress theory

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_Y}{2xF.S}$$

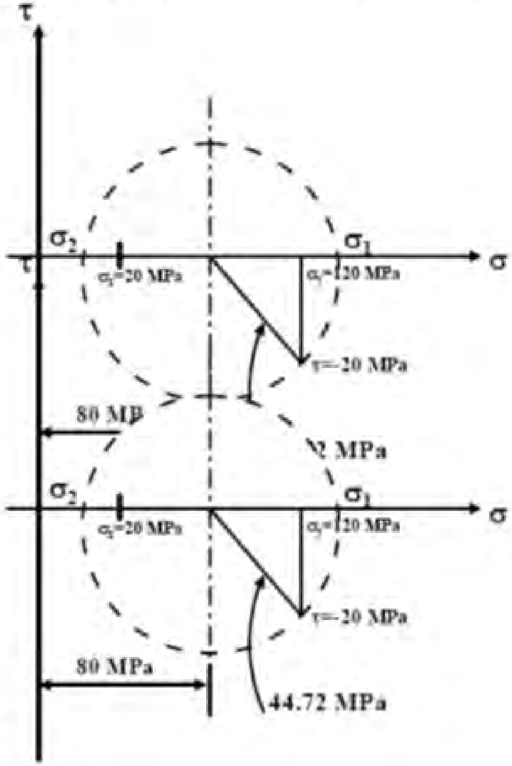
This gives F.S = 2.356.

(b) Maximum distortion energy theory

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 2(\sigma_Y / F.S)^2$$

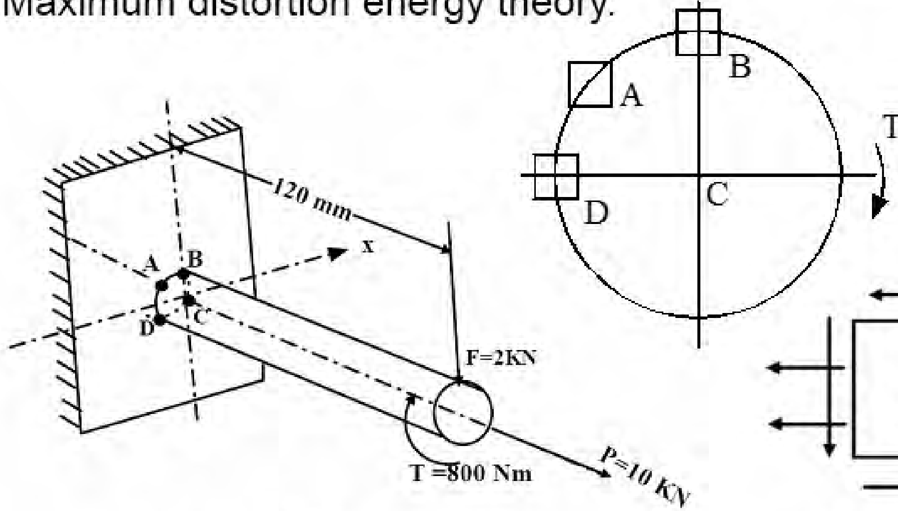
If $\sigma_3 = 0$ this gives F.S = 2.613.

Compare a) and b) we get $(F.S.)_M > (F.S.)_T$

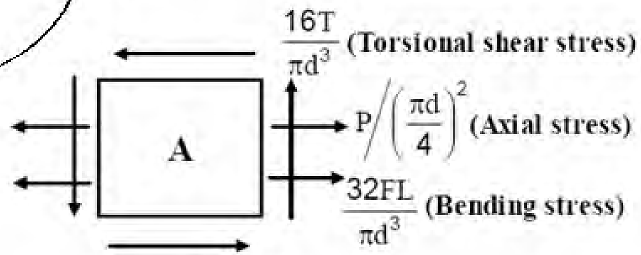


3.1.6.2

Q.3: A cantilever rod is loaded as shown in the **figure- 3.1.6.3**. If the tensile yield strength of the material is 300 MPa determine the rod diameter using (a) Maximum principal stress theory (b) Maximum shear stress theory (c) Maximum distortion energy theory.



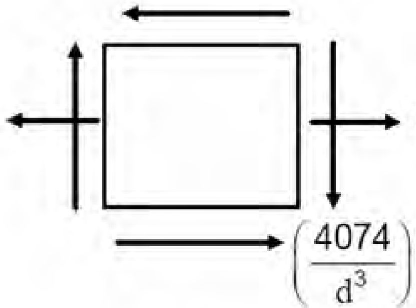
3.1.6.3



3.1.6.4

A.3: At the outset it is necessary to identify the mostly stressed element. Torsional shear stress as well as axial normal stress is the same throughout the length of the rod but the bearing stress is largest at the welded end. Now among the four corner elements on the rod, the element A is mostly loaded as shown in **figure-3.1.6.4**

Shear stress due to bending VQ/It is also developed but this is neglected due to its small value compared to the other stresses. Substituting values of T, P, F and L, the elemental stresses may be shown as in **figure-3.1.6.5**:



$$(\sigma_A + \tau_B) = \left(\frac{12732}{d^2} + \frac{2445}{d^3}\right)$$

$$\sigma_x = \tau - \text{torsional}$$
$$\sigma_y = \tau_B - \text{bending}$$

3.1.6.5

This gives the principal stress as

$$\sigma_{1,2} = \frac{1}{2}\left(\frac{12732}{d^2} + \frac{2445}{d^3}\right) \pm \sqrt{\frac{1}{4}\left(\frac{12732}{d^2} + \frac{2445}{d^3}\right)^2 + \left(\frac{4074}{d^3}\right)^2}$$

(a) Maximum principal stress theory, Setting $\sigma_1 = \sigma_Y$ we get d = 26.67 mm.

(b) Maximum shear stress theory,

Setting $\frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_Y}{2}$, we get d = 30.63 mm.

(c) Maximum distortion energy theory,

Setting $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 2(\sigma_Y)^2$

We get d = 29.36 mm.

3.1.7 Summary of this Lecture

Different types of loading and criterion for design of machine parts subjected to static loading based on different failure theories have been demonstrated. Development of yield surface and optimization of design criterion for ductile and brittle materials were illustrated.

Lecture

Theme 3

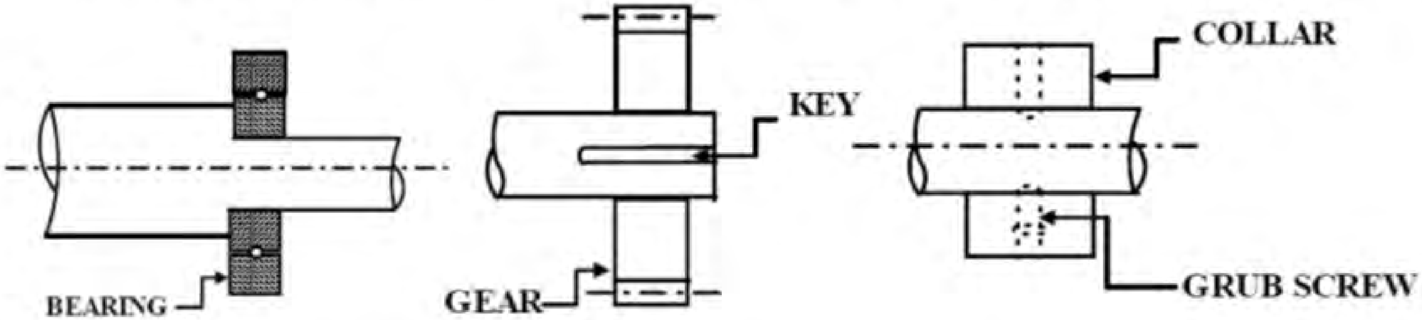
Design for Strength

3.2. Stress Concentration

3.2.1 Introduction

Before we studed in what way the choice of material for machine parts affects (ufluences) to distribution of stresses and strain in elements of machines. Now we cansider in what way the schope of the machine element influences (affects) to distribution of stresses and strain in machine parts. Euch designer must be able to give out instructions what material to take for creation of element and what elements hape is optimum.

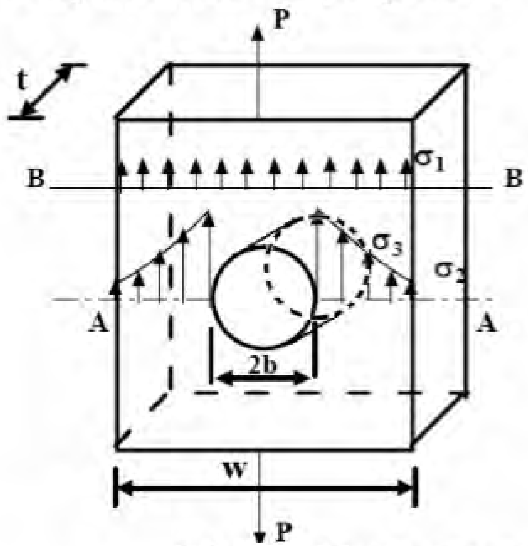
In developing a machine it is impossible to avoid changes in cross-section, holes,



3.2.1.1 - Some typical illustrations leading to stress concentration.

Any such discontinuity in a member affects the stress distribution in the neighbourhood and the discontinuity acts as a stress raiser.

Consider a plate with a centrally located hole and the plate is subjected to uniform tensile load at the ends. Stress distribution at a section A-A passing through the hole and another section BB away from the hole are shown in **figure- 3.2.1.2**. Stress distribution away from the hole is uniform but at AA there is a sharp rise in stress in the vicinity of the hole. Stress concentration factor k_t is defined as $k_t = \sigma_3/\sigma_{av}$, where σ_{av} at section AA is simply P/t(w−2b)=P/tw. This is the theoretical or geometric stress concentration factor and the factor is not affected by the material properties.



$$k_t = \frac{\sigma_3}{\sigma_{av}}$$

$$\sigma_{av} = \frac{P}{t(w-2b)} = \frac{P}{tw}$$

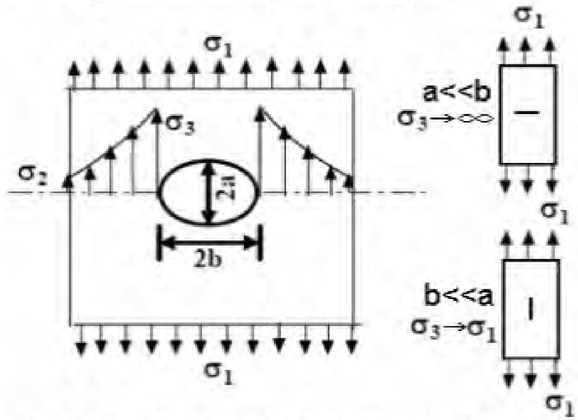
$$\boxed{\sigma_3 = \sigma_{max} = \sigma_{av} \cdot k_t}$$

3.2.1.2 - Stress concentration due to a central hole in a plate subjected to an uni-axial loading.

It is possible to predict the stress concentration factors for certain geometric shapes using theory of elasticity approach. For example, for an elliptical hole in an infinite plate, subjected to a uniform tensile stress σ_1 (**figure- 3.2.1.3**), stress z distribution around the discontinuity is disturbed and at points remote from the discontinuity the effect is insignificant. According to such an analysis

$$\sigma_3 = \sigma_1\left(1+\frac{2b}{a}\right)$$

If a=b the hole reduces to a circular one and therefore $\sigma_3 = 3\sigma_1$ which gives k_t=3. If, however 'b' is large compared to 'a' then the stress at the edge of transversecrack is very large and consequently k is also very large. If 'b' is small compared to a then the stress at the edge of a longitudinal crack does not rise and k_t =1.



3.2.1.3 - Stress concentration due to a central elliptical hole in a plate subjected to a uni-axial loading.

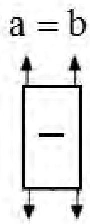
Stress concentration factors may also be obtained using any one of the following experimental techniques:

1. Strain gage method
2. Photoelasticity method
3. Brittle coating technique
4. Grid method

$$\sigma_{max} = \sigma_{av} \cdot k_t$$

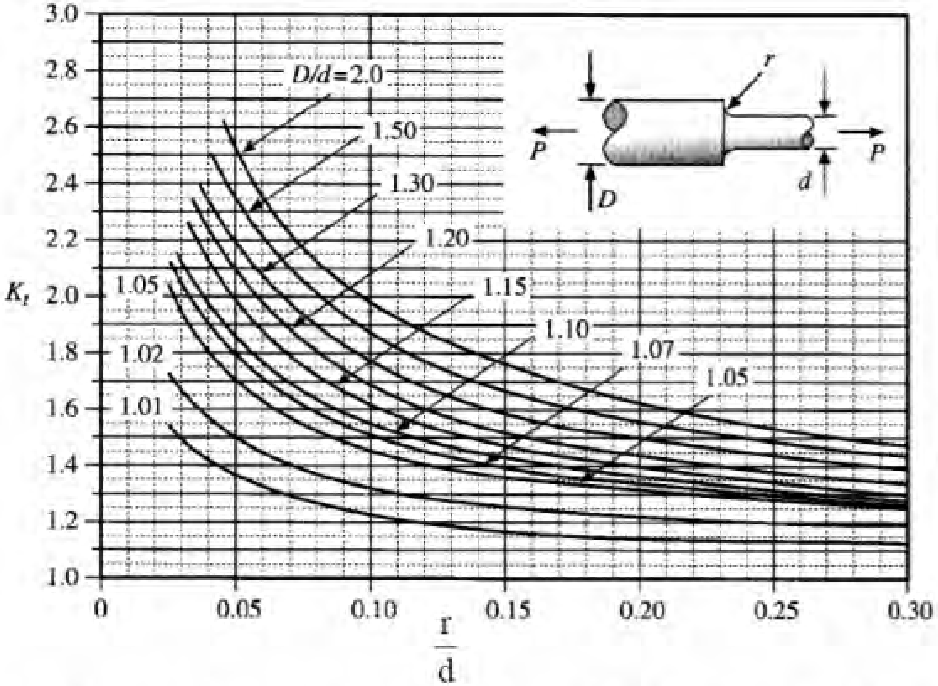
$$k_t = 1 \text{ without hole, } k_t \to \infty,\ b >> a$$

$$k_t = 3 \text{ round hole, } k_t \to 1,\ b << a$$



For more accurate estimation numerical methods like Finite element analysis may be employed.

Theoretical stress concentration factors for different configurations are available in handbooks. Some typical plots of theoretical stress concentration factors and r/d ratio for a stepped shaft are shown in **figure-3.2.1.4.**



3.2.1.4 - Variation of theoretical stress concentration factor with r/d of a stepped shaft for different values of D/d subjected to uni-axial loading.

$$\sigma_a = k_t \sigma_c$$

In design under fatigue loading, stress concentration factor is used in modifying the values of endurance limit while in design under static loading it simply acts as stress modifier. This means Actual stress σ_a= k_t × calculated stress σ_c. For ductile materials under static loading effect of stress concentration is not very serious but for brittle materials even for static loading it is important.

It is found that some materials are not very sensitive to the existence of notches or discontinuity. In such cases it is not necessary to use the full value of t k and instead a reduced value is needed. This is given by a factor known as fatigue strength reduction factor k_f and this is defined as

$$k_f = \frac{\text{Endurance limit of notch free specimens}}{\text{Endurance limit of notched specimens}}$$

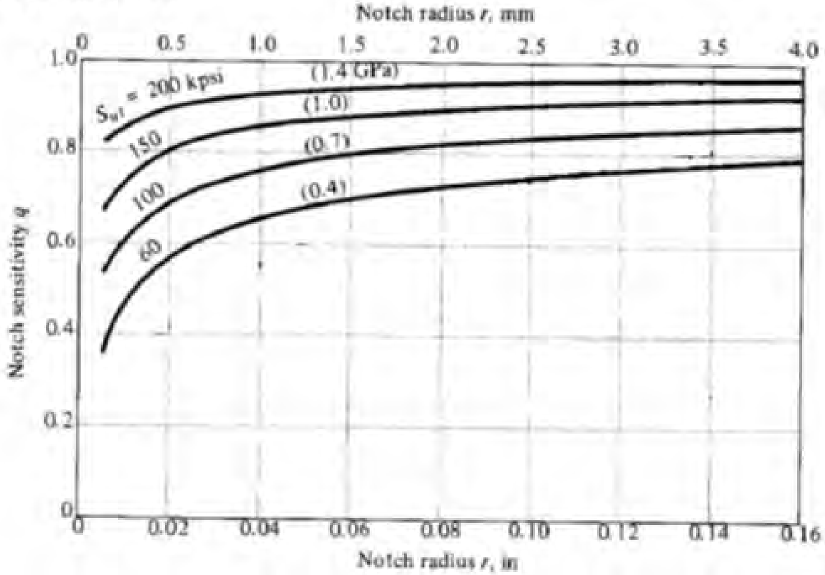
$$k_f = \frac{\sigma_f}{\sigma_t}$$

Another term called Notch sensitivity factor, q is often used in design and this is defined as

$$q = \frac{k_f - 1}{k_t - 1}$$

The value of 'q' usually lies between 0 and 1. If q=0, k_f =1 and this indicates no notch sensitivity. If however q=1, then f k_f = k_t and this indicates full notch sensitivity. Design charts for 'q' can be found in design hand-books and knowing k_t, k_f may be obtained. A typical set of notch sensitivity curves for steel is

3.2.1.5 - Variation of notch sensitivity with notch radius for steels of different ultimate tensile strength.

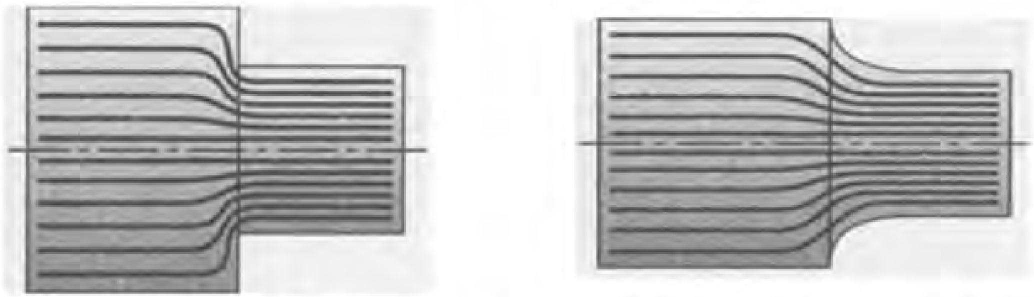


3.2.2 Methods of reducing stress concentration

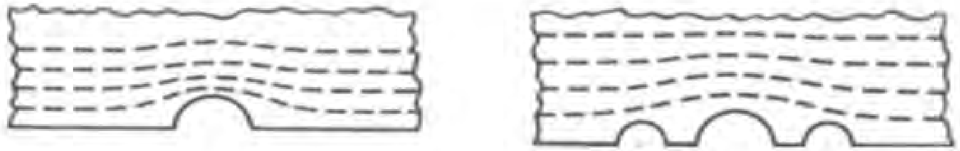
A number of methods are available to reduce stress concentration in machine parts. Some of them are as follows:

1. Provide a fillet radius so that the cross-section may change gradually.
2. Sometimes an elliptical fillet is also used.
3. If a notch is unavoidable it is better to provide a number of small notches rather than a long one. This reduces the stress concentration to a large extent.
4. If a projection is unavoidable from design considerations it is preferable to provide a narrow notch than a wide notch.
5. Stress relieving groove are sometimes provided.

These are demonstrated in **figure- 3.2.2.1.**

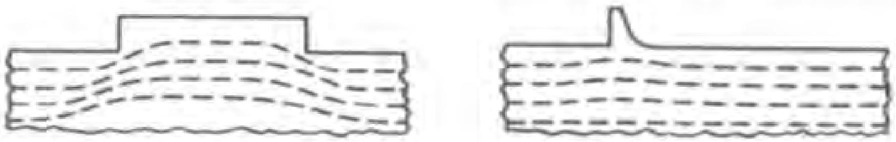


(a) Force flow around a sharp corner   Force flow around a corner with fillet: Low stress concentration.



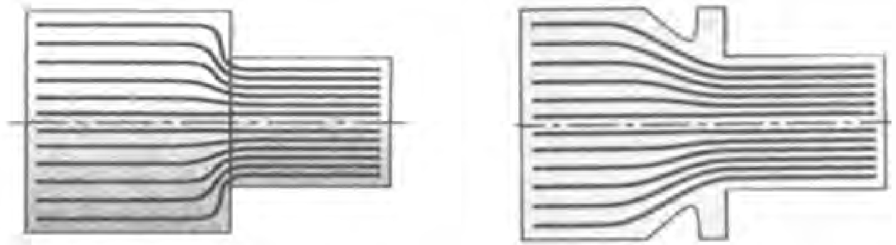
(b) Force flow around a large notch

Force flow around a number of small notches: Low stress concentration.



(c) Force flow around a wide projection

Force flow around a narrow projection: Low stress concentration.



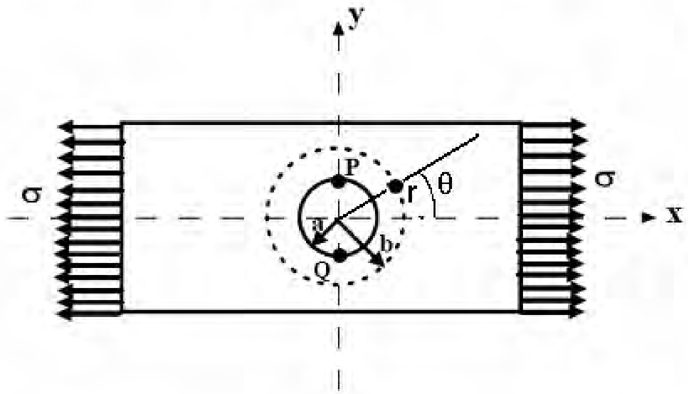
(d) Force flow around a sudden change in diameter in a shaft

Force flow around a stress relieving groove..

3.2.2.1 - Illustrations of different methods to reduce stress Concentration.

3.2.3 Theoretical basis of stress concentration

Consider a plate with a hole acted upon by a stress σ . St. Verant's principle states that if a system of forces is replaced by another statically equivalent system of forces then the stresses and displacements at points remote from the region concerned are unaffected. In **figure-3.2.3.1** 'a' is the radius of the hole and at r=b, b>>a the stresses are not affected by the presence of the hole.



3.2.3.1 - A plate with a central hole subjected to a uni-axial stress

Here, $\sigma_x = \sigma$, $\sigma_y = 0$, $\tau_{xy} = 0$

For plane stress conditions:
in polar coordinates system we get

$$\sigma_r = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta$$

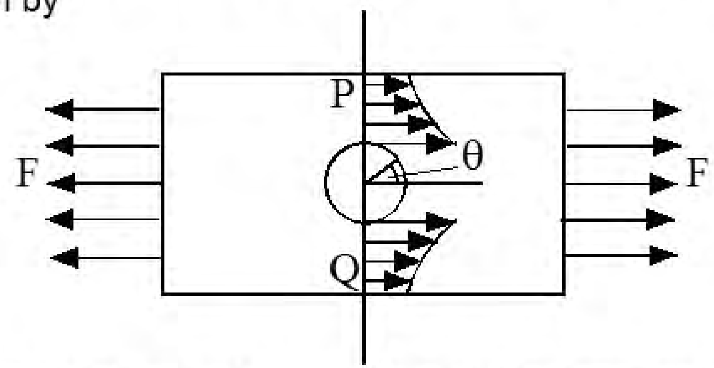
$$\sigma_\theta = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$

$$\tau_{r\theta} = (\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

We can this reduce to $\sigma_r = \sigma\cos^2\theta = \frac{\sigma}{2}(\cos 2\theta + 1) = \frac{\sigma}{2} + \frac{\sigma}{2}\cos 2\theta$

$$\sigma_\theta = \sigma\sin^2\theta = \frac{\sigma}{2}(1 - \cos 2\theta) = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta \qquad \tau_{r\theta} = -\frac{\sigma}{2}\sin 2\theta$$

such that 1st component in σ_r and σ_θ is constant and the second component varies with θ . Similar argument holds for $\tau_{r\theta}$ if we write $\tau_{r\theta} = (\sigma/2)\sin 2\theta$. The stress distribution within the ring with inner radius r_i=a and outer radius or r_o=b
due to 1st component can be analyzed using the solutions of thick cylinders and the effect due to the 2nd component can be analyzed following the Stress-function approach. Using a stress function of the form φ=R(r)cos2θ the stress distribution due to the 2nd component can be found and it was noted that the dominant stress is the Hoop Stress, given by



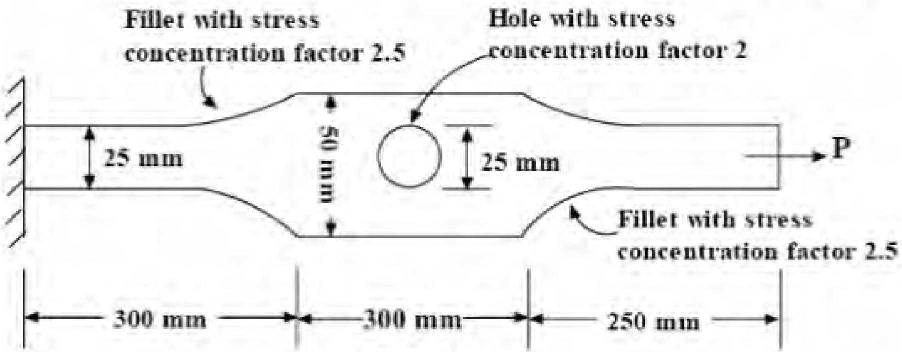
$$\sigma_\theta = \frac{\sigma}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta$$

This is maximum at $\theta = \pm\pi/2$ and the maximum value of $\sigma_\theta = \frac{\sigma}{2}\left(2 + \frac{a^2}{r^2} + \frac{3a^4}{r^4}\right)$

Therefore at points P and Q where r = a σ_θ is maximum and is given by $\sigma_\theta = 3\sigma$
i.e. stress concentration factor is 3. This result we had before when we got answer which is same to problem for a plate with a hole.

3.2.4 Problems with Answers

Q.1: The flat bar shown in **figure- 3.2.4.1** is 10 mm thick and is pulled by a force P producing a total change in length of 0.2 mm. Determine the maximum stress developed in the bar. Take E= 200 GPa.



3.2.4.1

$$t = 10\text{mm}$$
$$l = 0.2\text{mm}$$
$$E = 200\text{GPa}$$
$$\sigma_{max} = ?$$

A.1: Total change in length of the bar is made up of three components and this is given by

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$$

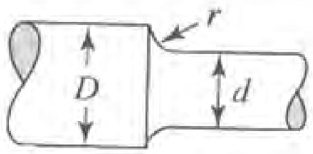
$$\Delta l = \frac{l_1 P_1}{S_1 E} + \frac{l_2 P_2}{S_2 E} + \frac{l_3 P_3}{S_3 E} = \left[\frac{0.3}{0.025x0.01} + \frac{0.3}{0.05x0.01} + \frac{0.25}{0.025x0.01}\right]\frac{P}{200x10^9} = 0.2x10^{-3}$$

This gives P=14.285 KN.

Stress at the shoulder $\sigma_s = k\frac{16666}{(0.05-0.025)x0.01}$ where k=2.

This gives σh = 114.28 MPa.

Q.2: Find the maximum stress developed in a stepped shaft subjected to a twisting moment of 100 Nm as shown in **figure- 3.2.4.2**. What would be the maximum stress developed if a bending moment of 150 Nm is applied.



r = 6 mm

d = 30 mm

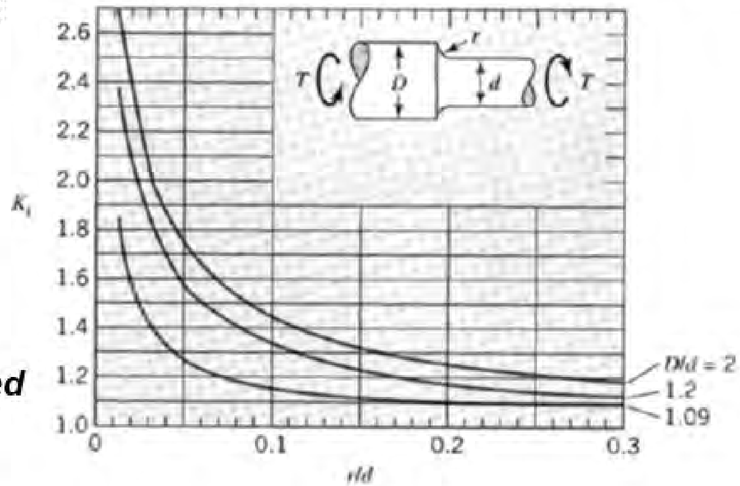
D = 40 mm.

3.2.4.2

A.2: Referring to the stress- concentration plots in **figure- 3.2.4.3** which we take from handbook, for stepped shafts subjected to torsion for r/d = 0.2 and D/d = 1.33, $K_t \approx 1.23$. Tby $\tau = \frac{16T}{\pi d^3}$. ear stress is given

Considering the smaller diameter and the stress concentration effect at the step, we ha $\tau_{max} = K_t \frac{16x100}{\pi(0.03)^3}$ hear stre

This gives τ_{max} = 23.201 MPa.



3.2.4.3 - Variation of theoretical stress concentration factor with r/d for a stepped shaft subjected to torsion.

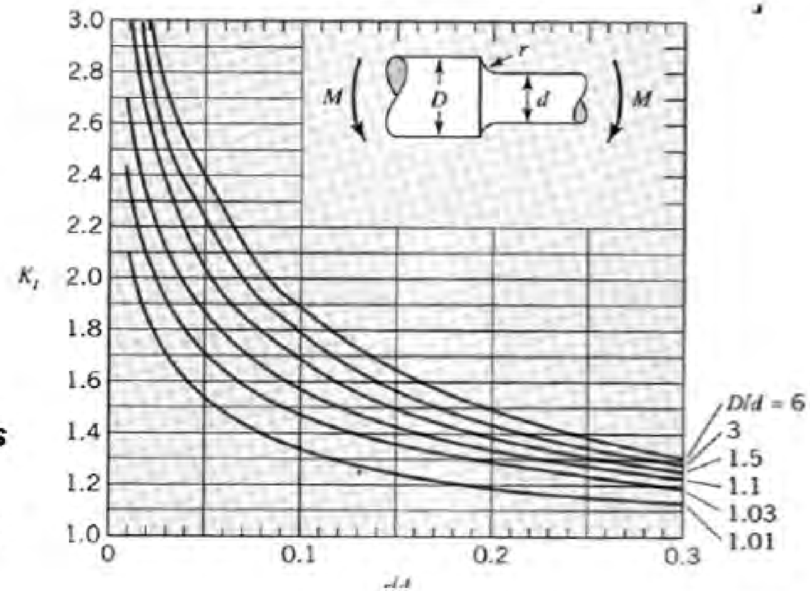
Similarly referring to stress-concentration plots in **figure- 3.2.4.4** which we take from handbook for stepped shaft subjected to bending , for r/d = 0.2 and D/d = 1.33, $K_t \approx 1.48$

Bending stress is given by $\sigma = \frac{32M}{\pi d^3}$

Considering the smaller diameter and the effect of stress concentration at the step, we have the maximum bending stress as

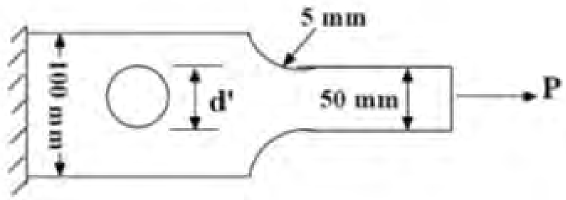
$$\sigma_{max} = K_t \frac{32x150}{\pi(0.03)^3}$$

This gives σ_{max} = 83.75 MPa.



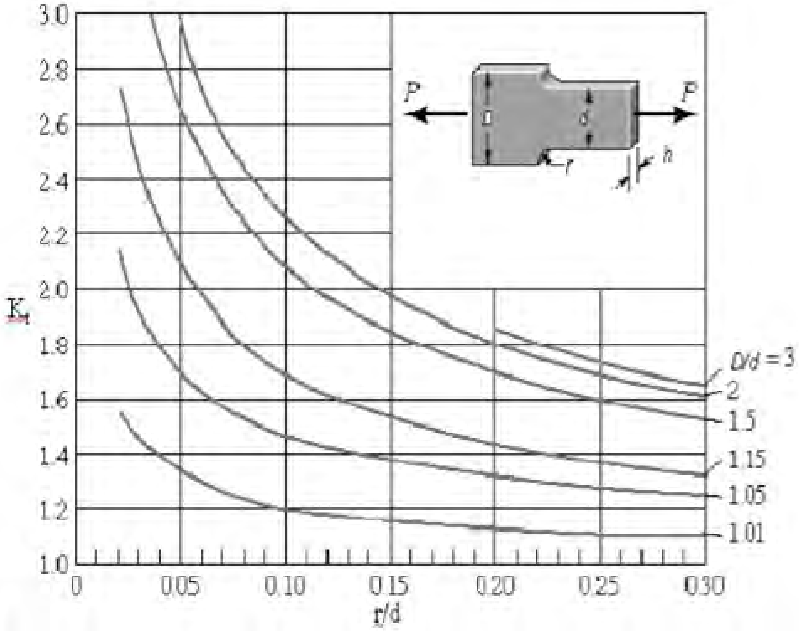
3.2.4.4 - Variation of theoretical stress concentration factor with r/d for a stepped shaft subjected to a bending moment.

Q.3: In the plate shown in **figure- 3.2.4.5** it is required that the stress concentration at Hole does not exceed that at the fillet. Determine the hole diameter.



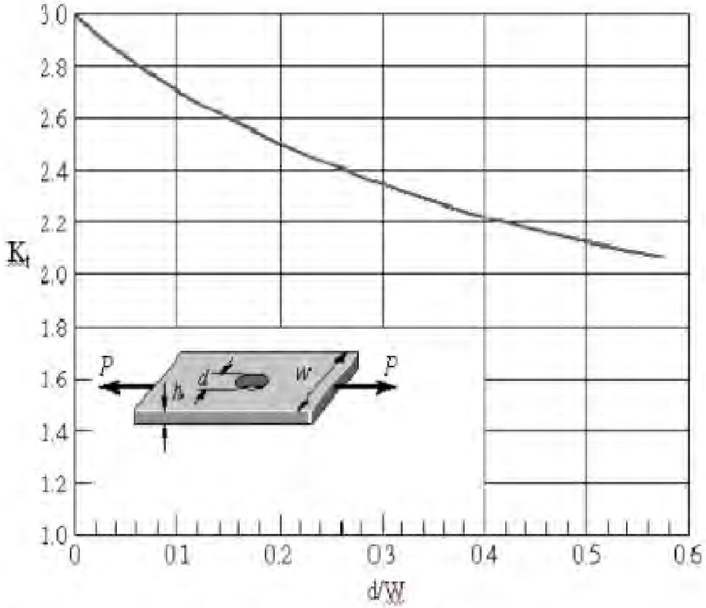
3.2.4.5

A.3: Referring to stress-concentration plots for plates with fillets under axial loading (figure- **3.2.4.6**) for r/d = 0.1 and D/d = 2, stress concentration factor, $K_t \approx 2.3$.



3.2.4.6 - Variation of theoretical stress concentration factor with r/d for a plate with fillets subjected to a uni-axial loading.

From stress concentration plots for plates with a hole of diameter 'd' under axial loading (**figure- 3.2.4.7**) we have for $K_t = 2.3$, $d'/D = 0.35$. This gives the hole diameter $d' = 35$ mm.



3.2.4.7 - Variation of theoretical stress concentration factor with d/W for a plate with a transverse hole subjected to a uni-axial loading.

3.2.5 Summary of this Lecture

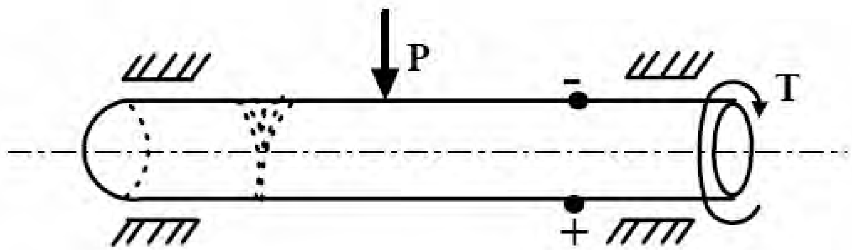
Stress concentration for different geometric configurations and its relation to fatigue strength reduction factor and notch sensitivity have been discussed. Methods of reducing stress concentration have been demonstrated and a theoretical basis for stress concentration was considered.

Lecture
Theme 3
Design for Strength

3.3. Design for dynamic loading

3.3.1 Introduction

Conditions often arise in machines and mechanisms when stresses fluctuate between a upper and a lower limit. For example in **figure-3.3.1.1**, the fiber on the surface of a rotating shaft subjected to a bending load, undergoes both tension and compression for each revolution of the shaft.



3.3.1.1 *- Stresses developed in a rotating shaft subjected to a bending load.*

Any fiber on the shaft is therefore subjected to fluctuating stresses. Machine elements subjected to fluctuating stresses usually fail at stress levels much below their ultimate strength and in many cases below the yield point of the material too. These failures occur due to very large number of stress cycle and are known as fatigue failure.

These failures usually begin with a small crack
which may develop at the points of discontinuity, an existing subsurface crack or surface faults. Once a crack is developed it propagates with the increase in stress cycle finally leading to failure of the component by fracture. There are mainly two characteristics of this kind of failures:

(a) Progressive development of crack.
(b) Sudden fracture without any warning since yielding is practically absent.

Fatigue failures are influenced by

(1) Nature and magnitude of the stress cycle.
(2) Endurance limit.
(3) Stress concentration.
(4) Surface characteristics.

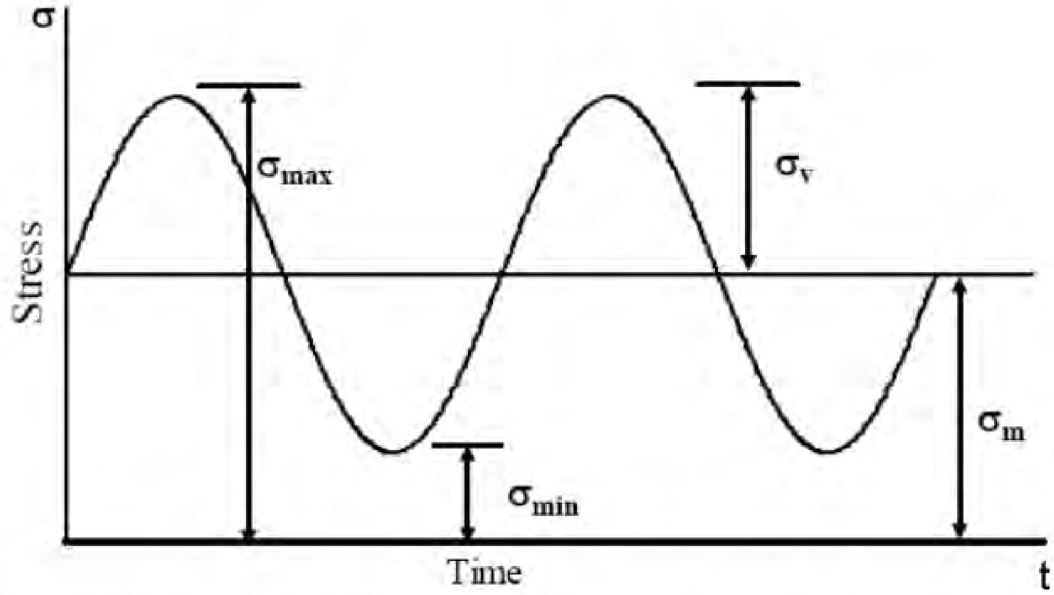
These factors are therefore interdependent. For example, by grinding and polishing, case hardening or coating a surface, the endurance limit may be improved. For machined steel endurance limit is approximately half the ultimate tensile stress. The influence of such parameters on fatigue failures will now be discussed in sequence.

3.3.2 Stress cycle

A typical stress cycle is shown in **figure- 3.3.2.1** where the maximum, minimum, mean and variable stresses are indicated. The mean and variable stresses are given by

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

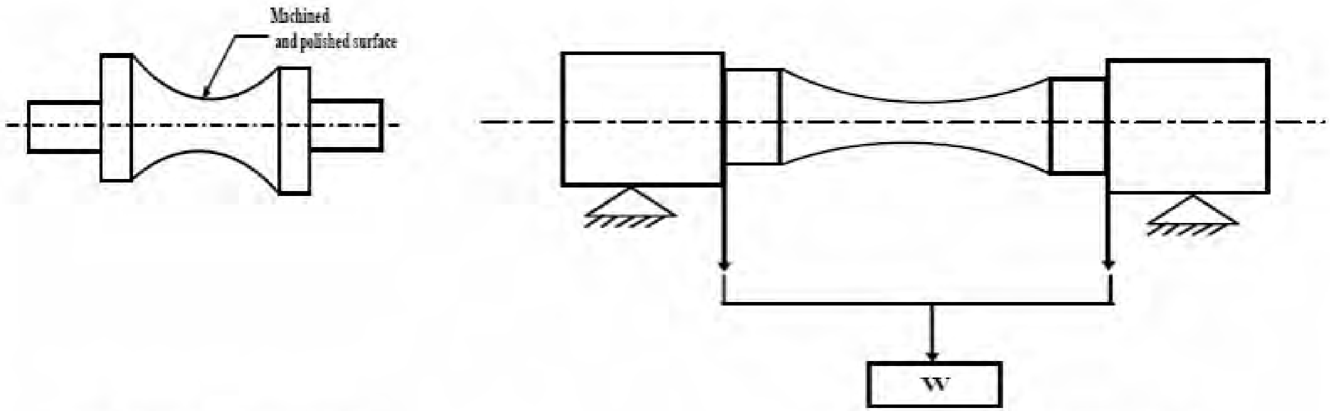
$$\sigma_{variable} = \frac{\sigma_{max} - \sigma_{min}}{2}$$



3.3.2.1 - A typical stress cycle showing maximum, mean and variable stresses.

3.3.3 Endurance limit

Figure- 3.3.3.1 shows the rotating beam arrangement along with the specimen.

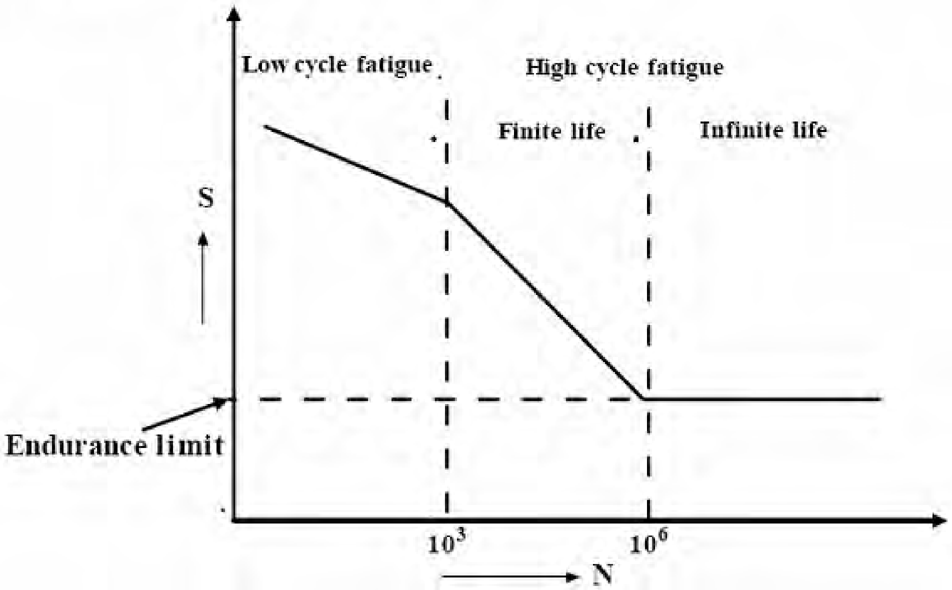


(a) Beam specimen (b) Loading arrangement

3.3.3.1 - A typical rotating beam arrangement.

The loading is such that there is a constant bending moment over the specimen ength and the bending stress is greatest at the center where the section is smallest. The arrangement gives pure bending and avoids transverse shear since bending moment is constant over the length. Large number of tests with varying bending loads are carried out to find the number of cycles to fail.

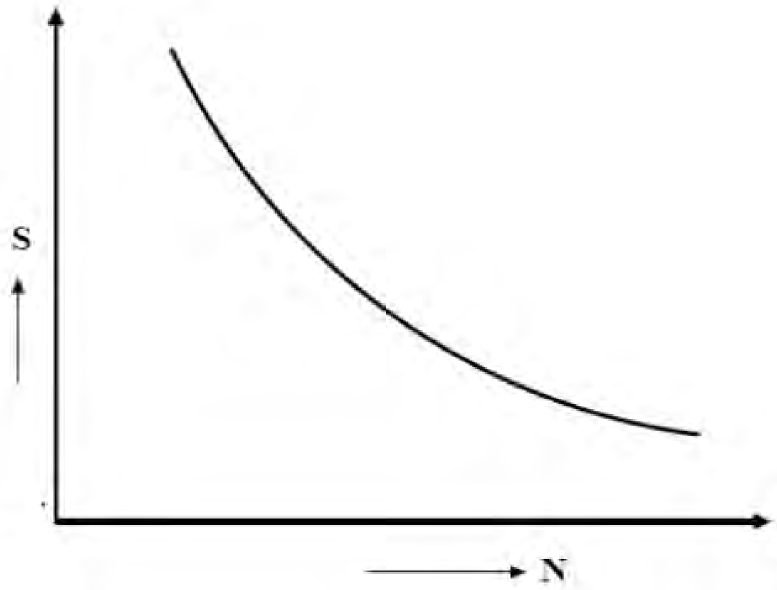
A typical plot of reversed stress (S) against number of cycles to fail (N) is shown in **figure-3.3.3.2**.The zone below 103 cycles is considered as low cycle fatigue, zone between 103 and 106 cycles is high cycle fatigue with finite life and beyond 106 cycles, the zone is considered to be high cycle fatigue with infinite life.



3.3.3.2 - A schematic plot of reversed stress (S) against number of cycles to fail (N) for steel.

The above test is for reversed bending. Tests for reversed axial, torsional or combined stresses are also carried out.For aerospace applications and non-metals axial fatigue testing is preferred.

For non-ferrous metals there is no knee in the curve as shown in **figure- 3.3.3.3** indicating that there is no specified transition from finite to infinite life.



3.3.3.3 *- A schematic plot of reversed stress (S) against number of cycles to fail (N) for non-metals, showing the absence of a knee in the plot.*

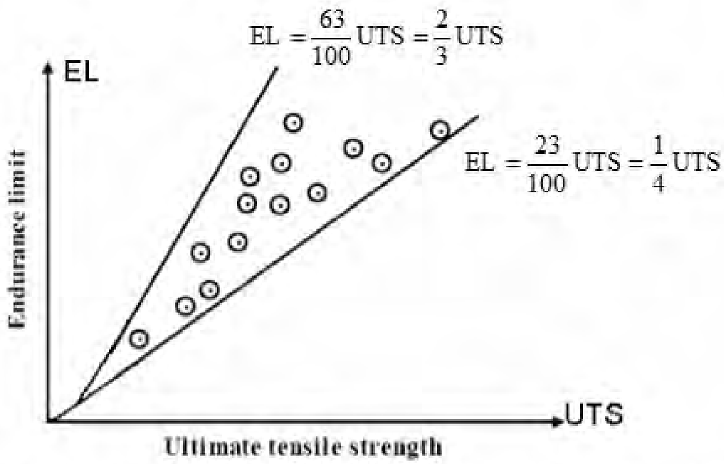
A schematic plot of endurance limit for different materials against the ultimate tensile strengths (UTS) is shown in **figure- 3.3.3.4**. The points lie within a narrow band and the following data is useful:

Steel Endurance limit ~ 35-60 % UTS

Cast Iron Endurance limit ~ 23-63 % UTS

The endurance limits are obtained from standard rotating beam experiments carried out under certain specific conditions. They need be corrected using a number of factors. In general the modified endurance limit σe' is given by

$$\sigma_e' = \sigma_e C_1 C_2 C_3 C_4 C_5 / K_f$$



3.3.3.4 - A schematic representation of the limits of variation of endurance limit with ultimate tensile strength.

1. C_1 is the size factor and the values may roughly be taken as

1. $C_1 = 1$, d≤7.6mm
2. $C_1 = 0.85$, 7.6mm≤d≤50mm
3. $C_1 = 0.75$, d≥50mm

For large size C_1= 0.6. Then data applies mainly to cylindrical steel parts.

2. C_2 is the loading factor and the values are given as

C_2 = 1, for reversed bending load.

C_2 = 0.85, for reversed axial loading for steel parts

C_2 = 0.78, for reversed torsional loading for steel parts.

3. C_3 is the surface factor and since the rotating beam specimen is given a mirror polish the factor is used to suit the condition of a machine part.
Since machining process rolling and forging contribute to the surface quality the plots of C_3 versus tensile strength or Brinnel hardness number for different production process, in **figure-3.3.3.5**, is useful in selecting the value of $C_3=C_{sur}$. Fig 3.3.3.5 we take from handbol

4. C_4 is the temperature factor and the values may be taken as follows:

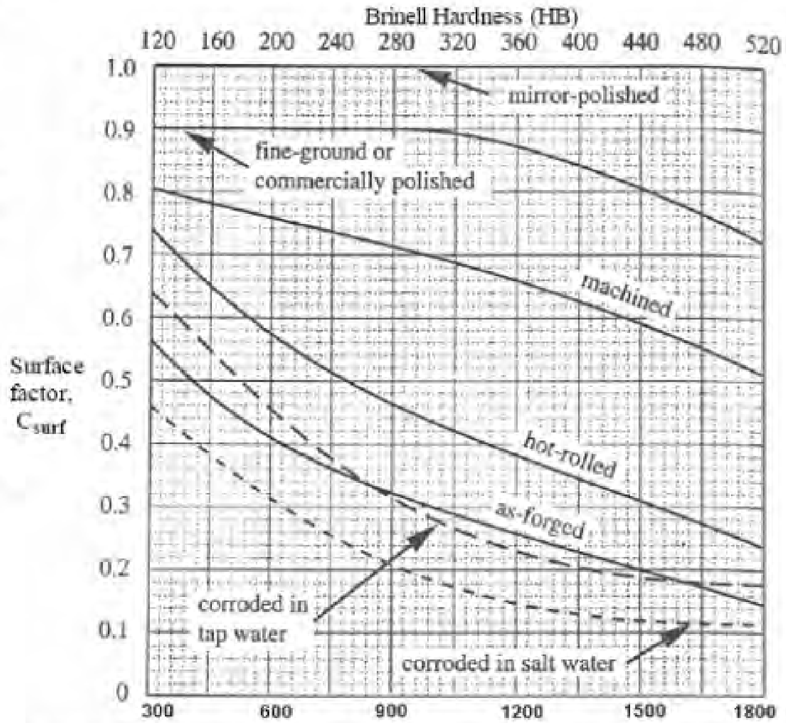
$C_4 = 1$, for $T \leq 450^oC$

$C_4 = 1-0.0058(T-450)$ for $450^o < T \leq 550^oC$

5. C_5 is the reliability factor and this is related to reliability percentage as follows:

Reliability %	C_5
50	1
90	0.897
99.99	0.702

6. K_f is the fatigue stress concentration factor, discussed in the next section.



3.3.3.5 - Variation of surface factor with tensile strength and Brinnel hardness for steels with different surface conditions (Ref.[2]).

3.3.4 Stress concentration

Stress concentration has been discussed in earlier lessons. However, it is important to realize that stress concentration affects the fatigue strength of machine parts severely and therefore it is extremely important that this effect be considered in designing machine parts subjected to fatigue loading. This is done by using fatigue stress concentration factor defined as

$$k_f = \frac{\text{Endurance limit of a notch free specimen}}{\text{Endurance limit of a notched specimen}}$$

The notch sensitivity 'q' for fatigue loading can now be defined in terms of k_f and the theoretical stress concentration factor k_t and this is given by

$$q = \frac{k_f - 1}{k_t - 1}$$

The value of q is different for different materials and this normally lies between 0 to 0.7. The index is small for ductile materials and it increases as the ductility decreases. Notch sensitivities of some common materials are given in **table- 3.3.4.1** . Which we can take from handbook

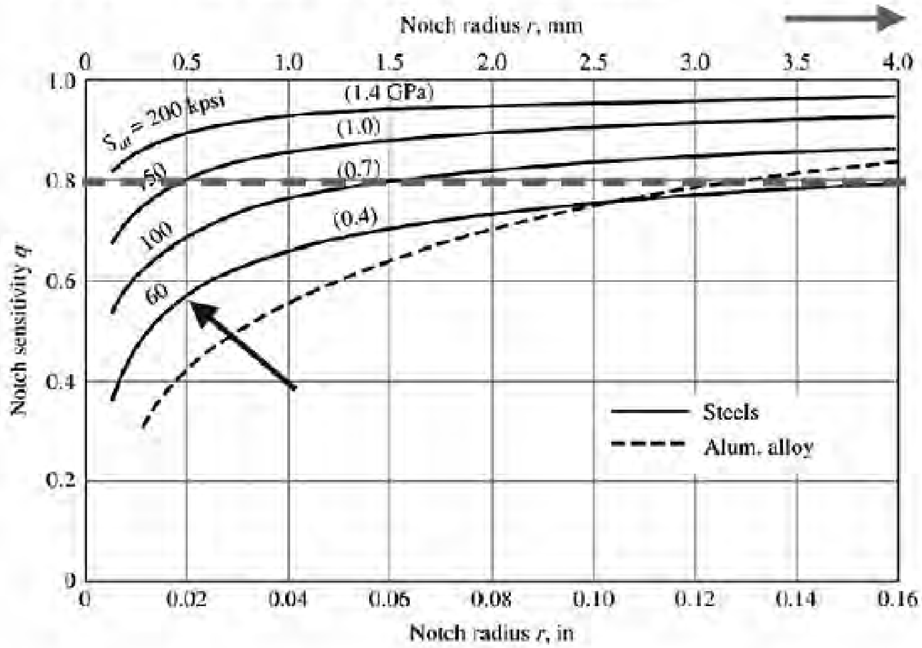
3.3.4.1 - Notch sensitivity of some common engineering materials.

Material	Notch sensitivity index
C-30 steel- annealed	0.18
C-30 steel- heat treated and drawn at 480°C	0.49
C-50 steel- annealed	0.26
C-50 steel- heat treated and drawn at 480°C	0.50
C-85 steel- heat treated and drawn at 480°C	0.57
Stainless steel- annealed	0.16
Cast iron- annealed	0.00-0.05
copper- annealed	0.07
Duraluminium- annealed	0.05-0.13

Notch sensitivity index q can also be defined as

$$q = \frac{1}{1+\left(\frac{a}{r}\right)^{1/2}} \qquad q = \frac{k_f - 1}{k_t + 1}$$

where, $\sqrt{a}$ is called the Nubert's constant that depends on materials and their heat treatments. A typical variation of q against notch radius r is shown in **figure-3.3.4.2** . This date we can take in handbook.



3.3.4.2 *- Variation of notch sensitivity with notch radius for steel and aluminium alloy with different ultimate tensile strengths*

3.3.5 Surface characteristics

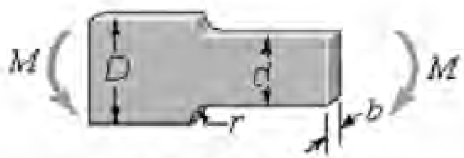
Fatigue cracks can start at all forms of surface discontinuity and this may include surface imperfections due to machining marks also. Surface roughness is therefore an important factor and it is found that fatigue strength for a regular surface is relatively low since the surface undulations would act as stress raisers.

It is, however, impractical to produce very smooth surfaces at a higher machining cost.

Another important surface effect is due to the surface layers which may be extremely thin and stressed either in tension or in compression. For example, grinding process often leaves surface layers highly stressed in tension. Since fatigue cracks are due to tensile stress and they propagate under these conditions and the formation of layers stressed in tension must be avoided. There are several methods of introducing pre-stressed surface layer in compression and they include shot blasting, peening, tumbling or cold working by rolling. Carburized and nitrided parts also have a compressive layer which imparts fatigue strength to such components. Many coating techniques have evolved to remedy the surface effects in fatigue strength reductions.

3.3.6 Problems with Answers

Q.1: A rectangular stepped steel bar is shown in **figure-3.3.6.1**. The bar is loaded in bending. Determine the fatigue stress-concentration factor if ultimate stress of the materials is 689 MPa.



r = 5mm
D = 50 mm
d = 40 mm
b = 1 mm

3.3.6.1

A.1:

From the geometry r/d = 0.125 and D/d = 1.25.

From the stress concentration chart in **figure- 3.2.4.6**

Stress- concentration factor $k_f \approx 1.7$

From **figure- 3.3.4.2**

Notch sensitivity index, q ≈ 0.88

The fatigue stress concentration factor k_f is now given by $q = \frac{k_f - 1}{k_t - 1}$

$k_f = 1 + q(k_t - 1) = 1.616$

3.3.7 Summary of this Lecture

Design of components subjected to dynamic load requires the concept of variable stresses, endurance limit, low cycle fatigue and high cycle fatigue with finite and infinite life. The relation of endurance limit with ultimate tensile strength is an important guide in such design. The endurance limit needs be corrected for a number of factors such as size, load, surface finish, temperature and reliability. The methods for finding these factors have been discussed and demonstrated in an example.

Lecture

Theme 4

Fasteners

4.1. Types of fasteners: Pins and keys

4.1.1 Introduction: Types of fasteners

A machine or a structure is made of a large number of parts and they need be joined suitably for the machine to operate satisfactorily. Parts are joined by fasteners and they are conveniently classified as permanent or detachable fasteners. They are often sub- divided under the main headings as follows:

Permanent fasteners: Riveted joints
Welded joints

Detachable joints: Threaded fasteners – screws, bolts and nuts, studs.
Cotter joints
Knuckle joints

Keys and Pin joints

Starting with the simple pin and key joints all the main fasteners will be discussed here.

4.1.2 Pin Joints

These are primarily used to prevent sliding of one part on the other, such as, to secure wheels, gears, pulleys, levers etc. on shafts. Pins and keys are primarily used to transmit torque and to prevent axial motion. In engineering practice the following types of pins are generally used.

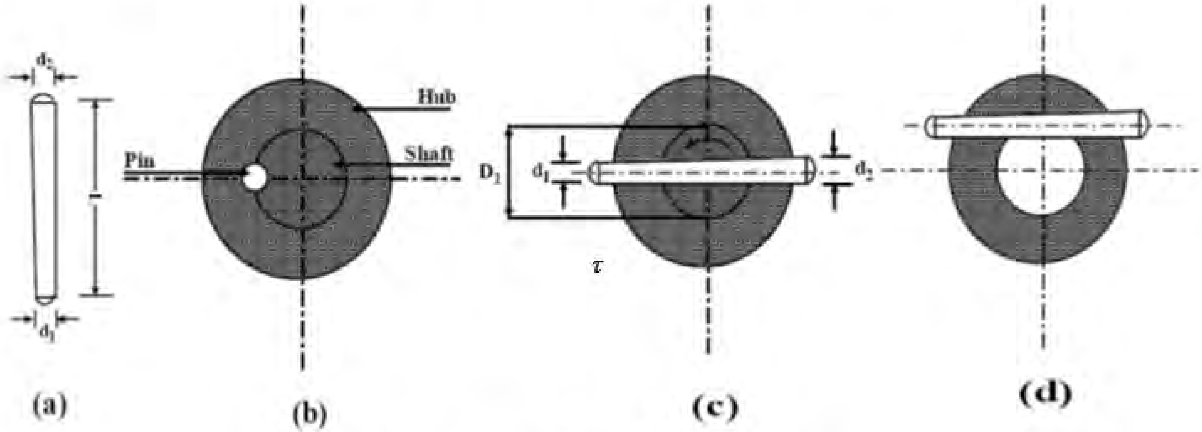
(a) Round pins (b) Taper pins (c) Dowel pins (d) Split pins

Round and taper pins are simple cylindrical pins with or without a taper and they offer effective means of fastening pulleys, gears or levers to a shaft. It may be fitted such that half the pin lies in the hub and the other half in the shaft as shown in **figure-4.1.2.1 (b)**. The pin may be driven through the hub and the shaft as in **figure- 4.1.2.1 (c)** or as in **figure- 4.1.2.1 (d)**. These joints give positive grip and the pins are subjected to a shear load. For example, for the shaft in the assembly shown in **figure- 4.1.2.1 (c)**, the pin is under double shear and we have

$$\tau\left(2\frac{\pi}{4}d^2\right).\frac{D_1}{2} = T$$

where d is the diameter of the pin at hub-shaft interface, $\tau$is the yield strength in shear of the pin material and T is the torque transmitted.

where d is the diameter of the pin at hub-shaft interface, τ is the yield strength in shear of the pin material and is the torque transmitted.

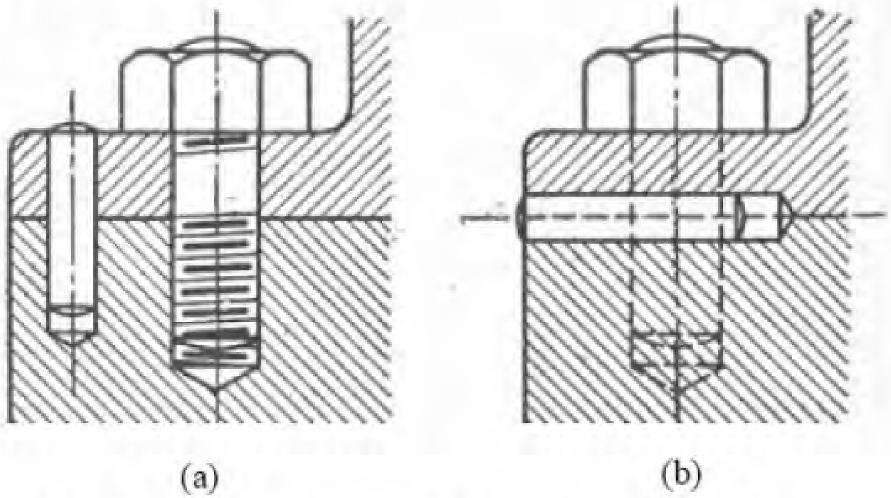


4.1.2.1 *- Different types of pin joints*

A taper pin is preferred over the straight cylindrical pins because they can be driven easily and it is easy to ream a taper hole.

Dowel pins

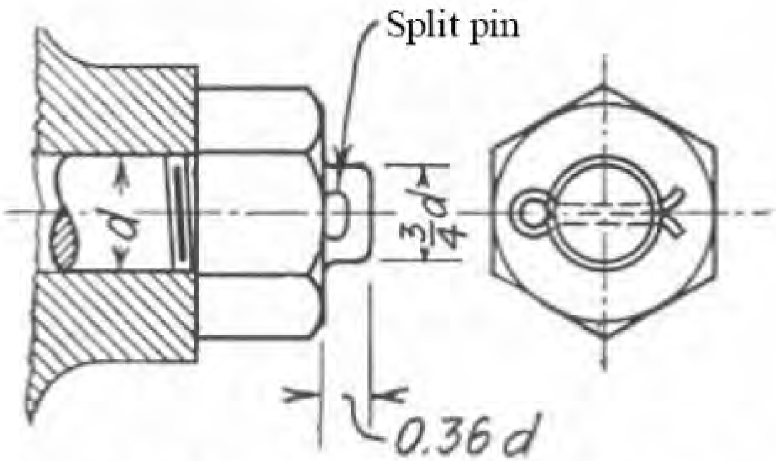
These are used to keep two machine parts in proper alignment. **Figure-4.1.2.2** demonstrates the use of dowel pins. Small cylindrical pins are normally used for this purpose.



4.1.2.2 *- Some uses of Dowel pins (Ref.[6]).*

Split pins

These are sometimes called cotter pins also and they are made of annealed iron or brass wire. They are generally of semi-circular cross section and are used to prevent nuts from loosening as shown in **figure- 4.1.2.3**. These are extensively used in automobile industry.



4.1.2.3 *- Typical use of a split pin (Ref.[6]).*

4.1.3 Keys

Steel keys are widely used in securing machine parts such as gears and pulleys. There is a large variety of machine keys and they may be classified under four broad headings:
Sunk keys, flat keys, saddle keys and pins or round keys

Sunk keys may be further classified into the following categories:

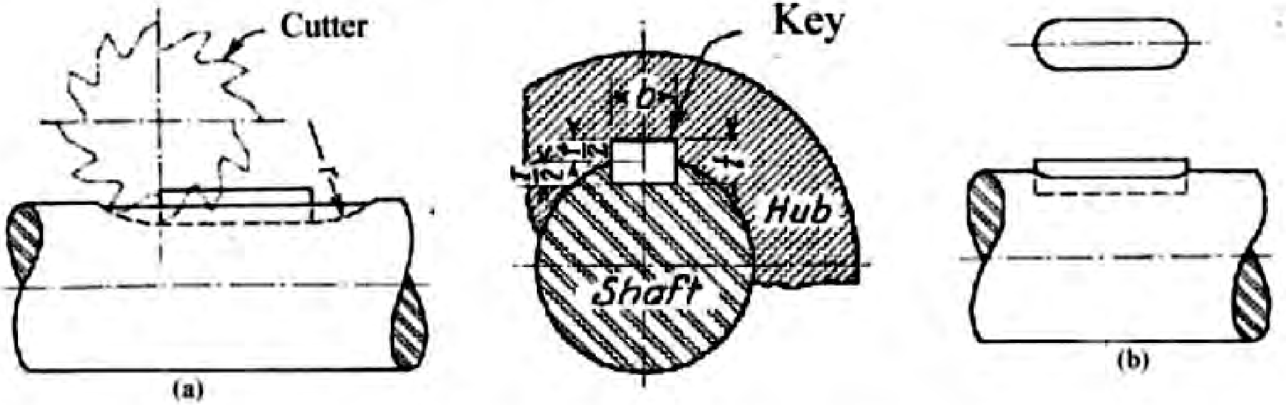
(a) Rectangular sunk keys

(b) Gib head sunk keys

(c) Feather keys

(d) Woodruff keys

Rectangular sunk keys are shown in **figure- 4.1.3.1**. They are the simplest form of machine keys and may be either straight or slightly tapered on one side. The parallel side is usually fitted into the shaft.



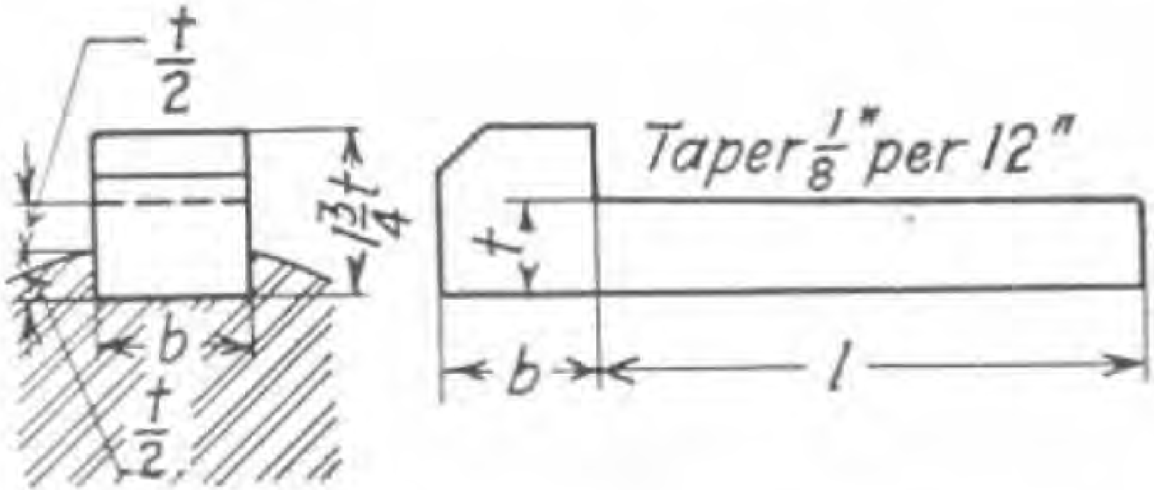
4.1.3.1 *- Rectangular sunk keys (Ref.[6]).*

The slots are milled as shown in **figure- 4.1.3.1(a)**. While transmitting torque a rectangular sunk key is subjected to both shear and crushing or bearing stresses.

Considering shear we may write $\tau.b.l.\frac{D}{2} = T$ where τ is the yield shear stress of the key material, D the shaft diameter and T is torque transmitted. Considering bearing stress we may write $\sigma_{br}.\frac{t.l}{2}.\frac{D}{2} = T$

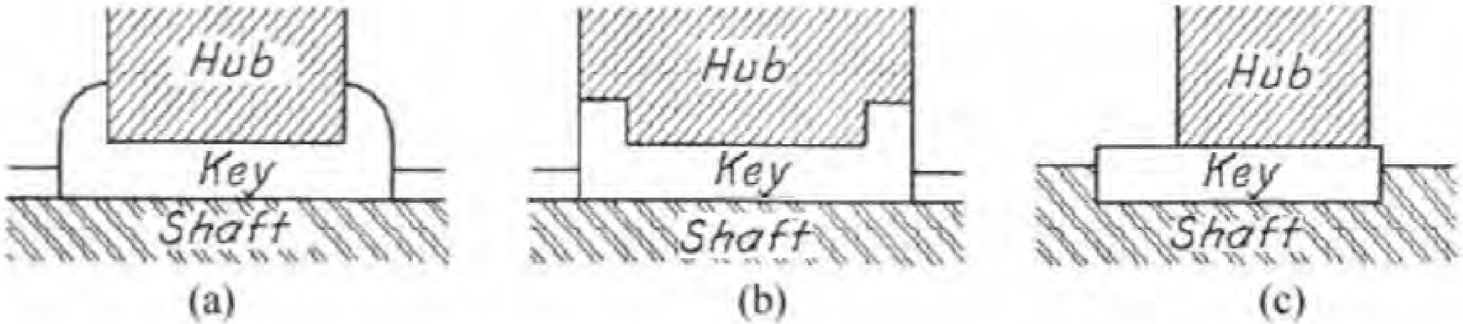
where σ_{br} is the bearing stress developed in the key. Based on these two criteria key dimensions may be optimized and compared with the standard key dimensions available in design hand books.

The **gib head keys** are ordinary sunk keys tapered on top with a raised head on one side so that its removal is easy. This is shown in **figure- 4.1.3.2**



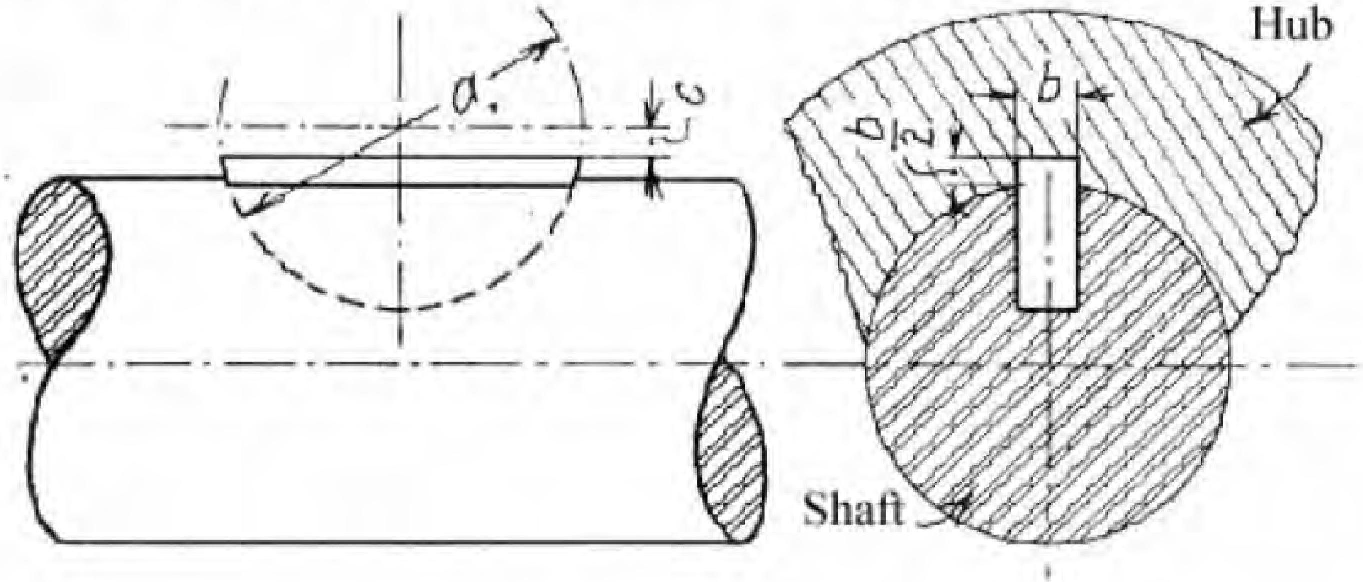
4.1.3.2 *- Gib head key (Ref.[6]).*

Some **feather key** arrangements are shown in **figure- 4.1.3.3**. A feather key is used when one component slides over another. The key may be fastened either to the hub or the shaft and the keyway usually has a sliding fit.



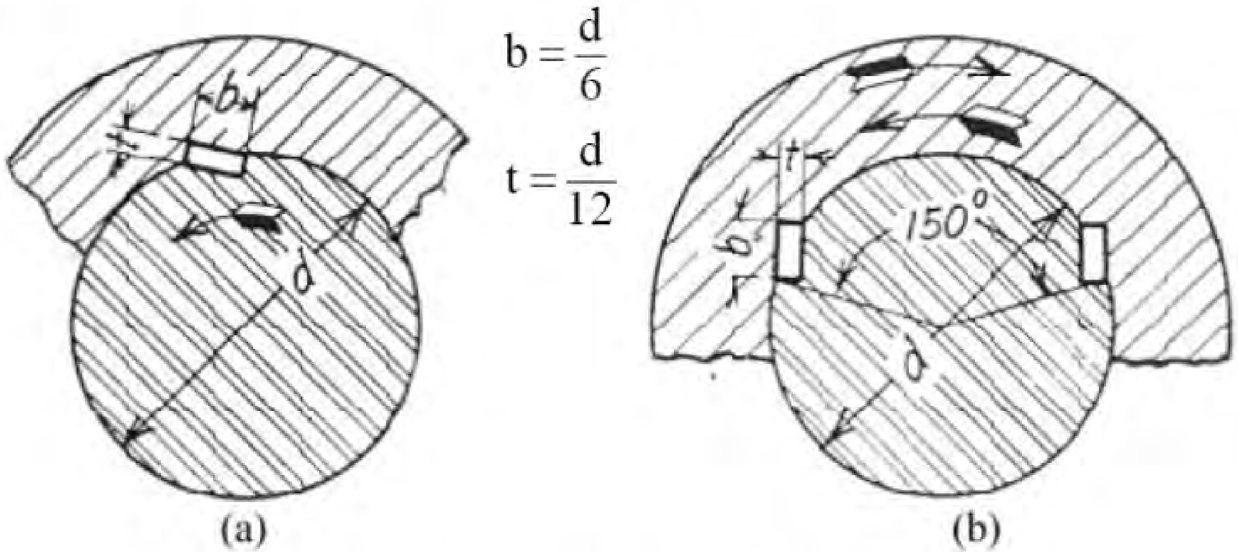
4.1.3.3 *- Some feather key arrangements (Ref.[6]).*

A **woodruff key** is a form of sunk key where the key shape is that of a truncated disc, as shown in **figure- 4.1.3.4**. It is usually used for shafts less than about 60 mm diameter and the keyway is cut in the shaft using a milling cutter, as shown in the **fiaure- 4.1.3.4**. It is widelv used in machine tools and automobiles due to



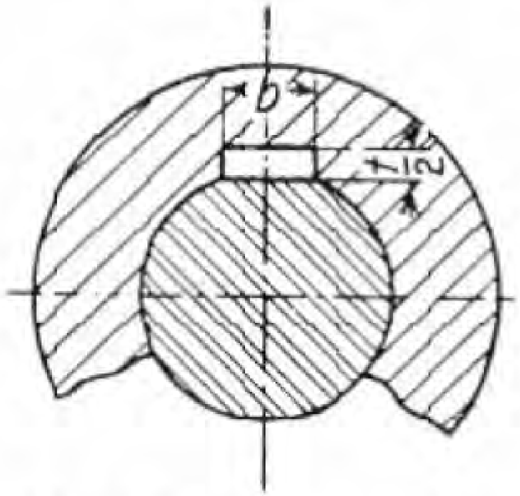
4.1.3.4 *- Woodruff key (Ref.[6]).*

Lewis keys, shown in **figure- 4.1.3.5**, are expensive but offer excellent service. They may be used as a single or double key. When they are used as a single key the positioning depends on the direction of rotation of the shaft. For heavy load two keys can be used as shown in **figure- 4.1.3.5 (b)**.

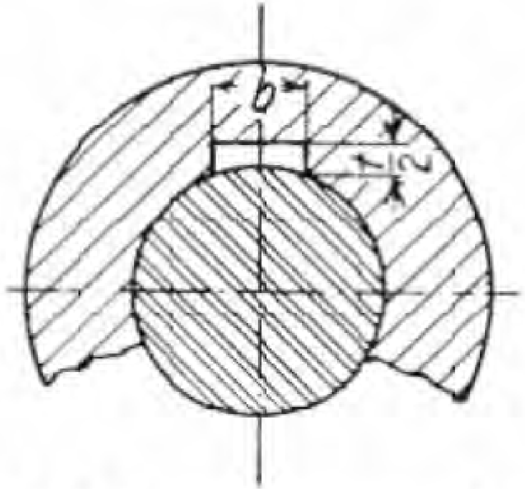


4.1.3.5 *- Lewis keys (Ref.[6]).*

A **flat key**, as shown in **figure- 4.1.3.6** is used for light load because they depend entirely on friction for the grip. The sides of these keys are parallel but the top is
slightly tapered for a tight fit. Theses keys have about half the thickness of sunk keys.

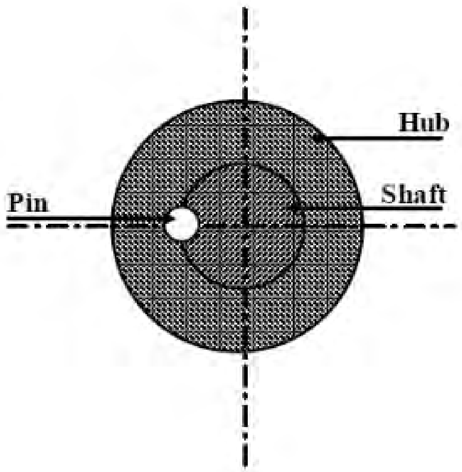


4.1.3.6 *- Flat key (Ref.[6])*.



4.1.3.7 *- Saddle key (Ref.[6]).*

A **saddle key**, shown in **figure- 4.1.3.7**, is very similar to a flat key except that the bottom side is concave to fit the shaft surface. These keys also have friction grip and therefore cannot be used for heavy loads. A simple pin can be used as a key to transmit large torques. Very little stress concentration occurs in the shaft in these cases. This is shown in **figure- 4.1.3.8 (b).**

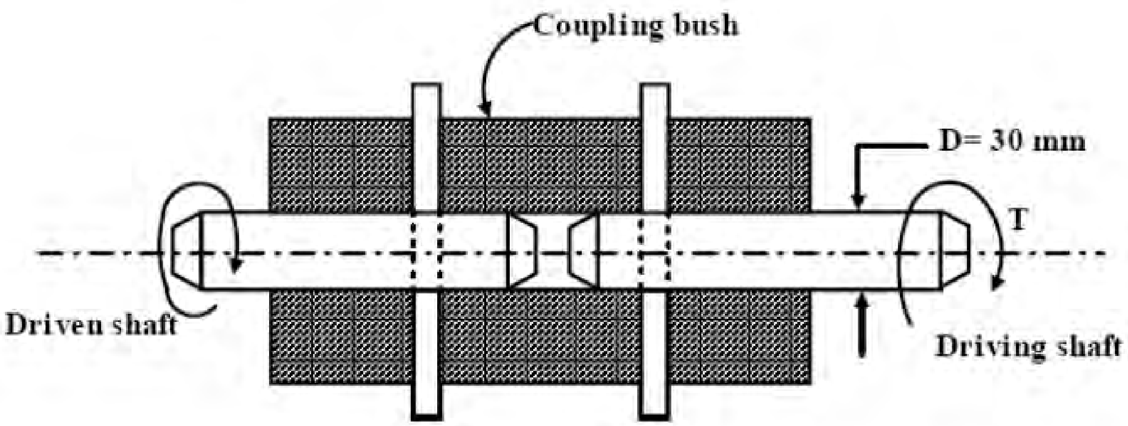


(b)

4.1.3.8.

4.1.4 Problems with Answers

Q.1: Two 30 mm diameter shafts are connected by pins in an arrangement shown in **figure-4.1.4.1**. Find the pin diameter if the allowable shear stress of the pins is 100 MPa and the shaft transmits 5 kW at 150 rpm.



4.1.4.1.

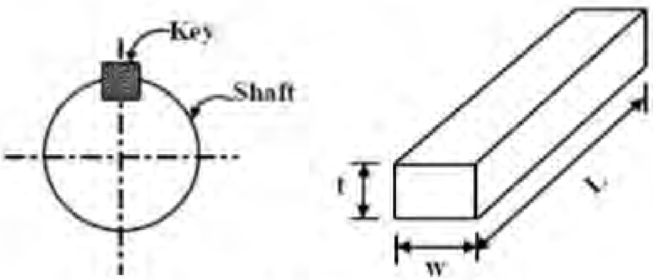
A.1:

The torque transmitted T= Power/$\left(\frac{2\pi N}{60}\right)$. Substituting power = $5x10^3$ Watts and N=150 rpm we have T = 318.3 Nm. The torque is transmitted from the driving shaft to the coupling bush via a pin. The torque path is then reversed and it is transmitted from the coupling bush to the driven shaft via another pin. Therefore both the pins transmit a torque of 318.3 Nm under double shear. We may then write $T = 2.\frac{\pi}{4}.d^2.\tau_y.\frac{D}{2}$. Substituting D=0.03 m, τ_y = 100 MPa and T= 318.3 MPa we have d=11.6 mm ≈ 12 mm.

Q.2: A heat treated steel shaft of tensile yield strength of 350 MPa has a diameter of 50 mm. The shaft rotates at 1000 rpm and transmits 100 kW through a gear. Select an appropriate key for the gear.

A.2:

Consider a rectangular key of width w, thickness t and length L as shown in **figure- 4.1.4.1**. The key may fail (a) in shear or (b) in crushing.



4.1.4.1.

Shear failure: The failure criterion is $T = \tau_y . w . L . \frac{d}{2}$

where torque transmitted is $T = \text{Power} / \left(\frac{2\pi N}{60} \right)$

respectively and τ_y is the yield stress in shear of the key material.

τ_y to be half of the tensile yield stress and substituting the values in equations (1) and (2) we have wL = 2.19 x 10^{-4} m^2.

Crushing failure: $T = \sigma_c . \frac{t.L}{2} . \frac{d}{2}$

Taking σ_c to be the same as σ_y and substituting values in equation (3) we have

tL= 2.19 x 10^{-4} m^2. Some standard key dimensions are reproduced in **table- 4.1.4.1**:

Shaft Diameter (mm)	30-38	38-44	44-50	50-58	58-65	65-75	75-85
Key width, w (mm)	10	12	14	16	18	20	22
Key depth, t (mm)	8	9	9	10	11	12	14
Key length, L (mm)	22-110	28-140	36-160	45-180	50-200	56-220	63-250

4.1.4.1

Based on the standard we may choose w=16 mm. This gives L = 13.6 mm. We may then choose the safe key dimensions as

w = 16 mm L = 45 mm t = 10 mm.

4.1.5 Summary of this Lesson

In this lesson firstly the types detachable of fasteners are discussed. Then types and applications of pin and key joints are discussed with suitable illustrations. A brief overview of key design is also included.

Lecture

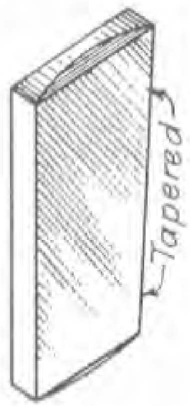
Theme 4

Fasteners

4.2. Cotter and knuckle joint

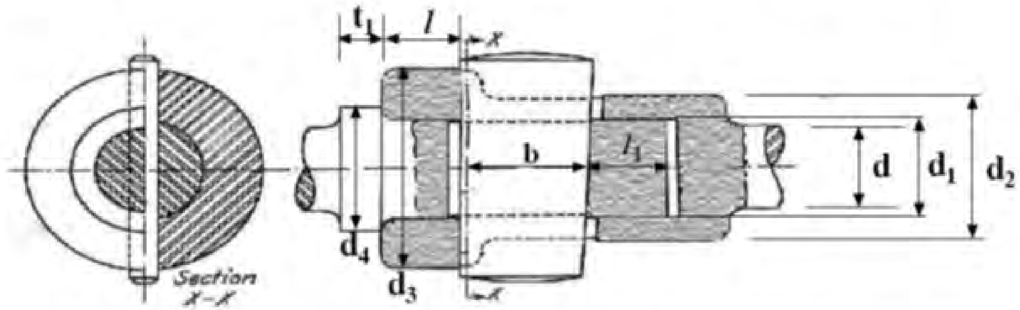
4.2.1 Cotter joint

A cotter is a flat wedge-shaped piece of steel as shown in **figure-4.2.1.1.** This is used to connect rigidly two rods which transmit motion in the axial direction, without rotation. These joints may be subjected to tensile or compressive forces along the axes of the rods.
Examples of cotter joint connections are: connection of piston rod to the crosshead of a steam engine, valve rod and its stem etc.



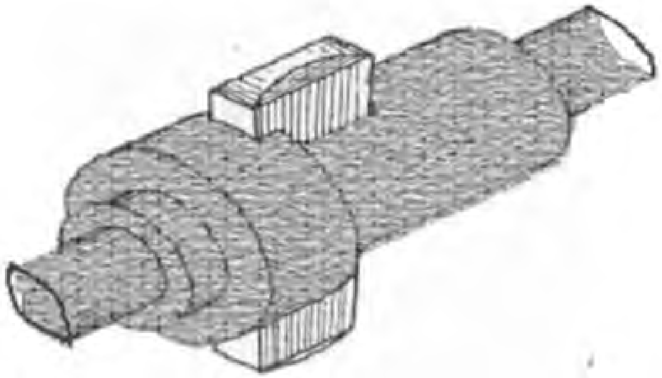
4.2.1.1 *- A typical cotter with a taper on one side only.*

A typical cotter joint is as shown in **figure-4.2.1.2**. One of the rods has a socket end into which the other rod is inserted and the cotter is driven into a slot, made in both the socket and the rod. The cotter tapers in width (usually 1:24) on one side only and when this is driven in, the rod is forced into the socket. However, if the taper is provided on both the edges it must be less than the sum of the friction angles for both the edges to make it self locking i.e $\alpha_1+\alpha_2<\varphi_1+\varphi_2$ where α_1, α_2 are the angles of taper on the rod edge and socket edge of the cotter respectively and φ_1, φ_2 are the corresponding angles of friction.



4.2.1.2 *- Cross-sectional views of a typical cotter joint*

This also means that if taper is given on one side only then $\alpha<\varphi_1+\varphi_2$ for self locking. Clearances between the cotter and slots in the rod end and socket allows the driven cotter to draw together the two parts of the joint until the socket end comes in contact with the cotter on the rod end.



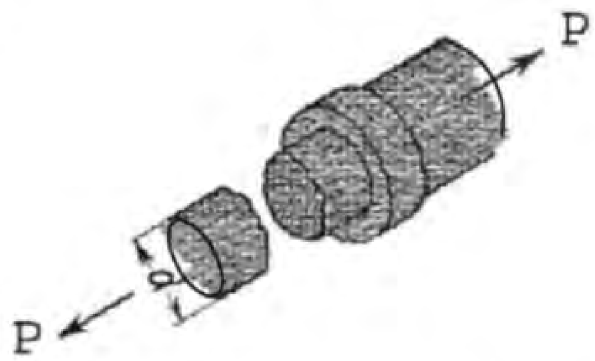
4.2.1.3 *- An isometric view of a typical cotter joint.*

4.2.2 Design of a cotter joint

If the allowable stresses in tension, compression and shear for the socket, rod and cotter be σ_t, σ_c and τ respectively, assuming that they are all made of the same material, we may write the following failure criteria:

1. Tension failure of rod at diameter d

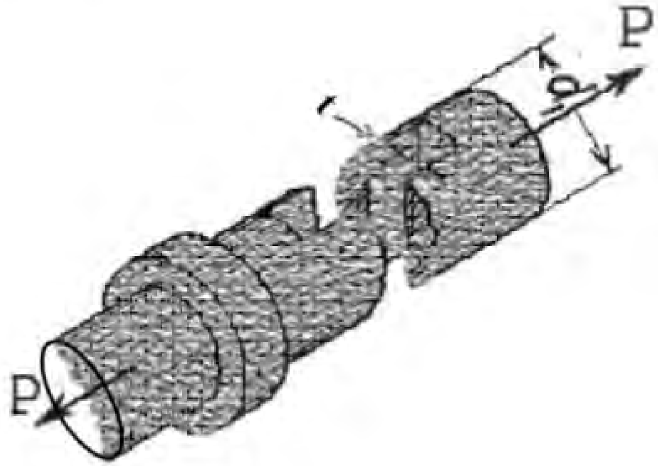
$$\frac{\pi}{4} d^2 \sigma_t = P$$



4.2.2.1 *- Tension failure of the rod/*

2. Tension failure of rod across slot

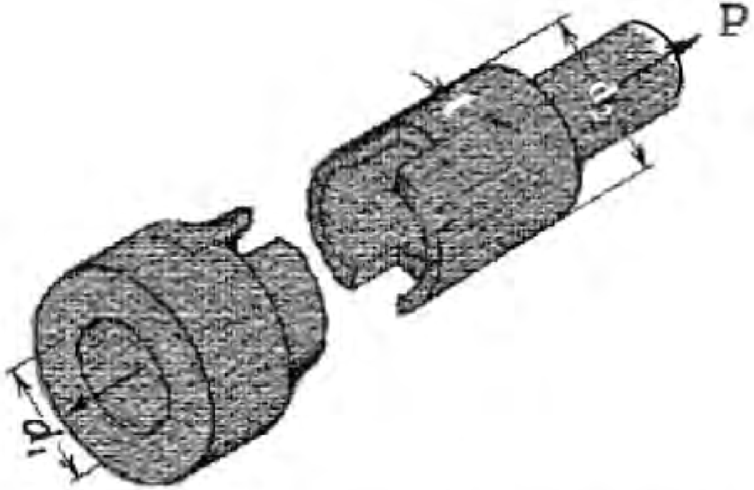
$$\left(\frac{\pi}{4}d_1^2 - d_1 t\right)\sigma_t = P$$



4.2.2.2 *- Tension failure of rod across slot*

3. Tensile failure of socket across slot

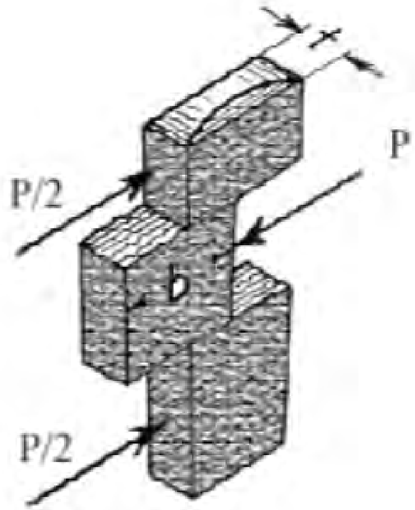
$$\left(\frac{\pi}{4}(d_2^2 - d_1^2) - (d_2 - d_1)t\right)\sigma_t = P$$



4.2.2.3 *- Tensile failure of socket across slot*

4. Shear failure of cotter

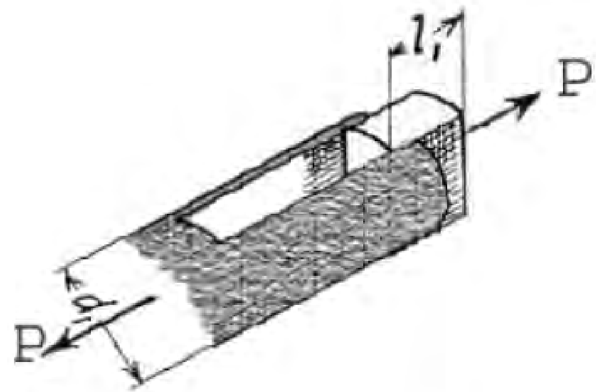
$$2bt\tau = P$$



4.2.2.4 *- Shear failure of cotter*

5. Shear failure of rod end

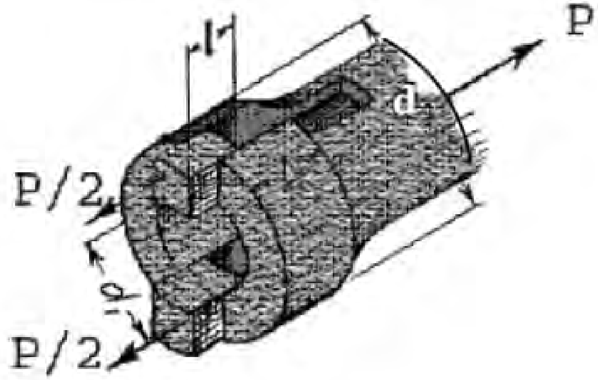
$$2l_1 d_1 \tau = P$$



4.2.2.5 *- Shear failure of rod end*

6. Shear failure of socket end

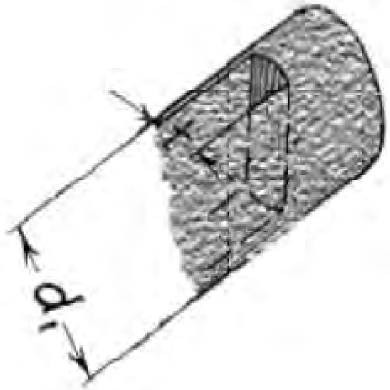
$$2l(d_3 - d_1)\tau = P$$



4.2.2.6 *- Shear failure of socket end*

7. Crushing failure of rod or cotter

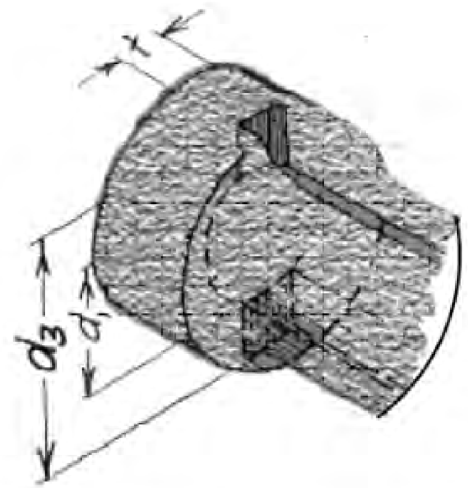
$$d_1 t \sigma_c = P$$



4.2.2.7 *- Crushing failure of rod or cotter*

8. Crushing failure of socket or rod

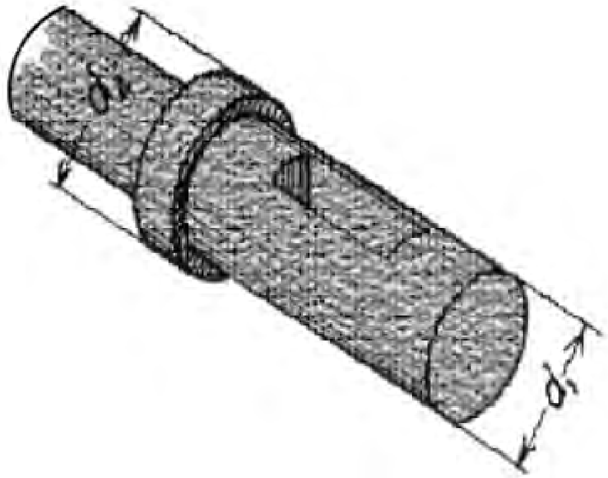
$$(d_3 - d_1)t\sigma_c = P$$



4.2.2.8 *- Crushing failure of socket or rod*

9. Crushing failure of collar

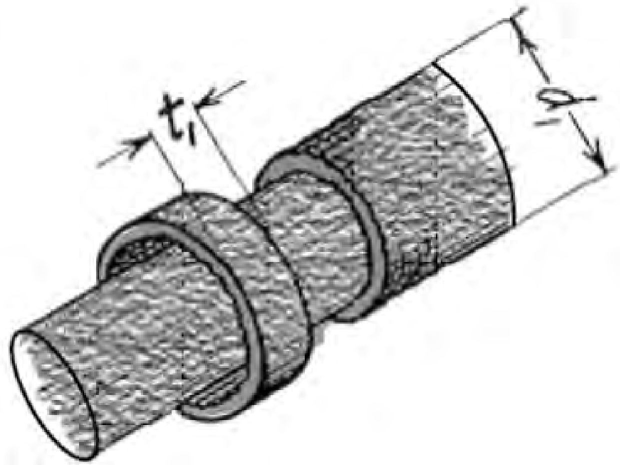
$$\left(\frac{\pi}{4}(d_4^2 - d_1^2)\right)\sigma_c = P$$



4.2.2.9 *- Crushing failure of collar*

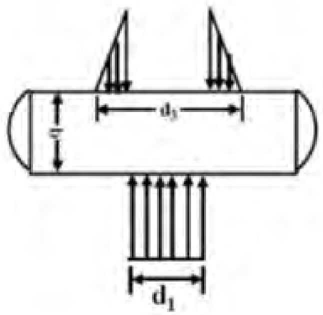
10. Shear failure of collar

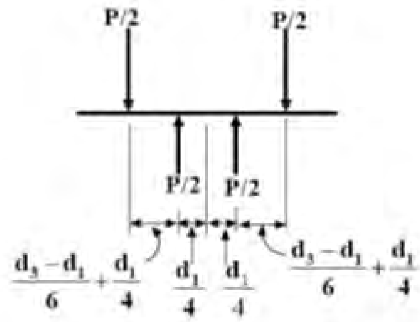
$$\pi d_1 t_1 \tau = P$$



4.2.2.10 *- Shear failure of collar.*

Cotters may bend when driven into position. When this occurs, the bending moment cannot be correctly estimated since the pressure distribution is not known. However, if we assume a triangular pressure distribution over the rod, as shown in figure-4.2.2.11 (a), we may approximate the loading as shown in figure-4.2.2.11 (b)





4.2.2.11 *- Bending of the cotter*

This gives maximum bending moment = $\frac{P}{2}\left(\frac{d_3-d_1}{6}+\frac{d_1}{4}\right)$ and

The bending stress, $\sigma_b = \dfrac{\frac{P}{2}\left(\frac{d_3-d_1}{6}+\frac{d_1}{4}\right)\frac{b}{2}}{\frac{tb^3}{12}} = \dfrac{3P\left(\frac{d_3-d_1}{6}+\frac{d_1}{4}\right)}{tb^2}$

Tightening of cotter introduces initial stresses which are again difficult to estimate. Sometimes therefore it is necessary to use empirical proportions to design the joint. Some typical proportions are given below:

$d_1 = 1.21.d$

$d_2 = 1.75.d$

$d_3 = 2.4\ d$

$d_4 = 1.5.d$

$t = 0.31d$

$b = 1.6d$

$l = l_1 = 0.75d$

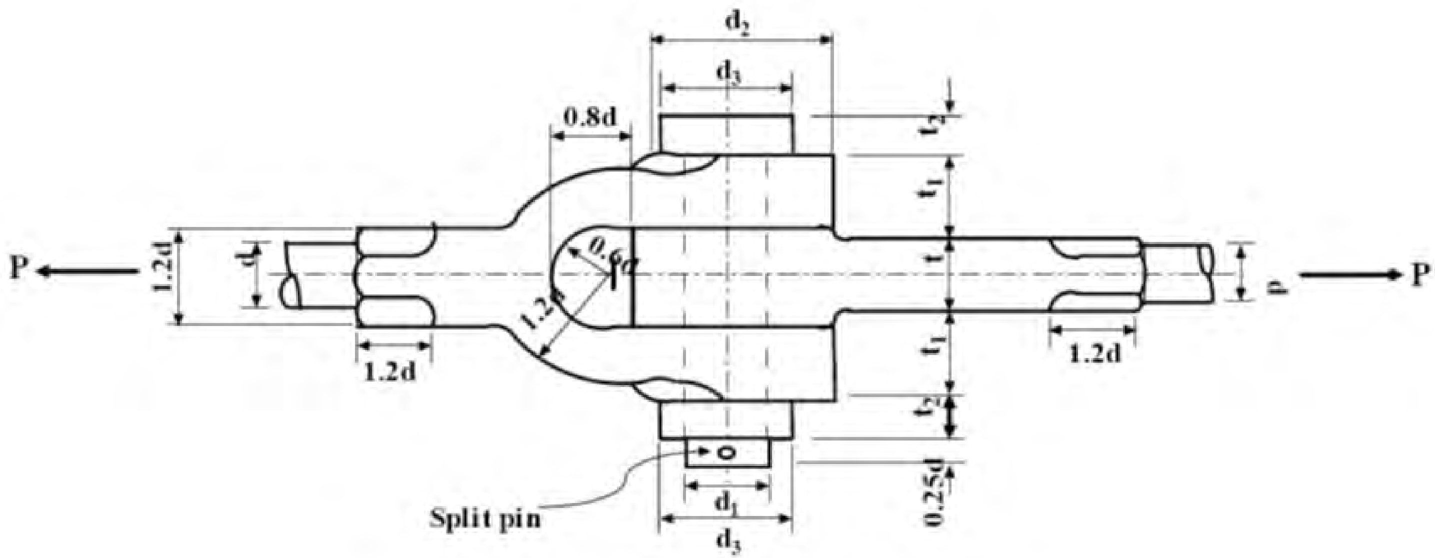
$t_1 = 0.45d$

s= clearance

A design based on empirical relation may be checked using the formulae based on failure mechanisms.

4.2.3 Knuckle Joint

A knuckle joint (as shown in **figure- 4.2.3.1**) is used to connect two rods under tensile load. This joint permits angular misalignment of the rods and may take compressive load if it is guided.



4.2.3.1 *- A typical knuckle joint*

These joints are used for different types of connections e.g. tie rods, tension links in bridge structure. In this, one of the rods has an eye at the rod end and the other one is forked with eyes at both the legs. A pin (knuckle pin) is inserted through the rod-end eye and fork-end eyes and is secured by a collar and a split pin.

d = diameter of rod

d_1= d, t =1.25=d, d_2 =2d, t_1 =0.75d, d_3 =1.5.d, t_2 =0.5d

Mean diameter of the split pin = 0.25 d

However, failures analysis may be carried out for checking. The analyses areshown below assuming the same materials for the rods and pins and the yield stresses in tension, compression and shear are given by σt, σc and τ.

1. Failure of rod in tension: $\frac{\pi}{4} d^2 \sigma_t = P$

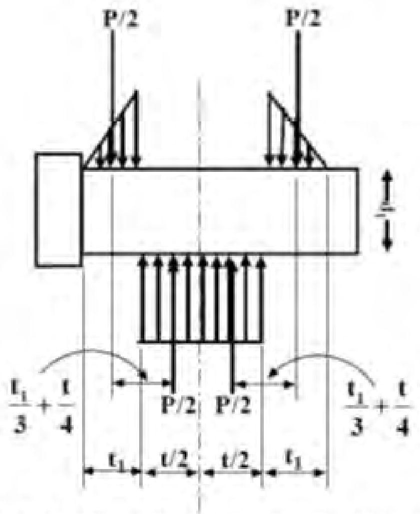
2. Failure of knuckle pin in double shear: $2\frac{\pi}{4} d_1{}^2 \tau = P$

3. Failure of knuckle pin in bending (if the pin is loose in the fork)

Assuming a triangular pressure distribution on the pin, the loading on the pin is shown in figure- 4.2.3.2.

Equating the maximum bending stress to tensile or compressive yield stress we have

$$\sigma_t = \frac{16P\left(\frac{t_1}{3}+\frac{t}{4}\right)}{\pi d_1^3}$$



4.2.3.2 *- Bending of a knuckle pin*

4. Failure of rod eye in shear:

$$(d_2 - d_1)t\tau = P$$

5. Failure of rod eye in crushing:

$$d_1 t\sigma_c = P$$

6. Failure of rod eye in tension:

$$(d_2 - d_1)t\sigma_t = P$$

7. Failure of forked end in shear:

$$2(d_2 - d_1)t_1\tau = P$$

8. Failure of forked end in tension:

$$2(d_2 - d_1)t_1\sigma_t = P$$

9. Failure of forked end in crushing:

$$2d_1 t_1\sigma_c = P$$

The design may be carried out using the empirical proportions and then the analytical relations may be used as checks.

For example using the 2nd equation we have $\tau = \frac{2P}{\pi d_1^2}$. We may now put value of d_1 from empirical relation and then find F.S. = $\frac{\tau_y}{\tau}$ which should be more than one.

4.2.4 Problems with Answers

Q.1: Design a typical cotter joint to transmit a load of 50 kN in tension or compression. Consider that the rod, socket and cotter are all made of a material with the following allowable stresses:

Allowable tensile stress σ_y = 150 MPa

Allowable crushing stress σ_c = 110 MPa

Allowable shear stress τ_y = 110 MPa.

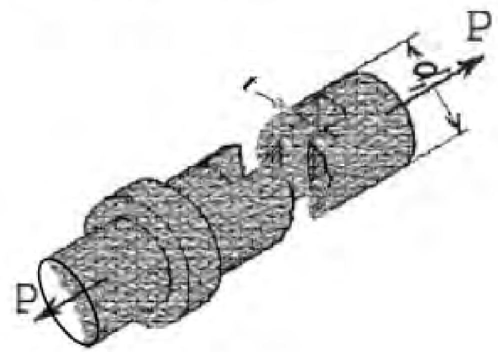
A.1: *Refer to figure- 4.2.1.2 and 4.2.2.1*

Axial load $P = \frac{\pi}{4} d^2 \sigma_y$. On substitution this gives d=20 mm. In general standard snaπ size in mm are

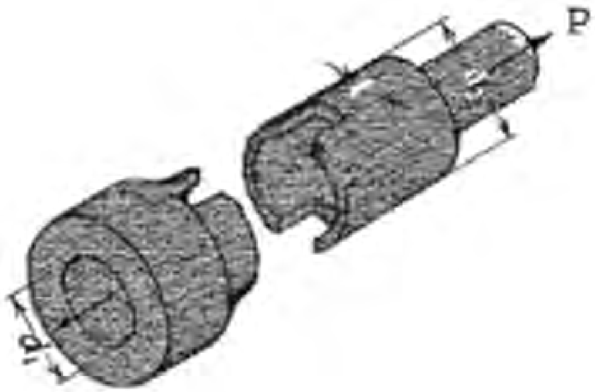
6 mm to 22 mm diameter	2 mm in increment
25 mm to 60 mm diameter	5 mm in increment
60 mm to 110 mm diameter	0 mm in increment
110 mm to 140 mm diameter	15 mm in increment
140 mm to 160 mm diameter	20 mm in increment
500 mm to 600 mm diameter	30 mm in increment

We therefore choose a suitable rod size to be 25 mm.

Refer to figure-4.2.2.2



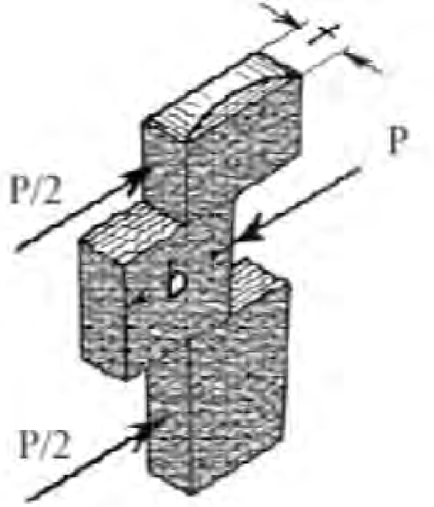
For tension failure across slot $\left(\frac{\pi}{4}d^2 - d_1 t\right)\sigma_y = P$. This gives $d_1 t = 1.58x10^{-4}$ m^2.From empirical relations we may take t=0.4d i.e. 10 mm and this gives d_1= 15.8 mm. Maintaining the proportion let d_1= 1.2 d = 30 mm.



Refer to figure-4.2.2.3

The tensile failure of socket across slot $\left\{\left(\frac{\pi}{4}d_2^2 - d_1^2\right) - (d_2 - d_1)t\right\}\sigma_y = P$

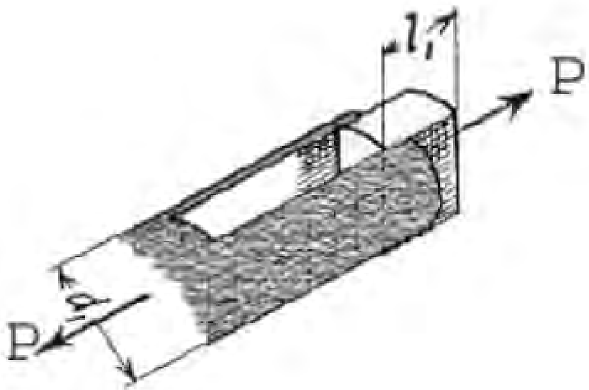
This gives d_2 = 37 mm. Let d_2 = 40 mm



Refer to figure-4.2.2.4

For shear failure of cotter $2bt\tau = P$. On substitution this gives b = 22.72 mm.

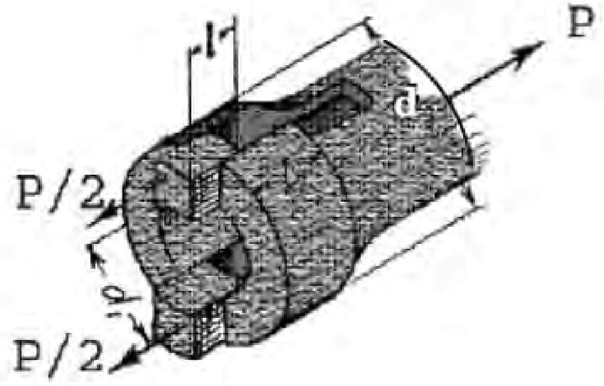
Let b = 25 mm.



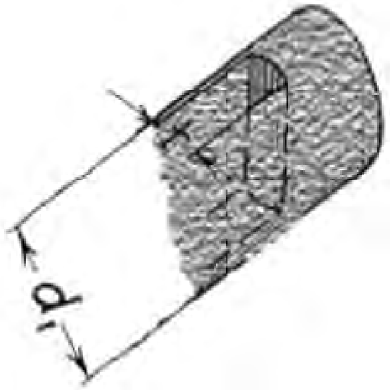
Refer to figure-4.2.2.5

For shear failure of rod end $2l_1d_1\tau = P$ and this gives $l_1 = 7.57$ mm. Let $l_1 =$ 10 mm.

Refer to figure-4.2.2.6

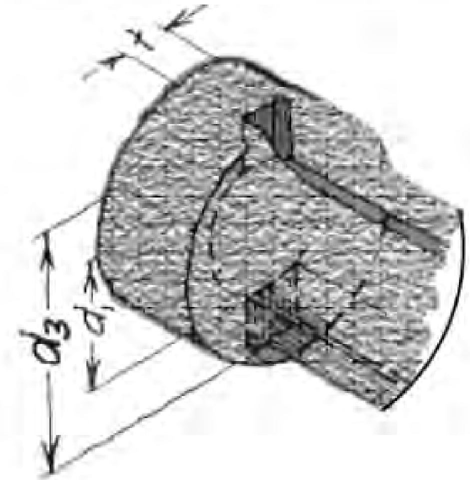


For shear failure of socket end $2l(d_2-d_1)\tau = P$. This gives l= 22.72 mm. Let l=25 mm



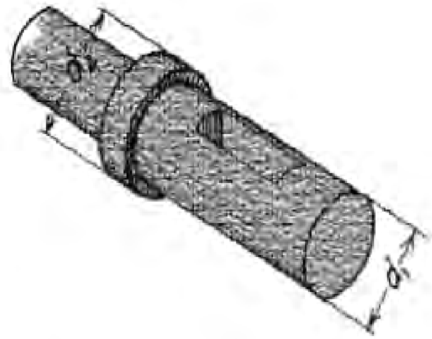
Refer to figure-4.2.2.8

For crushing failure of socket or rod $(d_3-d_1)t\sigma_c = P$. This gives $d_3 = 75.5$ mm. Let $d_3 = 77$ mm.



Refer to figure-4.2.2.9

For crushing failure of collar $\frac{\pi}{4}\left(d_4^2 - d_1^2\right)\sigma_c = P$. On substitution this gives d_4= 38.4 mm. Let d_4= 40 mm.



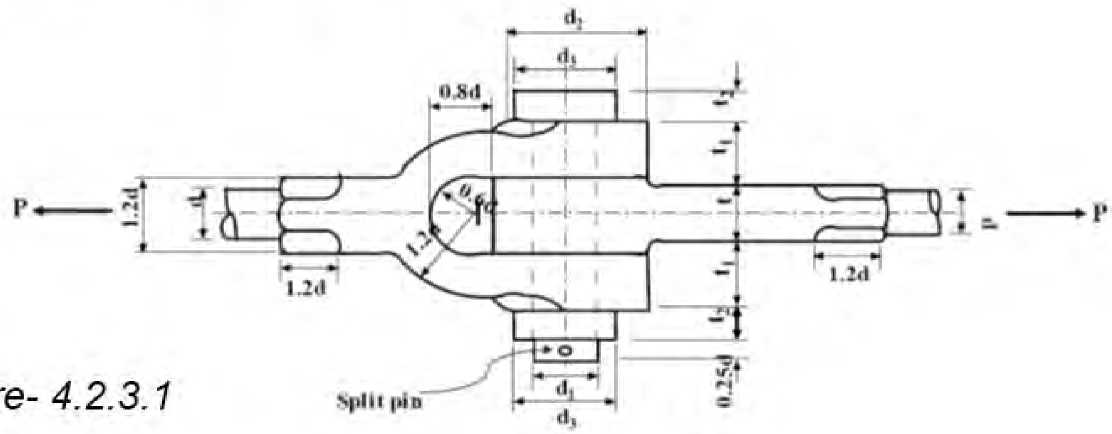
Refer to figure-4.2.2.10

For shear failure of collar $\pi d_1 t_1 \tau$ = P which gives t_1= 4.8 mm. Let t_1 = 5 mm.

Therefore the final chosen values of dimensions are

d= 25 mm; d_1= 30 mm; d_2 = 40 mm; d_3 = 77 mm; d_4 = 40 mm; t= 10 mm; t_1= 5 mm; *l*= 25 mm; l_1= 10 mm; b= 27 mm.

Q.2: Two mild steel rods are connected by a knuckle joint to transmit an axial force of 100 kN. Design the joint completely assuming the working stresses for both the pin and rod materials to be 100 MPa in tension, 65 MPa in shear and 150 MPa in crushing.



Refer to figure- 4.2.3.1

For failure of rod in tension, $P = \frac{\pi}{4} d^2 \sigma_y$. On substituting P=100 kN,

σ_y = 100 MPa we have d= 35.6 mm. Let us choose the rod diameter d = 40 mm which is the next standard size.

We may now use the empirical relations to find the necessary dimensions and then check the failure criteria.

d_1= 40 mm t= 50 mm

d_2 = 80 mm t_1= 30 mm;

d_3 = 60 mm t_2= 20 mm;

split pin diameter = 0.25 d_1 = 10 mm

To check the failure modes:

1. Failure of knuckle pin in shear: $P\Big/\left(2.\frac{\pi}{4}d_1^2\right) = \tau_y$ which gives τ_y = 39.8 MPa. This is less than the yield shear stress.

2. For failure of knuckle pin in bending: $\sigma_y = \dfrac{16P\left(\dfrac{t_1}{3}+\dfrac{t}{4}\right)}{\pi d_1^3}$. On substitution this gives σ_y = 179 MPa which is more than the allowable tensile yield stress of 100 MPa. We therefore increase the knuckle pin diameter to 55 mm which gives σ_y = 69 MPa that is well within the tensile yield stress.

3. For failure of rod eye in shear: $(d_2-d_1)t\tau = P$. On substitution $d_1 = 55$mm $\tau = 80$ MPa which exceeds the yield shear stress of 65 MPa. So d_2 should be at least 85.8 mm. Let d_2 be 90 mm.
4. For failure of rod eye in crushing: $d_1 t\sigma_c = P$ which gives $\sigma_c = 36.36$ MPa that is well within the crushing strength of 150 MPa.
5. Failure of rod eye in tension: $(d_2-d_1)t\sigma_t = P$. Tensile stress developed at the rod eye is then $\sigma_t = 57.14$ MPa which is safe.
6. Failure of forked end in shear: $2(d_2-d_1)t_1\tau = P$. Thus shear stress developed in the forked end is $\tau = 47.61$ MPa which is safe.
7. Failure of forked end in tension: $2(d_2-d_1)t_1\sigma_y = P$. Tensile strength developed in the forked end is then $\sigma_y = 47.61$ MPa which is safe.
8. Failure of forked end in crushing: $2d_1t_1\sigma_c = P$ which gives the crushing stress developed in the forked end as $\sigma_c = 42$ MPa. This is well within the crushing strength of 150 MPa.

Therefore the final chosen values of dimensions are:

$d_1 = 55$ mm	$t = 50$ mm	
$d_2 = 90$ mm	$t_1 = 30$ mm;	and $d = 40$ mm
$d_3 = 60$ mm	$t_2 = 20$ mm;	

4.2.5 Summary of this Lesson

In this lesson two well known joints viz. cotter and knuckle joints used in machinery are discussed. Their constructional detail and working principle have been described. Then the detailed design procedures of both these joints are given with suitable illustrations. Finally two examples, one on cotter joint and the other on cotter joint have been solved.

Lecture

Theme 5

Couplings

5.1. Types and uses of couplings

5.1.1 Introduction

Couplings are used to connect two shafts for torque transmission in varied applications. It may be to connect two units such as a motor and a generator or it may be to form a long line shaft by connecting shafts of standard lengths say 6-8m by couplings. Coupling may be rigid or they may provide flexibility and compensate for misalignment. They may also reduce shock loading and vibration. A wide variety of commercial shaft couplings are available ranging from a simple keyed coupling to one which requires a complex design procedure using gears or fluid drives etc. However there are two main types of couplings:

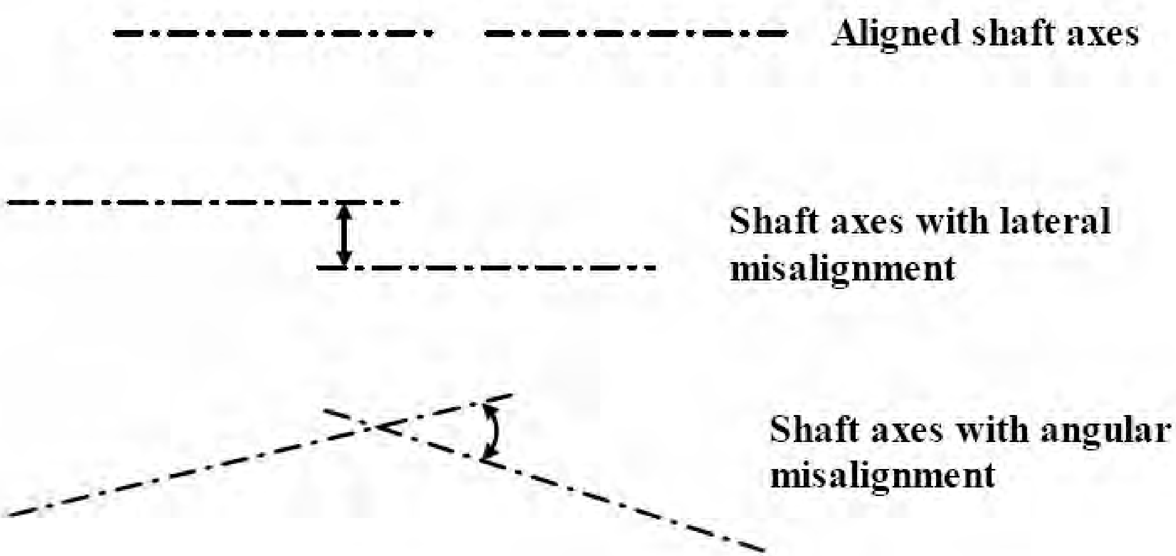
Rigid couplings

Flexible couplings

Rigid couplings are used for shafts having no misalignment while the flexible couplings can absorb some amount of misalignment in the shafts to be connected. In the next section we shall discuss different types of couplings and their uses under these two broad headings.

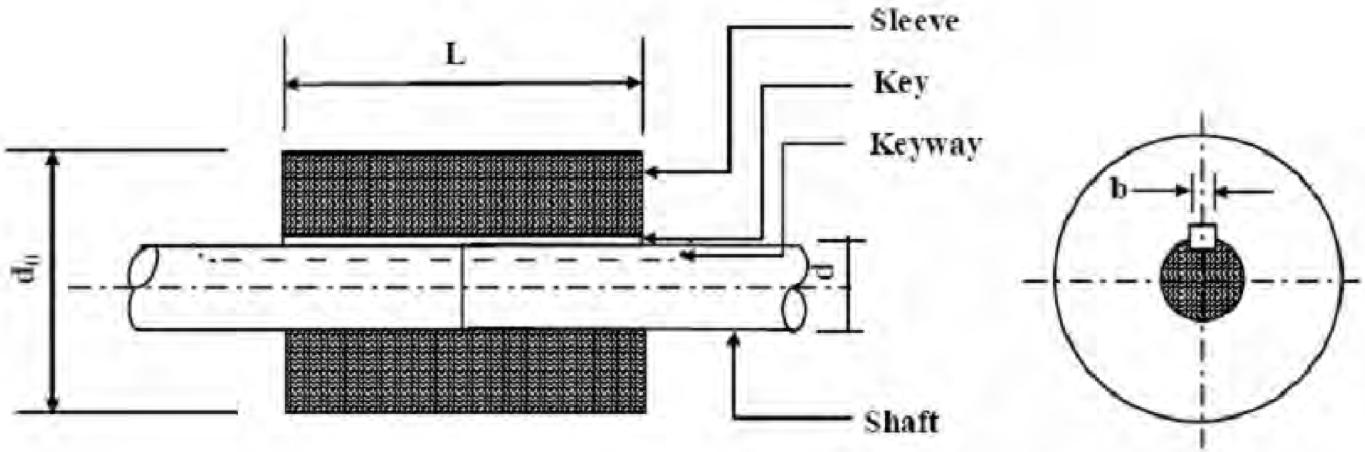
5.1.2 Types and uses of shaft couplings

5.1.2.1 **Rigid couplings** Since these couplings cannot absorb any misalignment the shafts to be connected by a rigid coupling must have good lateral and angular alignment. The types of misalignments are shown schematically in **figure-5.1.2.1.1**.



5.1.2.1.1. *- Types of misalignments in shafts*

5.1.2.1.1 Sleeve coupling One of the simple type of rigid coupling is a sleeve coupling which consists of a cylindrical sleeve keyed to the shafts to be connected. A typical sleeve coupling is shown in **figure- 5.1.2.1.1.1**.



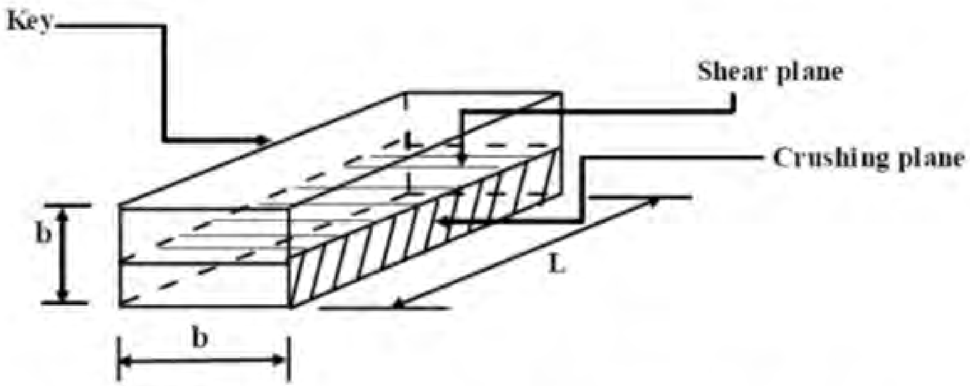
5.1.2.1.1.1 - A typical sleeve coupling

Normally sunk keys are used and in order to transmit the torque safely it is important to design the sleeve and the key properly. The key design is usually based on shear and bearing stresses. If the torque transmitted is T, the shaft radius is r and a rectangular sunk key of dimension b and length L is used then the induced shear stress τ (**figure- 5.1.2.1.1.2**) in the key is given by

$$\tau = T \Big/ \left(b \frac{L}{2} r \right)$$

and for safety

$$(2T/bLr) < \tau_y$$



***5.1.2.1.1.2** - Shear and crushing planes in the key.*

where τ_y is the yield stress in shear of the key material. A suitable factor of safety must be used. The induced crushing stress in the key is given as

$$\sigma_{br} = T \Big/ \left(\frac{b}{2} \frac{L}{2} r \right)$$ and for a safe design $$4T/(bLr) < \sigma_c$$

where σ_c is the crushing strength of the key material.

The sleeve transmits the torque from one shaft to the other. Therefore if d_i is the inside diameter of the sleeve which is also close to the shaft diameter d (say) and d_0 is outside diameter of the sleeve, the shear stress developed in the sleeve is $\tau_{sleeve} = \frac{16Td_0}{\pi\left(d_0^4 - d_i^4\right)}$ and the shear stress in the shaft is given by $\tau_{shaft} = \frac{16T}{\pi d_i^3}$. Substituting yield shear stresses of the sleeve and shaft materials for τ_{sleeve} and τ_{shaft} both d_i and d_0 may be evaluated.

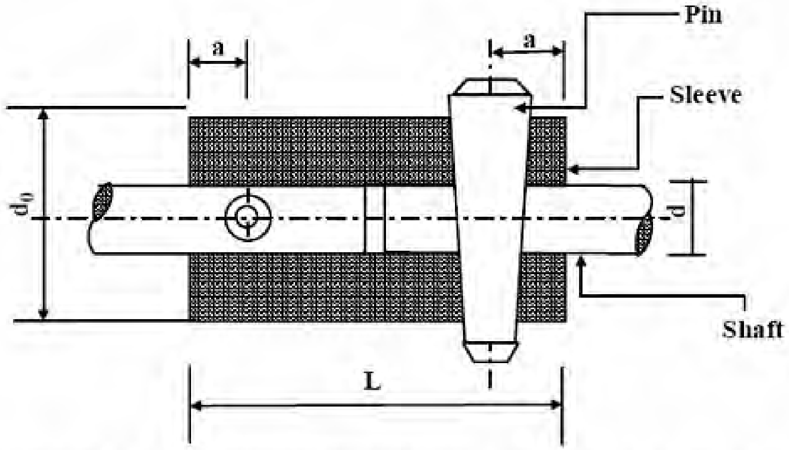
However from the empirical proportions we have:

$d_0 = 2d_i + 12.5$ mm and L=3.5d.

These may be used as checks.

5.1.2.1.2 Sleeve coupling with taper pins

Torque transmission from one shaft to another may also be done using pins as shown in **figure-5.1.2.1.2.1.**



5.1.2.1.2.1 *- A representative sleeve coupling with taper pins.*

The usual proportions in terms of shaft diameter d for these couplings are:
d_0 = 1.5d, L = 3d and a = 0.75d.
The mean pin diameter d_{mean} = 0.2 to 0.25 d. For small couplings d_{mean} is taken as 0.25d and for large couplings d_{mean} is taken as 0.2d. Once the dimensions are fixed we may check the pin

for shear failure using the relation $2\left(\frac{\pi}{4}d_{mean}^2\right)\tau\left(\frac{d}{2}\right)=T$.

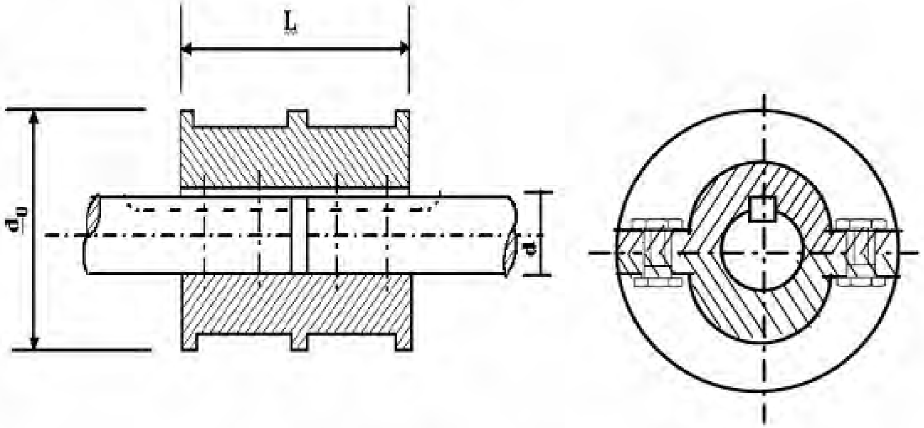
Here T is the torque and the shear stress τ must not exceed the shear yield stress of the pin material. A suitable factor of safety may be used for the shear yield stress.

5.1.2.1.3 Clamp coupling

A typical clamp coupling is shown in **figure-5.1.2.1.3.1**. It essentially consists of two half cylinders which are placed over the ends of the shafts to be coupled and are held together by through bolt.

The length of these couplings 'L' usually vary between 3.5 to 5 times the and the outside diameter 'd0' of the coupling sleeve between 2 to 4 times the shaft diameter d. It is assumed that even with a key the torque is transmitted due to the friction grip. If now the number of bolt on each half is n, its core diameter is dc and the coefficient of friction between the shaft and sleeve material is μ we may find the torque transmitted T as follows:

The clamping pressure between the shaft and the sleeve is given by



5.1.2.1.3.1 - A representative clamp coupling

$$p = \frac{n}{2} x \frac{\pi}{4} d_c^2 x \sigma_t \Big/ (dL/2)$$

where n is the total number of bolts, the number of effective bolts for each shaft is n/2 and σ_t is the allowable tensile stress in the bolt. The tangential force per unit area in the shaft periphery is F = μ p. The torque transmitted can therefore be given by $T = \frac{\pi dL}{2} \mu p . \frac{d}{2}$.

5.1.2.1.4 Ring compression type couplings

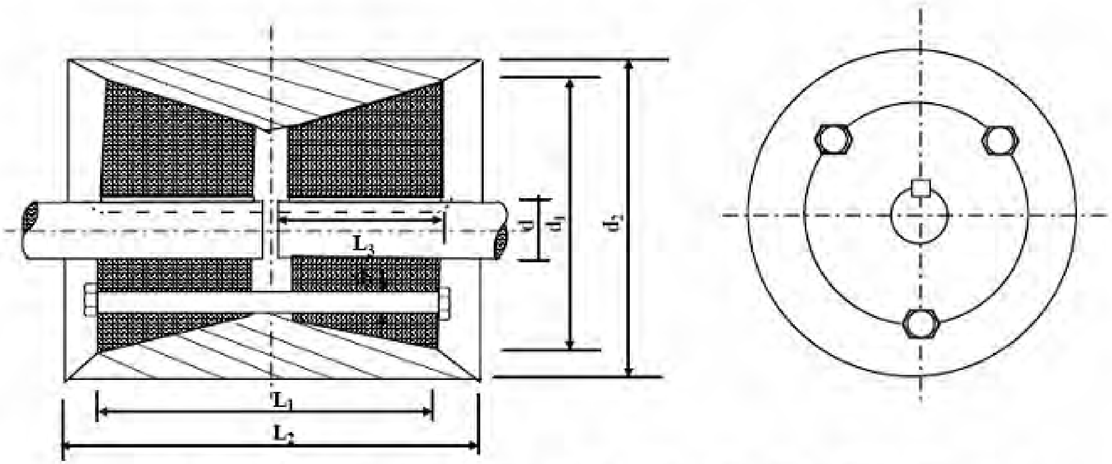
The coupling (**figure-5.1.2.1.4.1)** consists of two cones which are placed on the shafts to be coupled and a sleeve that fits over the cones. Three bolts are used to draw the cones towards each other and thus wedge them firmly between the shafts and the outer sleeve. The usual proportions for these couplings in terms of shaft diameter d are approximately as follows:

$d_1 = 2d + 15.24$ mm $\quad L_1 = 3d$

$d_2 = 2.45d + 27.94$ mm $\quad L_2 = 3.5d + 12.7$ mm

$d_3 = 0.23d + 3.17$ mm $\quad L_3 = 1.5d$

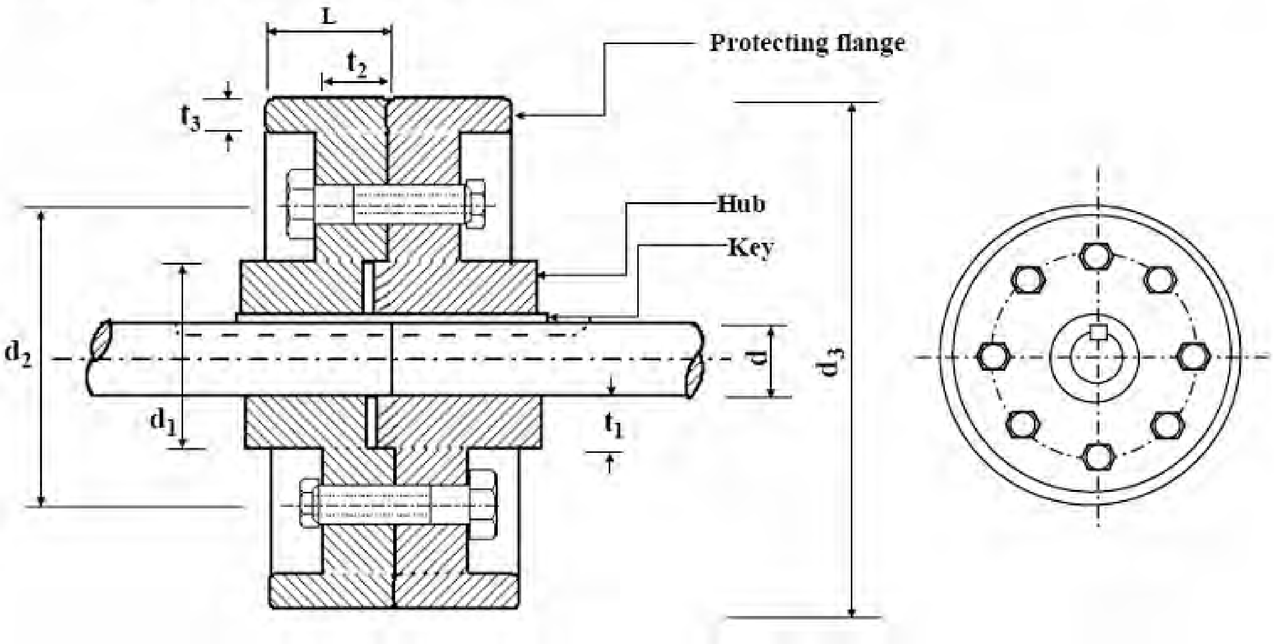
and the taper of the cone is approximately 1 in 4 on diameter.



5.1.2.1.4.1 - A representative ring compression type coupling.

5.1.2.1.4 Flange coupling

It is a very widely used rigid coupling and consists of two flanges keyed to the shafts and bolted. This is illustrated in **figure-5.1.2.1.4.2**.



***5.1.2.1.4.2** - A typical flange coupling*

Design details of such couplings will be discussed in the next lesson. The main features of the design are essentially

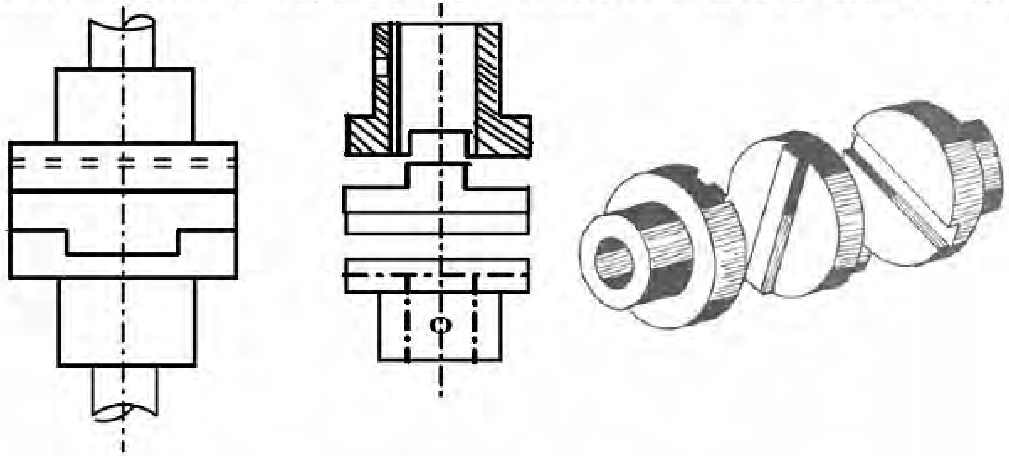
(a) Design of bolts (b) Design of hub (c) Overall design and dimensions.

5.1.2.2 Flexible coupling

As discussed earlier these couplings can accommodate some misalignment and impact. A large variety of flexible couplings are available commercially and principal features of only a few will be discussed here.

5.1.2.2.1 Oldham coupling

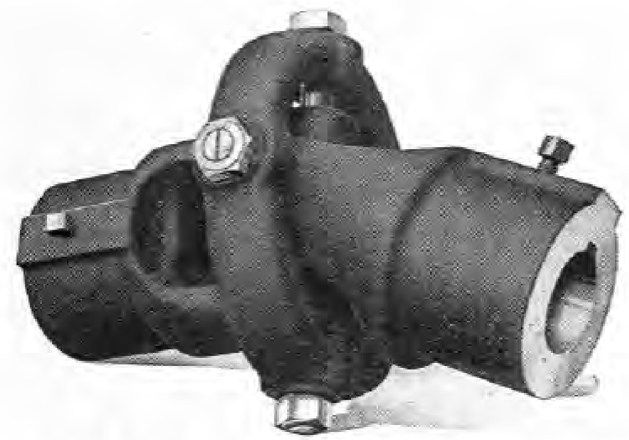
These couplings can accommodate both lateral and angular misalignment to some extent. An Oldham coupling consists of two flanges with slots on the faces and the flanges are keyed or screwed to the shafts. A cylindrical
piece, called the disc, has a narrow rectangular raised portion running across each face but at right angle to each other. The disc is placed between the flanges such that the raised portions fit into the slots in the flanges. The disc may be made of flexible materials and this absorbs some misalignment. A schematic representation is shown in **figure- 5.1.2.2.1.1.**



5.1.2.2.1.1 *- A schematic diagram of an Oldham coupling*

5.1.2.2.2 Universal joints

These joints are capable of handling relatively large angular misalignment and they are widely used in agricultural machinery, machine tools and automobiles.A typical universal joint is shown in **figure- 5.1.2.2.2.1.**There are many forms of these couplings, available commercially but they essentially consist of two forks keyed or screwed to the shaft. There is a center piece through which pass two pins with mutually perpendicular axes and they connect the two fork ends such that a large angular misalignment can be accommodated. The coupling, often known as, Hooke's coupling has no torsional rigidity nor can it accommodate any parallel offset.

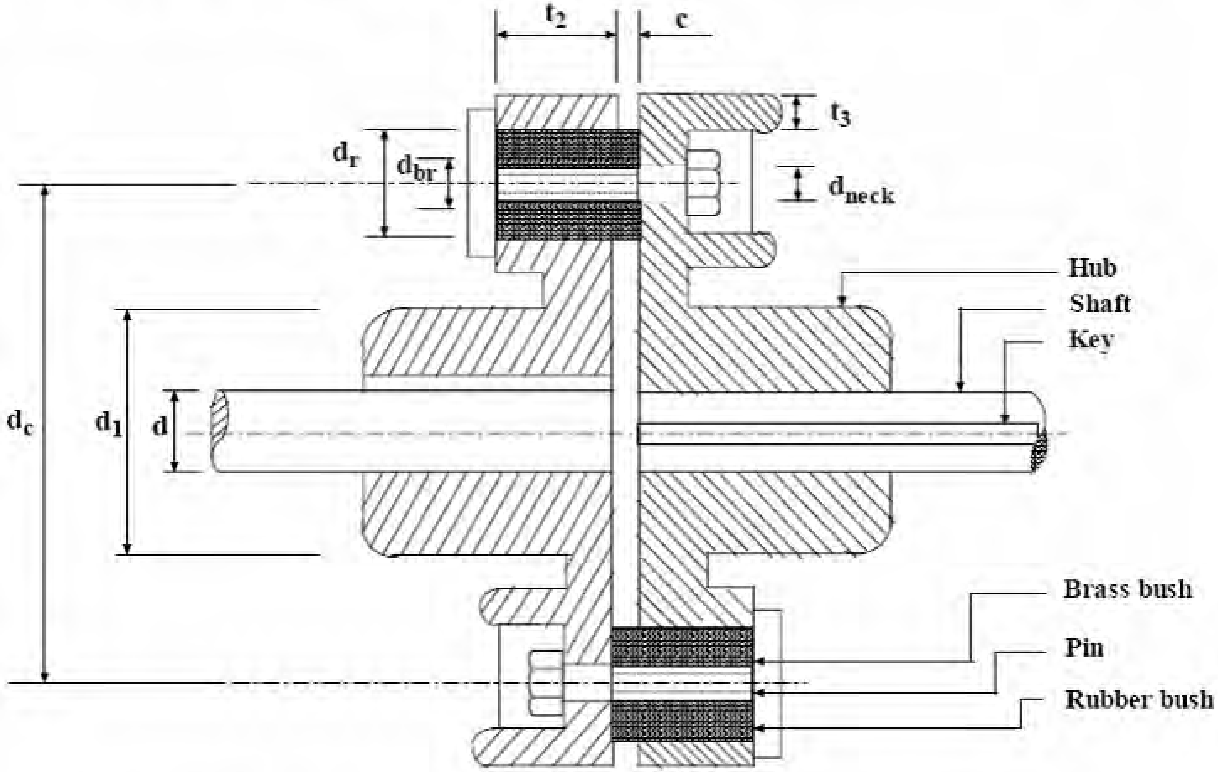


5.1.2.2.2.1 *- A typical universal joint*

5.1.2.2.2 Pin type flexible coupling

One of the most commonly used flexible coupling is a pin type flexible flange coupling in which torque is transmitted from one flange to the other through a flexible bush put around the bolt. This is shown in the next lesson and is shown in **figure-5.2.2.1.**

These are used when excessive misalignment is not expected such as a coupling between a motor and a generator or a pump mounted on a common base plate. Detail design procedure for these couplings will be discussed in the next lesson.



5.2.2.1 - A typical flexible coupling with rubber bushings.

5.1.3 Summary of this Lecture

Basic function of shaft couplings, their types and uses have been discussed in this lesson. Among the rigid couplings some details of sleeve couplings with key or taper pins, clamp couplings, ring compression type couplings and flange couplings have been described. Among the flexible couplings the Oldham coupling and universal joints are described and the functions of pin type flexible couplings are given briefly.

Lecture

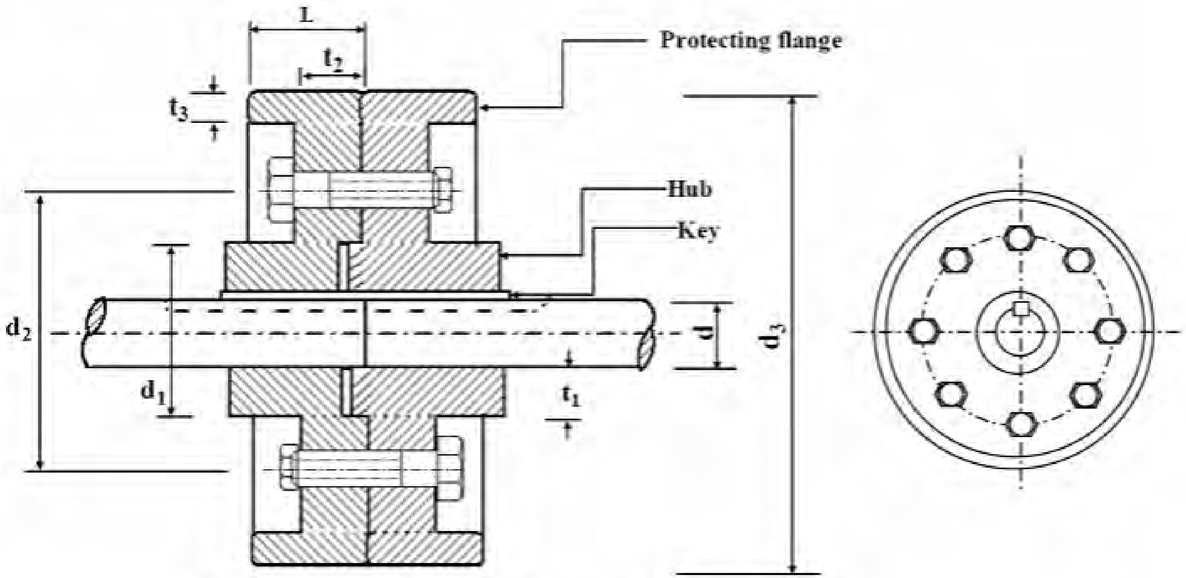
Theme 5

Couplings

5.2. Design procedures for rigid and flexible rubber-bushed couplings

5.2.1 Rigid Flange Coupling

A typical rigid flange coupling is shown in **Figure- 5.2.1.1.**



5. 2.1.1 - A typical flange coupling

If essentially consists of two cast iron flanges which are keyed to the shafts to be joined. The flanges are brought together and are bolted in the annular space between the hub and the protecting flange. The rotective flange is provided to guard the projecting bolt heads and nuts. The bolts are placed equi-spaced on a bolt circle diameter and the number of bolt depends on the shaft diameter d. A spigot 'A' on one flange and a recess on the opposing face is provided for ease of assembly.

The design procedure is generally based on determining the shaft diameter d for a given torque transmission and then following empirical relations different dimensions of the coupling are obtained. Check for different failure modes can then be carried out. Design procedure is given in the following steps:

(1) Shaft diameter'd' based on torque transmission is given by $d = \left(\frac{16T}{\pi\tau_s}\right)^{1/3}$

where T is the torque and τ_y is the yield stress in shear.

(2) Hub diameter d1 =1.75d +6.5mm

(3) Hub length L = 1.5d

But the hub length also depends on the length of the key. Therefore this length L must be checked while finding the key dimension based on shear and crushing failure modes.

(4) Key dimensions:

If a square key of sides b is used then b is commonly taken as d/4 . In that case, **for shear failure** we have $\left(\frac{d}{4}.L_k\right).\tau_y.\frac{d}{2} = T$ where τ_y is the yield stress in shear and Lk is the key length.

This gives $L_k = \frac{8T}{d^2\tau_y}$

If Lk determined here is less than hub length L we may assume the key length to be the same as hub length.

For crushing failure we have $\left(\frac{d}{8}.L_k\right)\sigma_c.\frac{d}{2} = T$ where σ_c is crushing stress induced in the key. This gives $\sigma_c = \frac{16T}{L_k d^2}$

and if $\sigma_c < \sigma_{cy}$, the bearing strength of the key material , the key dimensions chosen are in order.

(5) Bolt dimensions :

The bolts are subjected to shear and bearing stresses while transmitting torque.

Considering the shear failure mode we ha $n.\frac{\pi}{4}d_b{}^2\tau_{yb}\frac{d_c}{2} = T$

where n is the number of bolts, d_b the nominal bolt diameter, T is the torque transmitted, τ_{yb} is the shear yield strength of the bolt material and d_c is the bolt circle diameter. The bolt diameter may now be obtained if n is known. The number of bolts n is often given by the

following empirical relation: $n = \frac{4}{150}d + 3$

where d is the shaft diameter in mm. The bolt circle diameter must be such that it should provide clearance for socket wrench to be used for the bolts. The empirical relation takes care of this Considering crushing failure we have

$$n.d_b t_2 \sigma_{cyb}\frac{d_c}{2} = T$$

where t_2 is the flange width over which the bolts make contact and σ_{cyb} is the yield crushing strength of the bolt material. This gives t_2. Clearly the bolt length must be more than $2t_2$ and a suitable standard length for the bolt diameter may be chosen from hand book.

(6) A protecting flange is provided as a guard for bolt heads and nuts. The thickness t_3 is less than $t_2/2$. The corners of the flanges should be rounded.

(7) The spigot depth is usually taken between 2-3mm.

(8) Another check for the shear failure of the hub is to be carried out. For this failure mode we may write

$$\pi d_1 t_2 \tau_{yf} \frac{d_1}{2} = T$$

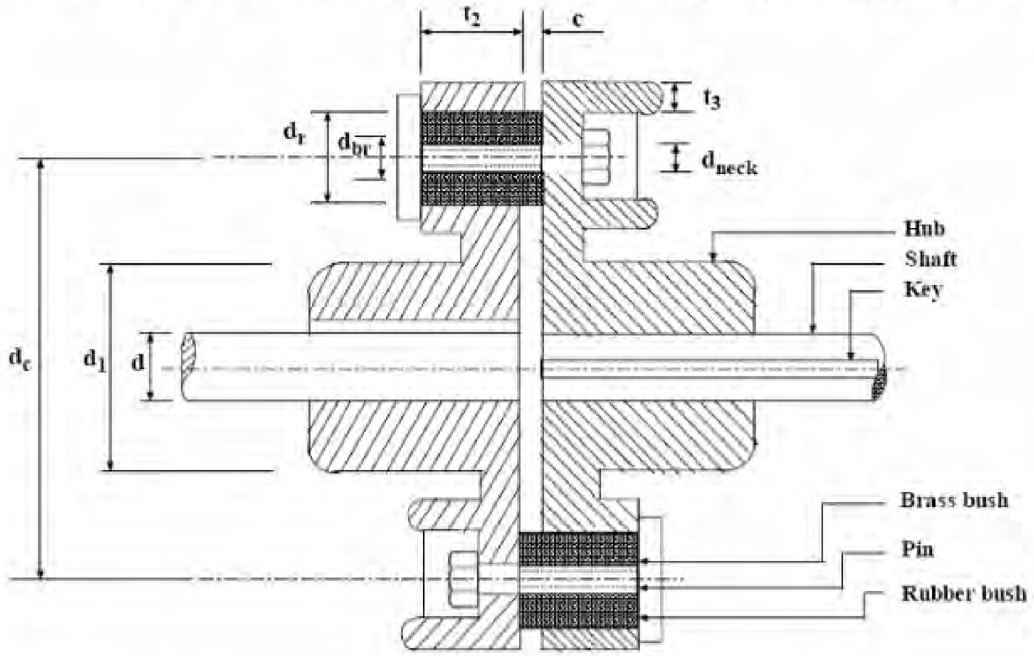
where d_1 is the hub diameter and τ_{yf} is the shear yield strength of the flange material.

Knowing τ_{yf} we may check if the chosen value of t_2 is satisfactory or not.

Finally, knowing hub diameter d_1, bolt diameter and protective thickness t_2 we may decide the overall diameter d_3.

5.2.2 Flexible rubber – bushed couplings

This is simplest type of flexible coupling and a typical coupling of this type is shown in **Figure- 5.2.2.1.**



5.2.2.1 - A typical flexible coupling with rubber bushings.

In a rigid coupling the torque is transmitted from one half of the coupling to the other through the bolts and in this arrangement shafts need be aligned very well.

However in the bushed coupling the rubber bushings over the pins (bolts) (as shown in **Figure-5.2.2.1**) provide flexibility and these coupling can accommodate some misalignment.

Because of the rubber bushing the design for pins should be considered carefully.

(1) Bearing stress Rubber bushings are available for different inside and out side diameters. However rubber bushes are mostly available in thickness between 6 mm to 7.5mm for bores upto 25mm and 9mm thickness for larger bores. Brass sleeves are made to suit the requirements. However, brass sleeve thickness may be taken to be 1.5mm. The outside diameter of rubber bushing d_r is given by

$$d_r = d_b + 2\,t_{br} + 2\,t_r$$

where d_b is the diameter of the bolt or pin , t_{br} is the thickness of the brass sleeve and t_r is the thickness of rubber bushing. Now write

$$n.d_r t_2 p_b \frac{d_c}{2} = T$$

where d_c is the bolt circle diameter and t_2 the flange thickness over the bush contact area. A suitable bearing pressure for rubber is 0.035 N/mm^2 and the number of pin is given by

$n = \frac{d}{25} + 3$. The d_c here is different from what we had for rigid flange bearings. This must be judged considering the hub diameters, out side diameter of the bush and a suitable clearance. From the above torque equation we may obtain bearing pressure developed and compare this with the bearing pressure of rubber for safely.

(2) Shear stress

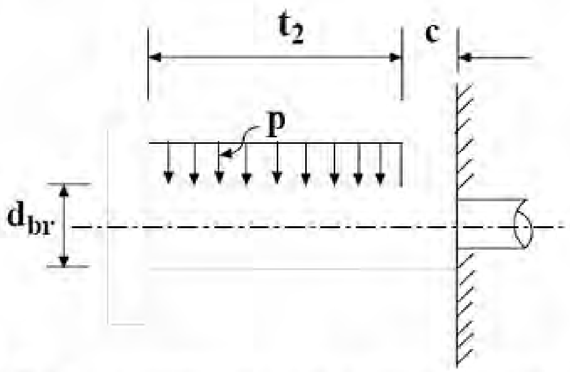
The pins in the coupling are subjected to shear and it is a good practice to ensure that the shear plane avoids the threaded portion of the bolt. Unlike the rigid coupling the shear stress due to torque transmission is given in terms of the tangential force F at the outside diameter of the rubber bush. Shear stress at the neck area is given by

$$\tau_b = \frac{p_b t_2 d_r}{\frac{\pi}{4} d_{neck}^2}$$

where d_{neck} is bolt diameter at the neck i.e at the shear plane.

(3) Bending Stress

The pin loading is shown in **Figure-5.2.2.2.**



5.2.2.2 - Loading on a pin supporting the bushings.

Clearly the bearing pressure that acts as distributed load $\mathbf{F=pt_2d}$ on rubber bush would produce bending of the pin. Considering an equivalent concentrated load the bending stress is

$$\sigma_b = \frac{32F(t_2/2)}{\pi d_{br}^3}$$

Knowing the shear and bending stresses we may check the pin diameter for principal stresses using appropriate theories of failure.

We may also assume the following empirical relations:

Hub diameter = 2d

Hub length = 1.5d

Pin diameter at the neck $= \frac{0.5d}{\sqrt{n}}$

5.2.3 Problems with Answers

Q.1: Design a typical rigid flange coupling for connecting a motor and a centrifugal pump shafts. The coupling needs to transmit 15 KW at 1000 rpm. The allowable shear stresses of the shaft, key and bolt materials are 60 MPa,50 MPa and 25 MPa respectively. The shear modulus of the shaft material may be taken as 84GPa. The angle of twist of the shaft should be limited to 1 degree in 20 times the shaft diameter.

A.1:

1. The shaft diameter based on strength may be given by

$d = \sqrt[3]{\frac{16T}{\pi\tau_y}}$ where T is the torque transmitted and τ_y is the allowable yield stress in shear.

Here T = Power/ $\left(\frac{2\pi N}{60}\right) = \frac{P}{\omega} \frac{15x10^3}{\left(\frac{2\pi x1000}{60}\right)} = 143Nm$, $\omega = \frac{2\pi N}{60}$

Where ω is rate of angular motion in radian, N is rate of angular motion in revolution number

And substituting $\tau_y = 60x10^6 Pa$ we have

$d = \left(\frac{16x143}{\pi x60x10^6}\right)^{\frac{1}{3}} = 2.29x10^{-2}m \approx 23mm.$

2. Let us consider a shaft of 25 mm which is a standard size.

From the rigidity point of view $\frac{T}{J} = \frac{G\theta}{L}$,

Substituting T = 143Nm , J = $\frac{\pi}{32}(0.025)^4 = 38.3\text{x}10^{-9}\,\text{m}^4, G = 84\text{x}10^9\,\text{Pa}$

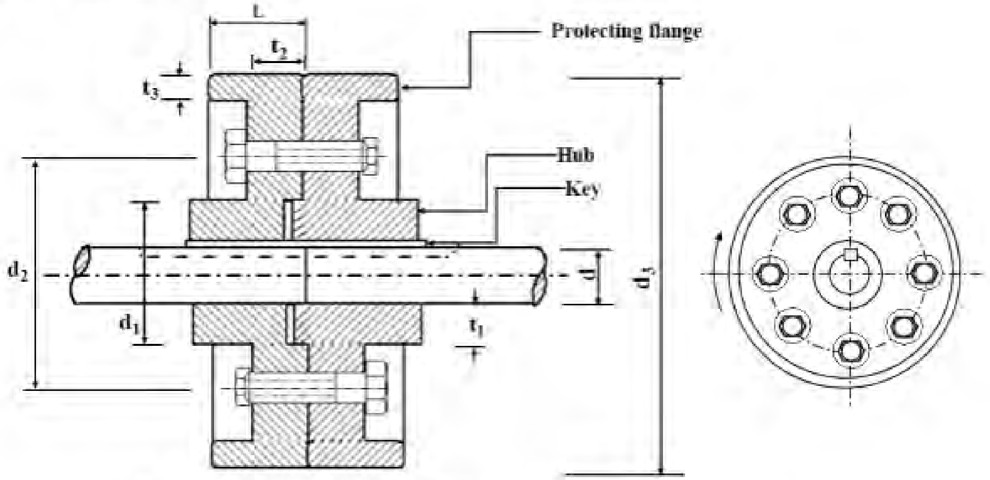
$$\frac{\theta}{L} = \frac{143}{38.3\text{x}10^{-9}\text{x}84\text{x}10^9}$$ = 0.044 radian per meter.

The limiting twist is 1 degree in 20 times the shaft diameter

which is $\frac{\frac{\pi}{180}}{20\text{x}0.025} = 0.035$ radian per meter

Therefore, the shaft diameter of 25mm is safe.

We now consider a typical rigid flange coupling as shown in **Figure 5.2.3.1.**



5. 2.3.1 *- A typical flange coupling*

3. **<u>Hub</u>-**

Using empirical relations

Hub diameter d_1 = 1.75d + 6.5 mm. This gives

d_1 = 1.75x25 + 6.5 = 50.25mm say d_1 = 51 mm

Hub length L=1.5d. This gives L = 1.5x25 = 37.5mm, say L= 38mm.

Hub thickness $t_1 = \frac{d_1 - d}{2} = \frac{51-25}{2} = 13\text{mm}$

4. Key –

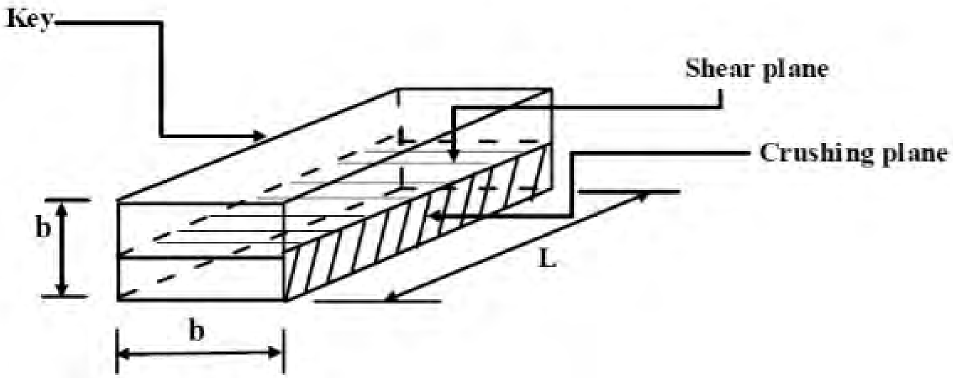
Now to avoid the shear failure of the key (refer to **Figure 5.1.2.1.1.2**)

$$\left(\frac{d}{4}L_k\right).\tau_y.\frac{d}{2}=T$$

where the key width w =d/4 and the key length is L_K

This gives $L_k = \frac{8T}{(\tau_y d^2)}$ i.e.

$$\frac{8x143}{50x10^6x(0.025)^2}=0.0366\ m=36.6\ mm$$



5.2.3.2 - Shear and crushing planes in the key.

The hub length is 37.5 mm. Therefore we take L_K = 37.5mm.

To avoid crushing failure of the key (Ref to **Figure 5.2.3.2**)

$(\frac{d}{8}L_k)\sigma.\frac{d}{2}$ = T where σ is the crushing stress developed in the key. This gives $\sigma=\frac{16T}{L_k d^2}$

Substituting T = 143Nm, L_k = 37.5 x 10^{-3} m and d = 0.025 m $\sigma=\frac{16x143x10^{-6}}{37.5x10^{-3}x(0.025)^2}=97.62MPa$

Assuming an allowable crushing stress for the key material to be 100MPa, the key design is safe. Therefore the key size may be taken as: a square key of 6.25 mm size and 37.5 mm long. However keeping in mind that for a shaft of diameter between 22mm and 30 mm a rectangular key of 8mm width, 7mm depth and length between 18mm and 90mm is recommended. We choose a standard key of 8mm width, 7mm depth and 38mm length which is safe for the present purpose.

5. Bolts.

To avoid shear failure of bolts $n\frac{\pi}{4}d_b^2\tau_{yb}\frac{d_c}{2}=T$

where number of bolts n is given by the empirical relation $n=\frac{4}{150}d+3$

where d is the shaft diameter in mm. which gives n=3.66 and we may take n=4 or more.

Here τ_{yb} is the allowable shear stress of the bolt and this is assumed to be 60 MPa.

d_c is the bolt circle diameter and this may be assumed initially based on hub diameter d_1=51 mm and later the dimension must be justified

Let d_c =65mm. Substituting the values we have the bolt diameter d_b as

$$d_b=\left(\frac{8T}{n\pi\tau_{yb}d_c}\right)^{\frac{1}{2}} \text{ i.e. } \left(\frac{8x143}{4\pi x25x10^6x65x10^{-3}}\right)^{\frac{1}{2}}=7.48x10^{-3}$$

which gives d_b = 7.48 mm. With higher factor of safety we may take d_b = 10 mm which is a standard size.

We may now check for crushing failure as $nd_bt_2\sigma_c\frac{d_c}{2}=T$

Substituting n=4, d_b=10mm,σ_c=100MPa,d_c=65mm&T=143Nm and this gives t_2=2.2mm.

However empirically we have $t_2=\frac{1}{2}t_1+6.5=13mm$

Therefore we take t_2=13mm which gives higher factor of safety.

6. **Protecting flange thickness.**

Protecting flange thickness t_3 is usually less than $\frac{1}{2}t_2$ we therefore take t_3 = 8mm since there is no direct load on this part.

7. **Spigot depth**

Spigot depth which is mainly provided for location may be taken as 2mm.

Check for the shear failure of the hub

To avoid shear failure of hub we have $\pi d_1 t_2 \tau_f \frac{d_1}{2} = T$

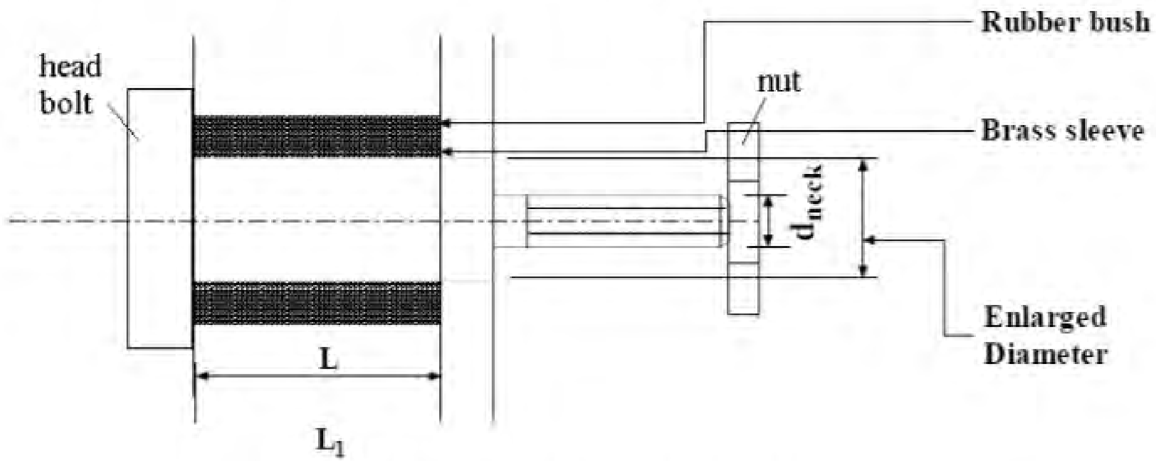
Substituting d_1=51mm, t_2=13mm and T = 143Nm, we have shear stress in flange τ_f as $\tau_f = \frac{2T}{(\pi d^2{}_1 t_2)}$

And this gives τ_f = 2.69 MPa which is much less than the yield shear value of flange material 60MPa.

Q.2: Determine the suitable dimensions of a rubber bush for a flexible coupling to connect of a motor and a pump. The motor is of 50 KW and runs at 300rpm. The shaft diameter is 50mm and the pins are on pitch circle diameter of 140mm. The bearing pressure on the bushes may be taken as 0.5MPa and the allowable shear and bearing stress of the pin materials are 25 MPa and 50 MPa respectively. The allowable shear yield strength of the shaft material may be taken as 60MPa.

A.2:

A typical pin in a bushed flexible coupling is as shown in **Figure-5.2.3.3.**



***5.2.3.1** - A typical pin for the bushings.*

There is an enlarged portion on which a flexible bush is fitted to absorb the misalignment. The threaded portion provided for a nut to tighten on the flange. Considering the whole pin there are three basic stresses developed in the pin in addition to the tightening stresses. There are (a) shear stresses at the unthreaded neck area (b) bending stress over the loaded portion (L) of the enlarged portion of the pin and (c) bearing stress.

However, before we consider the stresses we need to determine the pin diameter and length. Here the torque transmitted $T = \frac{50x10^3}{\left(\frac{2\pi x3000}{60}\right)} = 159Nm$ Based on torsional shear the shaft diameter $d = \left(\frac{16T}{\pi\tau_y}\right)^{\frac{1}{3}}$

Substituting T=159Nm and τ_y = 60MPa, we have d = 23.8mm. Let the shaft diameter be 25mm. From empirical relations we have. Pin diameter at the neck $d_{neck} = \frac{0.5d}{\sqrt{n}}$ where the number of pins $n = \frac{4d}{150} + 3$.

Substituting d = 25 mm we have n = 3.67 (say) 4 d_{neck} = 6.25 (say) 8mm

On this basis the shear stress at the neck = $\frac{T}{\left[\frac{\pi}{4}d^2_{neck}n\frac{d_c}{2}\right]}$ which gives

11.29 MPa and this is much less than yield stress of the pin material.

There is no specific recommendation for the enlarged diameter based on d_{neck} but the enlarged diameters should be enough to provide a neck for tightening. We may choose $d_{enlarged}$ = 16mm which is a standard size. Therefore we may determine the inner diameter of the rubber bush as d_{bush} = Enlarged diameter of the pin + 2x brass sleeve thickness. A brass sleeve of 2mm thickness is sufficient and we have d_{bush} = 20mm. Rubber bush of core diameter up to 25mm are available in thickness of 6mm. Therefore we choose a bush of core diameter 20mm and thickness 6mm. In order to determine the bush length we have $T = npLd_{bush}\frac{d_c}{2}$

where p is the bearing pressure, (Ld_{bush}) is the projected area and d_c is the pitch circle diameter. Substituting T= 159Nm, p = 0.5MPa, d_{bush} = 0.02m and d_c = 0.14m we have L = 56.78 mm.

The rubber bush chosen is therefore of 20mm bore size, 6mm wall thickness and 60 mm long.

5.2.4 Summary of this Lecture

Detailed design procedure of a rigid flange coupling has been discussed in which failure modes of different parts such as the shaft, key, bolts and protecting flange are described. Design details of a flexible coupling using rubber bushings have also been discussed. Here the failure modes of the flexible rubber bushings have been specially considered. Some typical problems have also been solved.

Lecture

Theme 6

Power Screws

6.1. Power Screw drives and their efficiency

6.1.1 Introduction

A power screw is a drive used in machinery to convert a rotary motion into a linear motion for power transmission. It produces uniform motion and the design of the power screw may be such that

(a) Either the screw or the nut is held at rest and the other member rotates as it moves axially. A typical example of this is a screw clamp.

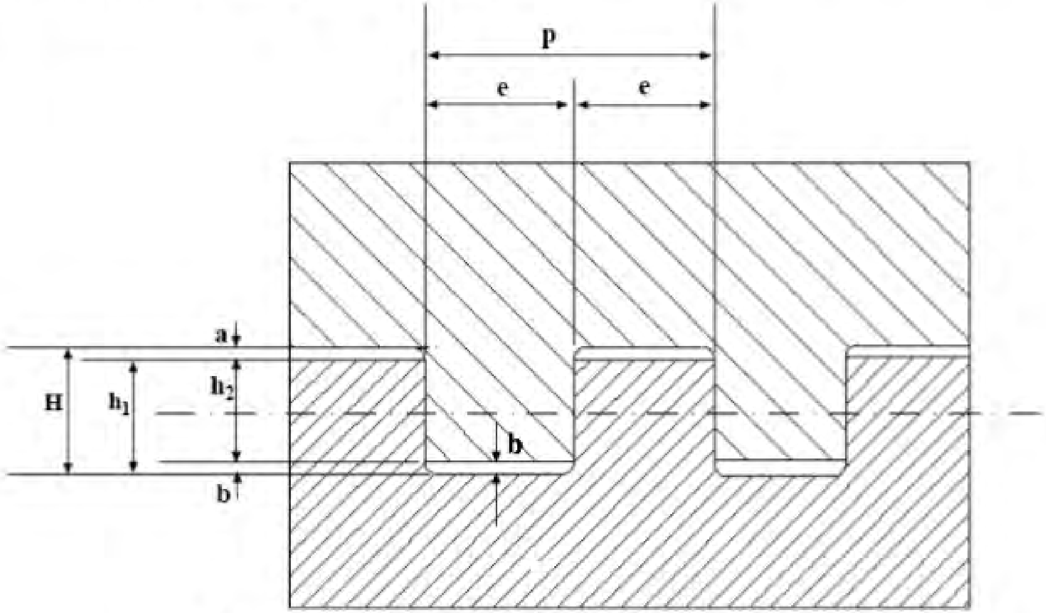
(b) Either the screw or the nut rotates but does not move axially. A typical example for this is a press.

Other applications of power screws are jack screws, lead screws of a lathe, screws for vices, presses etc.

Power screw normally uses square threads but ACME or Buttress threads may also be used. Power screws should be designed for smooth and noiseless transmission of power with an ability to carry heavy loads with high efficiency. We first consider the different thread forms and their proportions:

Square threads-

The thread form is shown in **figure-6.1.1.1**. These threads have high efficiency but they are difficult to manufacture and are expensive. The proportions in terms of pitch are: h_1= 0.5 p ; h_2 = 0.5 p - b ; H = 0.5 p + a; e = 0.5 p a and b are different for different series of threads.



6.1.1.1 *– Some details of square thread form*

There are different series of this thread form and some nominal diameters, corresponding pitch and dimensions a and b are shown in **table-6.1.1.1** as per I.S. 4694-1968.

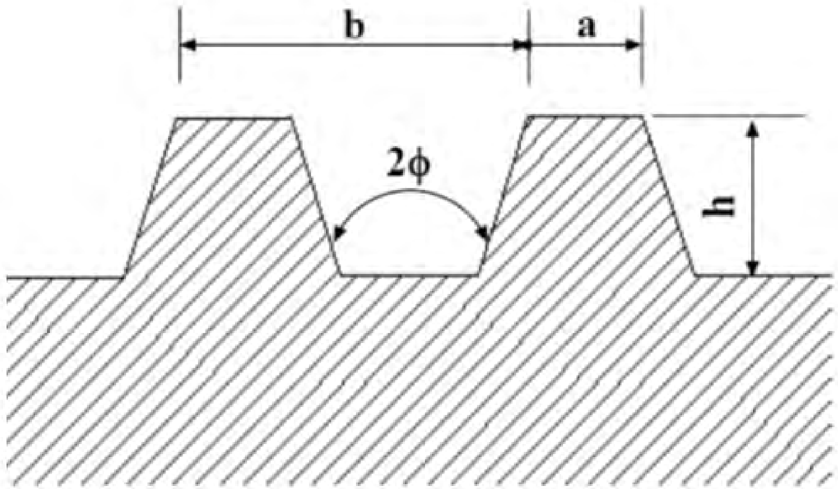
6.1.1.1 – Dimensions of three different series of square thread form.

Fine Series					Normal Series					Coarse Series				
Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)	Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)	Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)
10-22	2	2	0.25	0.25	22-28	2	5	0.25	0.5	22-28	2	8	0.25	0.5
22-62	2	3	0.25	0.25	30-36	2	6	0.25	0.5	30-38	2	10	0.25	0.5
115-175	5	6	0.25	0.5	115-145	5	14	0.5	1	115-130	5	22	0.5	1
250-300	10	12	0.25	0.5	240-260	10	22	0.5	1	250-280	10	40	0.5	1
420-500	20	18	0.5	1	270-290	10	24	0.5	1	290-300	10	44	0.5	1

According to IS-4694-1968, a square thread is designated by its nominal diameter and pitch, as for example, SQ 10 x 2 designates a thread form of nominal diameter 10 mm and pitch 2 mm.

Acme or trapezoidal threads

The acme thread form is shown in **figure- 6.1.1.2**. These threads may be used in applications such as lead screw of a lathe where loss of motion cannot be tolerated. The included angle 2φ = 29° and other proportions are $a = \frac{p}{2.7}$ and $h = 0.25\ p + 0.25\ mm$ where p is pitch



6.1.1.2 *– Some details of acme or trapezoidal thread forms.*

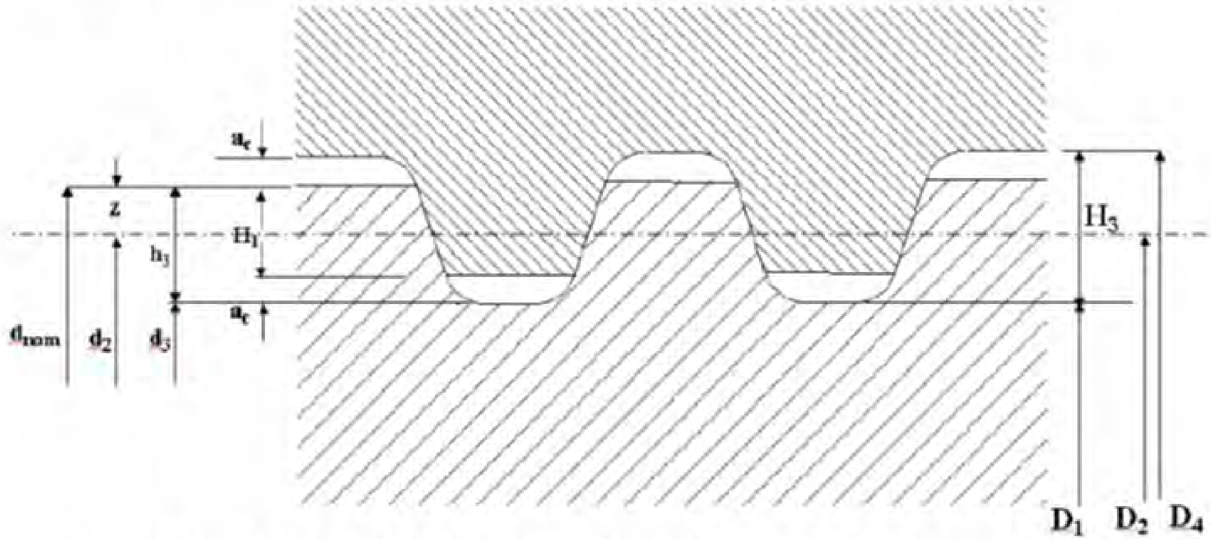
A metric trapezoidal thread form is shown in **figure- 6.1.1.3** and different proportions of the thread form in terms of the pitch are as follows:

Included angle = 30° ; H_1= 0.5 p ; z= 0.25 p+H_1/2 ; H_3= h_3 = H_1+ a_c = 0.5 p + ac

a_c is different for different pitch, for example

a_c = 0.15 mm for p = 1.5 mm ; a_c = 0.25 mm for p = 2 to 5 mm;

a_c = 0.5 mm for p = 6 to 12 mm ; a_c = 1 mm for p = 14 to 44 mm.



6.1.1.3 *- Some details of a metric Trapezoidal thread form.*

Some standard dimensions for a trapezoidal thread form are given in **table- 6.1.1.2** as per IS 7008 (Part II and III) - 1973:

6.1.1.2 - Dimensions of a trapezoidal thread form.

Nominal Diameter (mm)	8	10	5	25	50	75	100	150	200	250	300
pitch (mm)	1.5	2	4	5	8	10	12	16	18	22	24

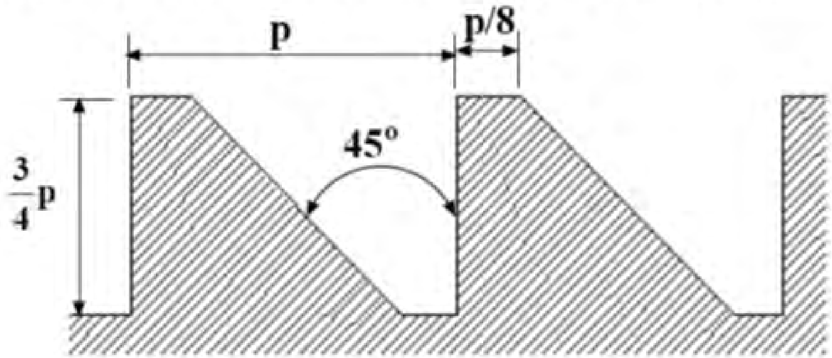
According to IS7008-1973 trapezoidal threads may be designated as, for example, Tr 50 x 8 which indicates a nominal diameter of 50 mm and a pitch of 8 mm.

Buttress thread

This thread form can also be used for power screws but they can transmit power only in one direction. Typical applications are screw jack, vices etc. A Buttress thread form is shown in **figure- 6.1.1.4**. and the proportions are shown in the figure in terms of the pitch.

On the whole the square threads have the highest efficiency as compared to other thread forms but they are less sturdy than the trapezoidal thread forms and the adjustment for wear is difficult for square threads.

When a large linear motion of a power screw is required two or more parallel threads are used. These are called multiple start power drives.



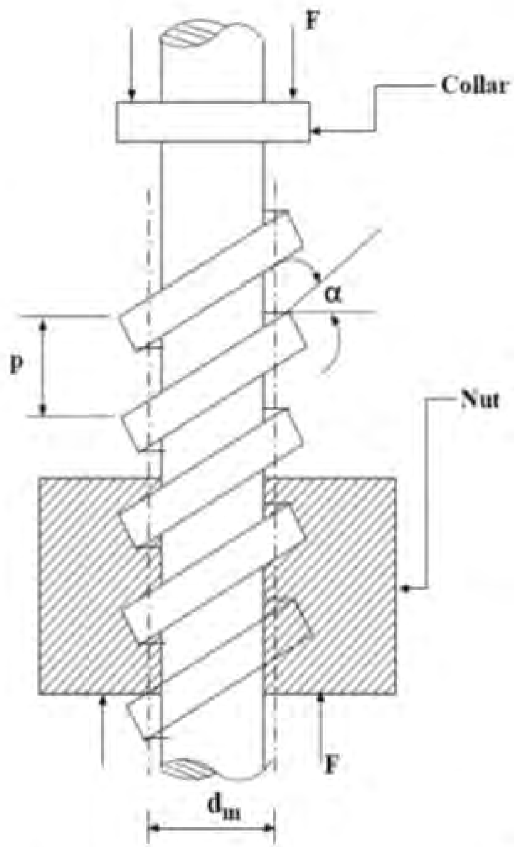
***6.1.1.4** – Some details of a Buttress thread form*

6.1.2 Efficiency of a power screw

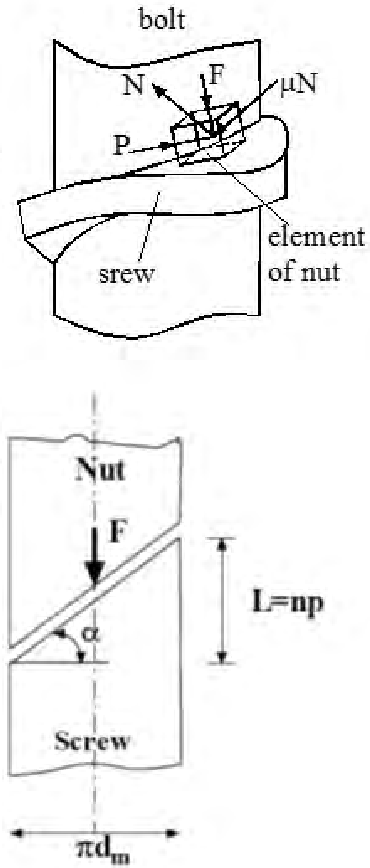
A square thread power screw with a single start is shown in **figure- 6.1.2.1**. Here p is the pitch, α the helix angle, dm the mean diameter of thread and F is the axial load. A developed single thread is shown in **figure- 6.1.2.2** where L = n p for a multi-start drive, n being the number of starts. In order to analyze the mechanics of the power screw we need to consider two cases:

(a) Raising the load

(b) Lowering the load.



***6.1.2.1** – A square thread power screw*



***6.1.2.2** - Development of a single thread*

Raising the load

This requires an axial force P as shown in **figure- 6.1.2.3**. Here N is the normal reaction and μN is the frictional force.

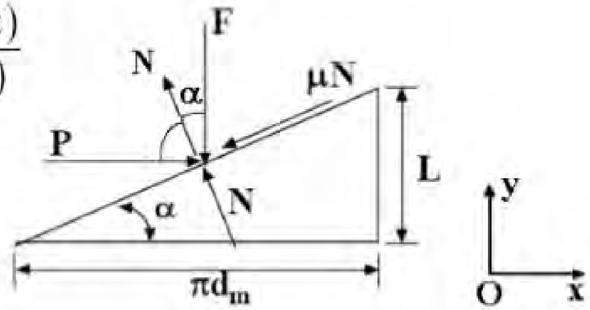
For equilibrium

y: $P - \mu N \cos \alpha - N \sin \alpha = 0$

x: $F + \mu N \sin \alpha - N \cos \alpha = 0$

This gives $N = F/(\cos\alpha - \mu \sin\alpha)$, $P = \dfrac{F(\mu\cos\alpha + \sin\alpha)}{(\cos\alpha - \mu\sin\alpha)}$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$



***6.1.2.3** - Forces at the contact surface for raising the load.*

Torque transmitted during raising the load is then given by $T_R = P\dfrac{d_m}{2} = F\dfrac{d_m}{2}\dfrac{(\mu\cos\alpha + \sin\alpha)}{(\cos\alpha - \mu\sin\alpha)}$

Since $\tan\alpha = \dfrac{L}{\pi d_m}$ we have

$$T_R = F\frac{d_m}{2}\frac{(\mu\pi d_m + L)}{(\pi d_m - \mu L)}$$

The force system at the thread during lowering the load is shown in **figure- 6.1.2.4**. For equilibrium we have the equations

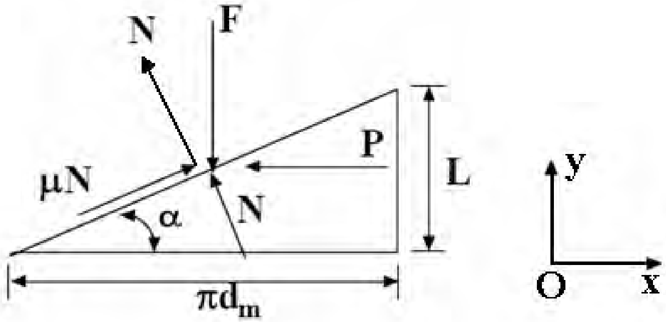
$P - \mu N \cos\alpha + N \sin\alpha = 0$

$F - N \cos\alpha - \mu N \sin\alpha = 0$

This gives

$$N = F/(\cos\alpha + \mu\sin\alpha)$$

$$P = \frac{F(\mu\cos\alpha - \sin\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$



6.1.2.4 *- Forces at the contact surface for lowering the load.*

Torque required to lower the load is given by

$$T_L = P\frac{d_m}{2} = F\frac{d_m}{2}\frac{(\mu\cos\alpha - \sin\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$

And again taking $\tan\alpha = \frac{L}{\pi d_m}$ we have $T_L = F\frac{d_m}{2}\frac{(\mu\pi d_m - L)}{(\pi d_m + \mu L)}$

Condition for self locking

The load would lower itself without any external force if

$$\mu \pi d_m < L$$

and some external force is required to lower the load if

$$\mu \pi d_m \geq L$$

This is therefore the **condition for self locking**.

Efficiency of the power screw is given by

$$\eta = \frac{\text{Work output}}{\text{Work input}}$$

Here work output = F. L

Work input = P. πd_m

This gives $\eta = \frac{F}{P} \tan \alpha$

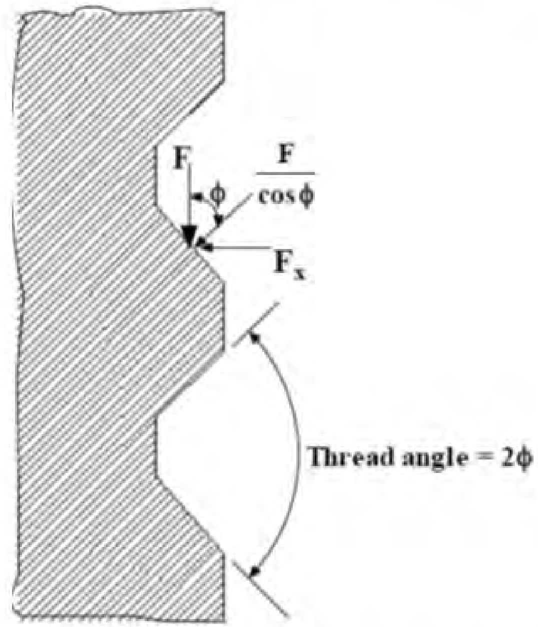
$$w = F \cdot L. \quad \text{Force} \times \frac{\text{vertical}}{\text{dis tan ce}}$$

$$w = P \cdot \pi d_m. \quad \text{Force} \times \text{dis tan ce}$$

The above analysis is for square thread and for trapezoidal thread some modification is required. Because of the thread angle the force normal to the thread surface is increased as shown in **figure- 6.1.2.5.** The torque is therefore given by

$$T = F\frac{d_m}{2}\frac{(\mu\pi d_m \sec\phi + L)}{(\pi d_m - \mu L \sec\phi)}$$

This considers the increased friction due to the wedging action. The trapezoidal threads are not preferred because of high friction but often used due to their ease of machining.



6.1.2.5 *– Normal force on a trapezoidal thread surface*

Bursting effect on the nut

Bursting effect on the nut is caused by the horizontal component of the axial load F on the screw and this is given by (**figure- 6.1.2.5**)

$F_x = F \tan \phi$

For an ISO metric nut $2\phi = 60^o$ and $F_x = 0.5777$ F.

Collar friction

If collar friction μc is considered then another term $\mu F d_c/2$ must be added to torque expression. Here dc is the effective friction diameter of the collar. Therefore we may write the torque required to raise the load as

$$T = F\frac{d_m}{2}\frac{(\mu\pi d_m + L)}{(\pi d_m - \mu L)} + \mu_c F\frac{d_c}{2}$$

6.1.3 Problems with Answers

Q.1: The C-clamp shown in **figure-6.1.3.1** uses a 10 mm screw with a pitch of 2 mm. The frictional coefficient is 0.15 for both the threads and the collar. The collar has a frictional diameter of 16 mm. The handle is made of steel with allowable bending stress of 165 MPa. The capacity of the clamp is 700 N.

(a) Find the torque required to tighten the clamp to full capacity.

(b) Specify the length and diameter of the handle such that it will not bend unless the rated capacity of the clamp is exceeded. Use 15 N as the handle force. T=F·L

$d_{nom} = 10mm$

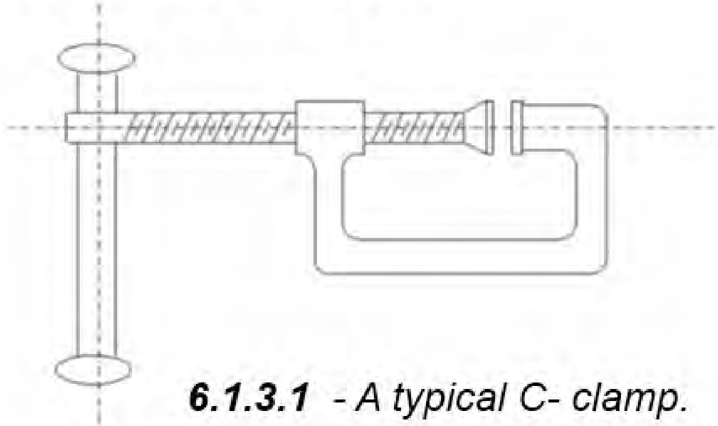
$P = 2mm$

$\mu = 0.15$

$d_c = 16mm$

$\sigma_y = 165$

$F = 700N$



6.1.3.1 *- A typical C- clamp.*

A.1.

1. Nominal diameter of the screw, d = 10 mm.

Pitch of the screw, p = 2 mm.

Choosing a square screw thread we have the following dimensions:

Root diameter, $d_3 = d_{nominal} - 2h_3$ = 7.5 mm (since a_c = 0.25mm and $h_3 = 0.5p + a_c$)

Pitch diameter, $d_2 = d_{nominal} - 2z$ = 8 mm. (since z = 0.5 p)

Mean diameter, dm = (7.5+8)/2 = 7.75 mm.

Torque, $T = F\frac{d_m}{2}\frac{(\mu\pi d_m + L)}{(\pi d_m - \mu L)} + \mu_c F\frac{d_c}{2}$

Here F = 700 N, $\mu = \mu_c$ = 0.15, L = p = 2 mm (assuming a single start screw thread) and d_c = 16 mm. Substituting these date. This gives T=1.48Nm. Ans.

Equating the torque required and the torque applied by the handle of length L we have 1.48 = 15 L since the assumed handle force is 15 N. This gives L= 0.0986 m. Let the handle length be 100 mm.

The maximum bending stress that may be developed in the handle is

$\sigma = \frac{My}{I} = \frac{32M}{\pi d^3}$ where d is the diameter of the handle.

Taking the allowable bending stress as 165 MPa we have

$$d = \left(\frac{32M}{\pi\sigma_y}\right)^{1/3} = \left(\frac{32x1.48}{\pi x165x10^6}\right)^{1/3} = 4.5x10^{-3} m = 4.5 \text{ mm}$$

With a higher factor of safety let d = 10 mm.

Q.2. A single square thread power screw is to raise a load of 50 KN. A screw thread of major diameter of 34 mm and a pitch of 6 mm is used. The coefficient of friction at the thread and collar are 0.15 and 0.1 respectively. If the collar frictional diameter is 100 mm and the screw turns at a speed of 1 rev s^{-1} find

(a) the power input to the screw.

(b) the combined efficiency of the screw and collar.

A.2. (a) Mean diameter, $d_m = d_{major} - p/2 = 34\text{-}3 = 31$ mm.

Torque $T = F\frac{d_m}{2}\frac{(\mu\pi d_m + L)}{(\pi d_m - \mu L)} + \mu_c F\frac{d_c}{2}$, substituting

Here $F=5 \times 10^3$ N, $d_m=31$ mm, $\mu=0.15$, $\mu_c=0.1$, L=p= 6 mm and $d_c=100$ mm

Therefore $T = 50x10^3 x\frac{0.031}{2}\left(\frac{0.15\pi x 0.031 + 0.006}{\pi x 0.031 - 0.15x0.006}\right) + 0.1x50x10^3 x\frac{0.1}{2}$ = <u>416 Nm</u>. Ans.

(b) The torque to raise the load only (T_0) may be obtained by substituting $\mu = \mu_c = 0$ in the torque equation. This gives

$$T_0 = F\frac{d_m}{2}\left(\frac{L}{\pi d_m}\right) = \frac{FL}{2\pi} = \frac{50x10^3 x0.006}{2\pi} = 47.75$$

Therefore $\eta = \frac{FL/2\pi}{T} = \frac{47.75}{416} = 0.1147$ i.e. 11.47%

6.1.4 Summary of this Lecture

Power screw drive in machinery is firstly discussed and some details of the thread forms used in such drives are given. The force system at the contact surface between the screw and the nut is analyzed and the torque required to raise and lower a load, condition for self locking and the efficiency of a power screw are derived. Typical problems on power screw drives are taken up and discussed.

Lecture

Theme 6

Power Screws

6.2. Design of power screws

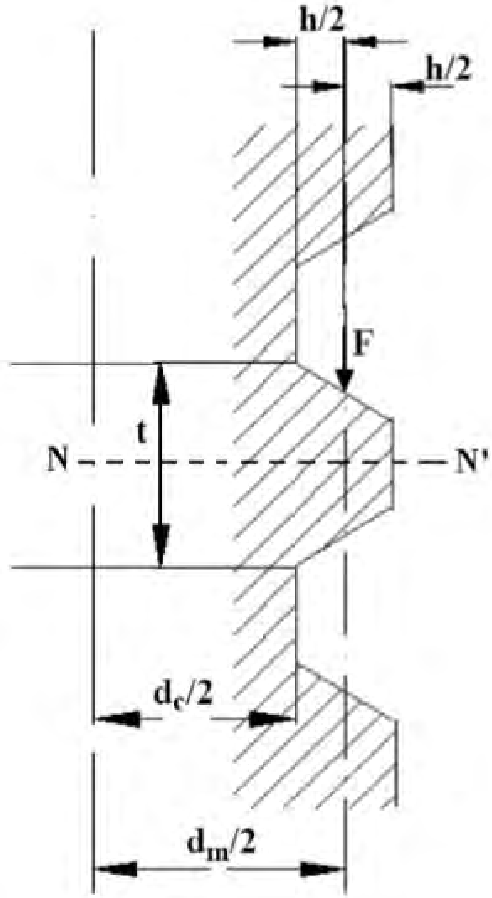
6.2.1 Stresses in power screws

Design of a power screw must be based on the stresses developed in the constituent parts. A power screw is subjected to an axial load and a turning moment. The following stresses would be developed due to the loading:

a) **Compressive stress** is developed in a power screw due to axial load. Depending on the slenderness ratio it may be necessary to analyze for buckling. The compressive stress σ_c is given by σ_c=P/pd_c2 where d_c is the core diameter and if slenderness ratio λ is more than 100 or so buckling criterion must be used. λ is defined as λ = L/K where I=AK2 and L is the length of the screw. Buckling analysis yields a critical load P_c and if both ends are assumed to be hinged critical load is given by $P_C = \pi^2 \frac{EI}{L^2}$. In general the equation may be written as $P_C = n\pi^2 \frac{EI}{L^2}$ where n is a constant that depends on end conditions.

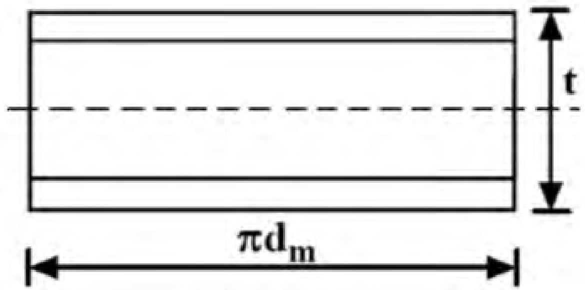
b) Torsional shear stress is developed in the screw due to the turning moment and this is given by $\tau = 16T/\pi\delta_c^3$ where T is the torque applied.

c) Bending stresses are developed in the screw thread and this is illustrated in **figure-6.2.1.1**.

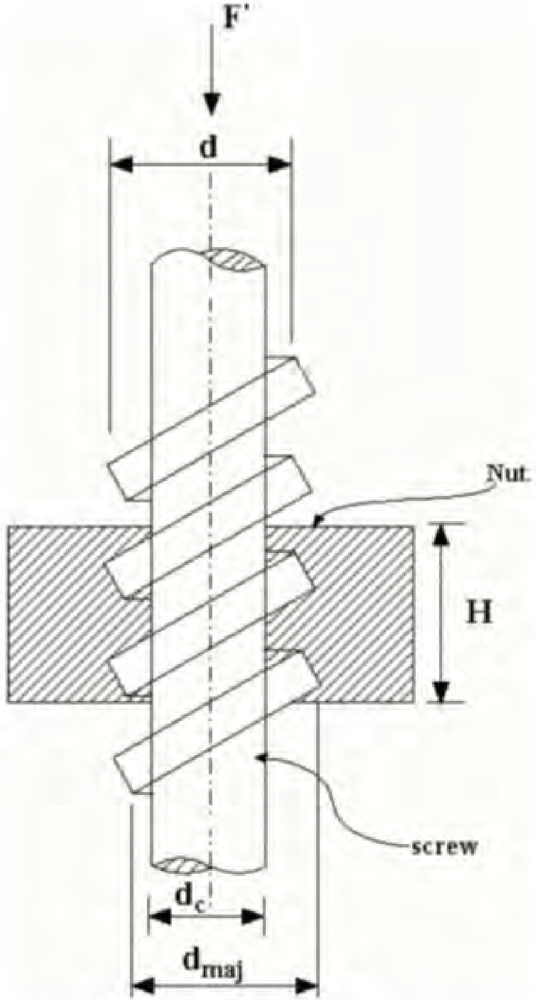


6.2.1.1 *- Loading and bending stresses in screw threads*

The bending moment $M=F'h/2$ and the bending stress on a single thread is given by $\sigma_b=My/I$. Here $y=t/2$, $I=\pi d_m t^3/12$ and F' is the load on a single thread. **Figure-6.2.1.2** shows a developed thread and **figure-6.2.1.3** shows a nut and screw assembly. This gives the bending stress at the thread root to be $\sigma_b=3F'h/\pi d_m t^2$. This is clearly the most probable place for failure.



***6.2.1.2** - Dimensions of a developed thread*



***6.2.1.3** - A screw and nut assembly*

Assuming that the load is equally shared by the nut threads

d) Bearing stress σ_br at the threads is given by

$$\sigma_{br} = \frac{F' / n'}{\pi d_m h}$$

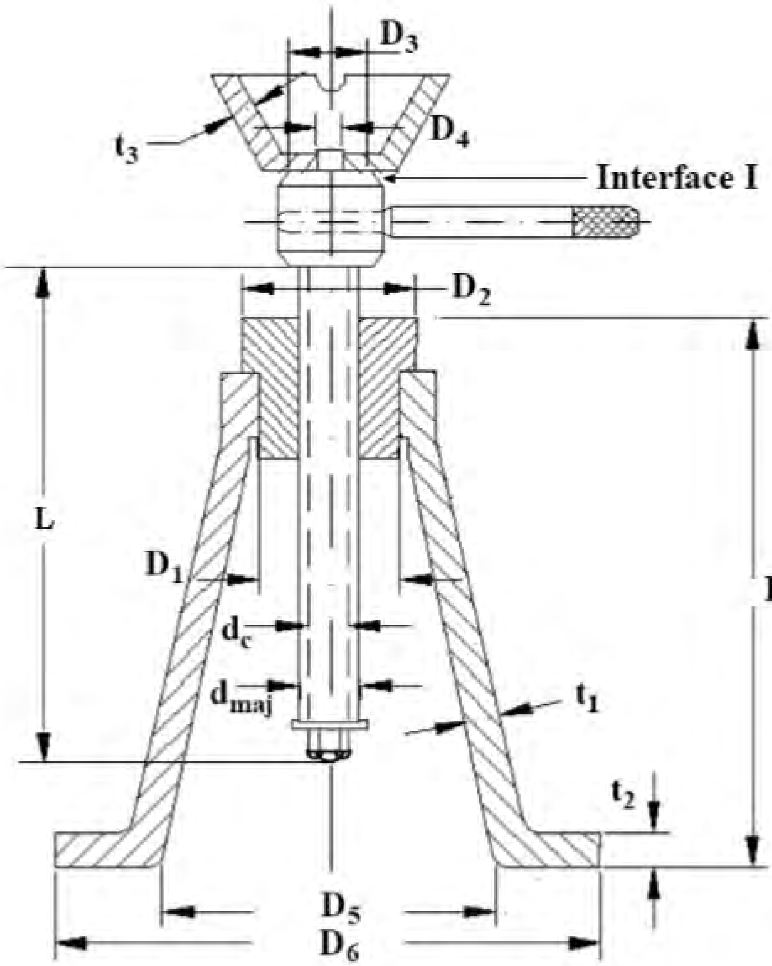
e) Again on similar assumption **shear stress** τ at the root diameter is given by

$$\tau = \frac{F' / n'}{\pi d_c t}$$

Here n is the number of threads in the nut. Since the screw is subjected to torsional shear stress in addition to direct or transverse stress combined effect of bending, torsion and tension or compression should be considered in the design criterion.

6.2.2 Design procedure of a Screw Jack

A typical screw jack is shown in **figure-6.2.2.1** . It is probably more informative to consider the design of a jack for a given load and lift. We consider a reasonable value of the load to be 100KN and lifting height to be 500mm. The design will be considered in the following steps:



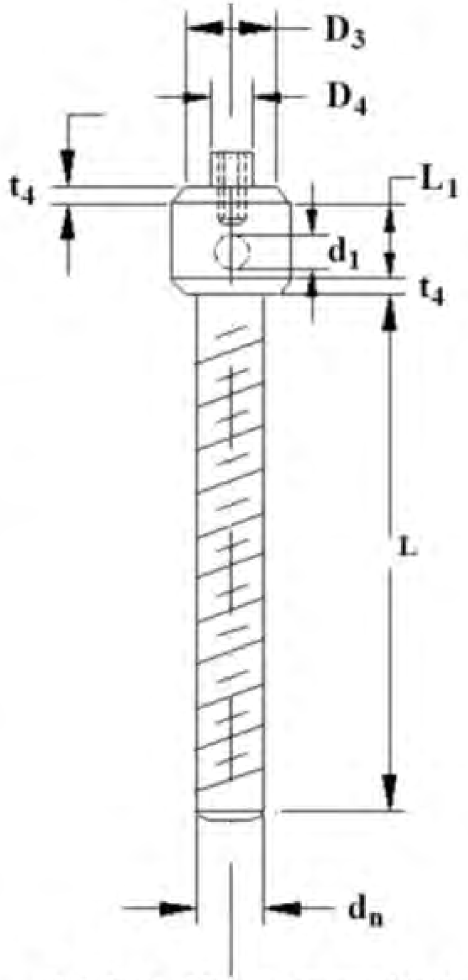
6.2.2.1 *- A typical screw jack*

1. Design of the screw

A typical screw for this purpose is shown in **figure-6.2.2.2.**

Let us consider a mild steel screw for which the tensile and shear strengths may be taken to be approximately 448MPa and 224 MPa respectively. Mild steel being a ductile material we may take the compressive yield strength to be also close to 448MPa. Taking a very high factor of safety of 10 due to the nature of the application and considering the axial compression the core diameter of the screw dc is given by $d_c = \sqrt{\dfrac{100\text{x}10^3}{\dfrac{\pi}{4}\left(\dfrac{448\text{x}10^6}{10}\right)}}$

which gives $d_c \approx 54$ mm.



6.2.2.2 *- The screw with the provision for tommy bar attachment*

From the chart of normal series square threads in **table- 6.1.1.1** the nearest standard nominal diameter of 70 mm is chosen, with pitch p=10 mm.
Therefore, core diameter d_c = 60 mm , Major diameter d_{maj} = 70mm , Mean diameter d_m = 65 mm , Nominal diameter d_n = 70mm.

The torque required to raise the load is given by $T = \frac{Fd_m}{2}\left(\frac{l+\mu\pi d_m}{\pi d_m - \mu l}\right)$

Where l = np, n being the number of starts. Here we have a single start screw and hence l = p =10mm, d_m = 65mm, F = $100X10^3$N
Taking a safe value of μ for this purpose to be 0.26 and substituting the values we get T = 1027 Nm.

6.1.1.1 – Dimensions of three different series of square thread form.

Fine Series					Normal Series					Coarse Series				
Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)	Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)	Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)
10-22	2	2	0.25	0.25	22-28	2	5	0.25	0.5	22-28	2	8	0.25	0.5
22-62	2	3	0.25	0.25	30-36	2	6	0.25	0.5	30-38	2	10	0.25	0.5
115-175	5	6	0.25	0.5	115-145	5	14	0.5	1	115-130	5	22	0.5	1
250-300	10	12	0.25	0.5	240-260	10	22	0.5	1	250-280	10	40	0.5	1
420-500	20	18	0.5	1	270-290	10	24	0.5	1	290-300	10	44	0.5	1

Check for combined stress

The screw is subjected to a direct compressive stress σc and a torsional shear stress τ . The stresses are given by

$$\sigma_c = \frac{4F}{\pi d_c^{\ 2}} = \frac{4x100x10^3}{\pi x(0.06)^2} = 35.3MPa$$

$$\tau = \frac{16T}{\pi d_c^{\ 3}} = \frac{16x1027}{\pi x(0.060)^3} = 24.22MPa$$

The principal stress can be given by

$$\sigma_{1,2} = \frac{35.3}{2} \pm \sqrt{\left(\frac{35.3}{2}\right)^2 + (24.22)^2} = 47.6MPa \text{ and } -12.31 \text{ MPa}$$

and maximum shear stress τ_{max} = 29.96 MPa.

The factor of safety in compression = 448/12.31=36.4 and in shear = 224/29.96=7.48. Therefore the screw dimensions are safe. Check for buckling and thread stress are also necessary. However this can be done after designing the nut whose height and number of threads in contact is needed to determine the free length of the screw.

2. Design of the nut

A suitable material for the nut, as shown in **figure- 6.2.2.3**, is phosphor bronze which is a Cu-Zn alloy with small percentage of Pb and the yield stresses may be taken as

Yield stress in tension σ_{ty} = 125MPa,

Yield stress in compression σ_{cy} = 150MPa,

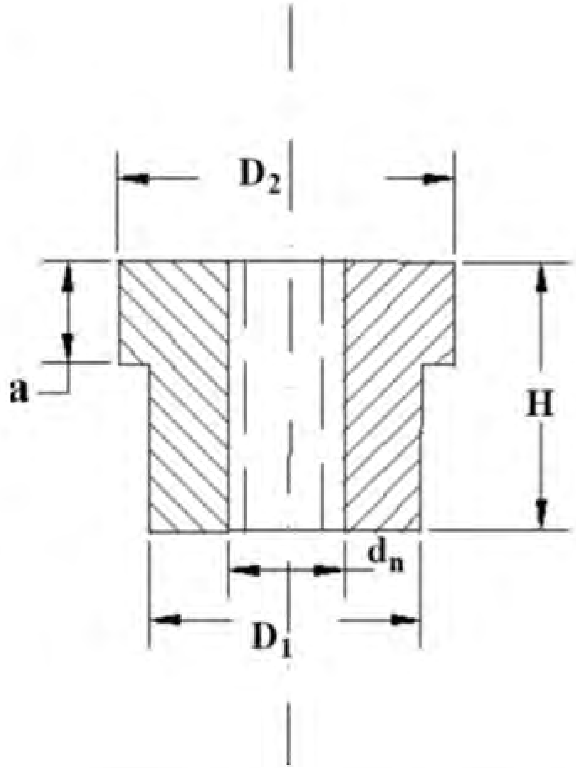
Yield stress in shear τ_y = 105MPa,

Safe bearing pressure P_b = 15MPa.

Considering that the load is shared equally by all threads bearing failure may be avoided if

$$F = \frac{\pi}{4}\left(d_{maj}^{\ 2} - d_c^{\ 2}\right)P_b n^{/}$$

where $n^{/}$ is the number of threads in contact. Substituting values in the above equation we have $n^{/}$ = 6.52. Let $n^{/}$ =8. Therefore H = $n^{/}$p = 8X10 = 80mm.



6.2.2.3 - A phosphor bronze nut for the screw jack

The nut threads are also subjected to crushing and shear. Considering crushing failure we have

$$F = n' \frac{\pi}{4}\left(d_{maj}^{2} - d_{c}^{2}\right)\ \sigma_C$$

This gives σ_c = 12.24 MPa which is adequately safe since σ_{cy} = 150 MPa and therefore crushing is not expected. To avoid shearing of the threads on the nut we may write $F = \pi d_{maj}\ t\ n'\ \tau$ where t is the thread thickness which for the square thread is p/2 ie 5. This gives τ=11.37MPa and since τ_y=105MPa shear failure of teeth is not expected. Due to the screw loading the nut needs to be checked for tension also and we may write

$$CF = \frac{\pi}{4}\left(D_1^{2} - d_c^{2}\right)\sigma_{ty}$$

A correlation factor C for the load is used to account for the twisting moment. With C=1.3 and on substitution of values in the equation D_1 works out to be 70mm. But D_1 needs to be larger than d_{maj} and we take D_1 = 100mm.

<u>We may also consider crushing</u> of the collar of the nut and to avoid this we may write

$$F = \frac{\pi}{4}\left(D_2^2 - D_1^2\right)\sigma_{cy} = \pi\left[\left(\frac{D_2}{2}\right)^2 - \left(\frac{D_1}{2}\right)\right]\sigma_{cy}.$$

Substituting values we have D_2 = 110 mm. To allow for the collar margin we take D_2 =120mm. **Considering shearing** of the nut collar $\pi D_1 a \tau_y = F$. Substituting values we have a = 4mm Let a = 15mm.

3. Buckling of the Screw.

Length L of the screw = Lifting height + H.

This gives L= 500+80 = 580 mm

With the nominal screw diameter of 70mm ,

I = $\frac{\pi(0.07)^4}{64} = 1.178X10^{-6}$ and K = $\sqrt{\frac{I}{A}} = \sqrt{\frac{1.178X10^{-6}}{\frac{\pi}{4}(0.07)^2}} = 0.0175\text{mm.}$

The slenderness ration

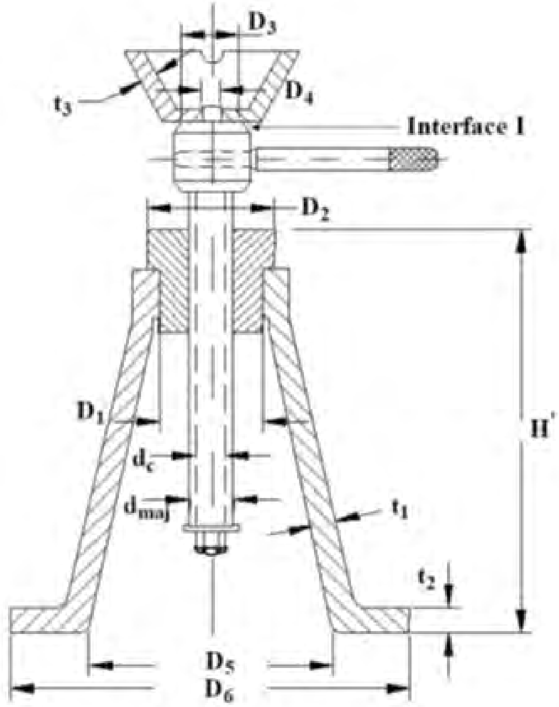
$$\lambda = \frac{L}{K} = \frac{0.58}{0.0175} \simeq 33$$

This value of slenderness ratio is small (< 40) and **the screw may be treated as a short column . No buckling of the screw is therefore expected.**

4. Tommy bar

A typical tommy bar for the purpose is shown in **figure-6.2.2.4.a**.

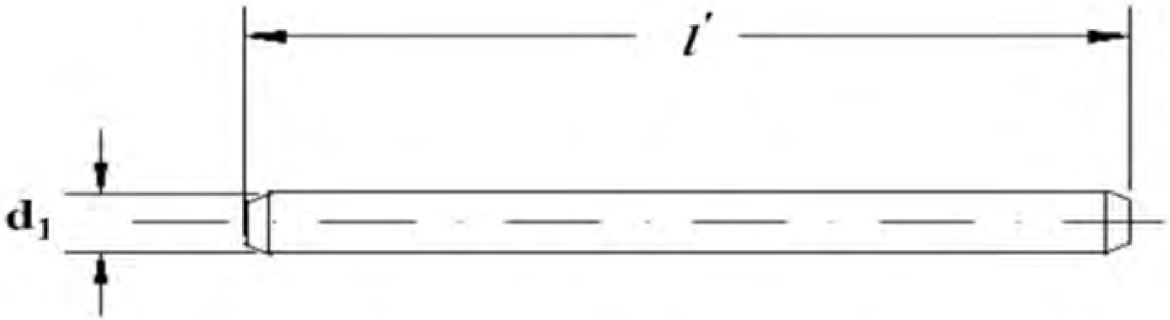
Total torsional moment without the collar friction is calculated in section 6.2.2.1 and T = 1027 Nm. The collar friction in this case (see **figure-6.2.2.1**) occurs at the interface I. However in order to avoid rotation of the load when the screw rotates a loose fitting of the cup is maintained.



6.2.2.1 *- A typical screw jack*

6.2.2.4.a *- A typical tommy bar with a holding end.*

Length $l^/$ of the tommy bar = l_1 + and we may write the torque T as 3D $T= F1l^/$ Where F_1 is the maximum force applied at the tommy bar end and this may be taken as approximately 400 N . This gives $l^/$=1027/400 = 2.56m. This length of the tommy bar is too large and one alternative is to place the tommy bar centrally and apply force at both the ends. This alternative design of the tommy bar is also shown in **figure-6.2.2.4.b**



6.2.2.4.b *- A typical centrally located tommy bar*

The bar is subjected to a bending moment and its maximum value may be taken as1027 Nm. This means to avoid bending we may write $\frac{\pi}{32} d_1^3 \sigma_{ty} = 1027$ where d_1 is the tommy bar diameter as shown in **figure- 6.2.2.4.b** If we choose a M.S bar of σ_{ty} = 448MPa the tommy bar diameter d_1 works out to be $\underline{d_1 = 0.0285m}$.

Let $\underline{d_1 = 30\ mm}$ and we choose $\underline{d_2 = 40mm.}$

5. Other dimensions

D_3 = (1.5 to 1.7) d. Let D_3 = 112 mm.

$D_4 = D_3/2$= 56 mm.

Let L_1 = 100 mm and t_4 = 10 mm.

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