МИНИСТЕРСТВО ОБРАЗОВАНИЯ РЕСПУБЛИКИ БЕЛАРУСЬ

Белорусский национальный технический университет

Кафедра «Теоретическая механика»

А. В. Чигарев

МАШИНОСТРОИТЕЛЬНЫЙ ДИЗАЙН ДЛЯ МЕХАТРОННЫХ СИСТЕМ

MECHANICAL ENGINEERING DESIGN FOR MECHATRONIC SYSTEMS

Курс лекций в форме презентаций

В 2 частях

Часть 2

Минск
БНТУ
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Рассмотрены методы расчета механических компонентов машин, представляющих собой мехатронные системы. Лекции построены в виде презентаций так, что каждая страница на бумажном носителе соответствует слайду в проекторе. Пособие предназначено для студентов специальностей по направлениям мехатроники, робототехники, информационных систем машиностроения, приборостроения и других технических специальностей. Может быть полезно магистрантам, аспирантам, инженерам соответствующих специальностей.

This course of lectures in the form presentation will introduce the mechanical engineering students to the design methods of mechanical elements of machines using only analytical tools of classical mechanics and theory of solid body accordance with International and National Standards for machine design.

Methods of these courses are used by students for studying disciplines “Mechatronics”, “Design of mechatronic systems”, which offered in Belarusian National Technical University for students which to take the profession “Computer mechatronics” and other technical professions.

This course will be useful for engineers which are practiced in field of machine design.

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Lecture
Theme 1

1.1 Definitions and classifications for machine design

1.1.1 Introduction

Design is to formulate a plan to satisfy a particular need and to create something with a physical reality. Consider for an example, design of a table. A number of factors need be considered first:

(a) The purpose for which the table is to be designed such as whether it is to be used as an easy table, an office table or a dining table.

(b) Whether the table is to be designed for a grownup person or a child.

(c) Material for the table, its strength and cost need to be determined.

(d) Finally, the aesthetics of the designed table.

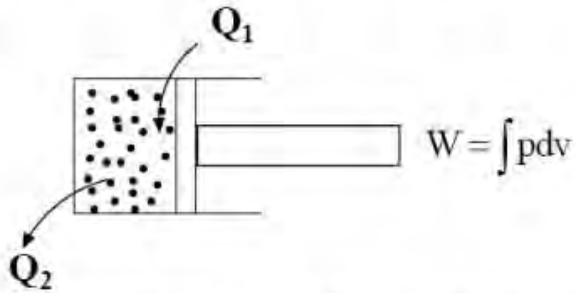
Almost everyone is involved in design, in one way or the other, in our daily lives because problems are posed and they need to be solved.

1.1.2 Basic concept of machine design

Decision making comes in every stage of design. Consider two cars of different makes. They may both be reasonable cars and serve the same purpose but the designs are different. The designers consider different factors and come to certain conclusions leading to an optimum design. Market survey gives an indication of what people want. Existing norms play an important role. Once a critical decision is made, the rest of the design features follow. For example, once we decide the engine capacity, the shape and size, then the subsequent course of the design would follow. A bad decision leads to a bad design and a bad product.

Design may be for different products and with the present specialization and knowledge bank, we have a long list of design disciplines e.g. ship design, plan design, building design, robot design, mechatroniks system and so on.

Here we are concerned with machine design. We now define a machine as a combination of resisting bodies with successfully constrained relative motions which is used to transform other forms of energy into mechanical energy or transmit and modify available energy to do some useful work. If it converts heat into mechanical energy we then call it a heat engine. This is illustrated in figure-1.1.2.1.



1.1.2.1- Conversion of heat to mechanical energy in a piston cylinder arrangement.

In this example for case heat energy $Q_1 > Q_2$, W – energy (work) piston is moved to right, for case heat energy $Q_1 < Q_2$, piston is moved to left.

In many cases however, the machines receive mechanical energy and modify it so that a specific task is carried out, for example a hoist, a bicycle or a hand-winch.

1.1.3 Types of design

There may be several types of design such as

Adaptive design
This is based on existing design, for example, standard products or systems adopted for a new application. Conveyor belts, control system of machines and mechanisms or haulage systems are some of the examples where existing design systems are adapted for a particular use.

Developmental design
Here we start with an existing design but finally a modified design is obtained. A new model of a car is a typical example of a developmental design .

New design
This type of design is an entirely new one but based on existing scientific principles. No scientific invention is involved but requires creative thinking to solve a problem. Examples of this type of design may include designing a small vehicle for transportation of men and material on board a ship or in a desert. Some research activity may be necessary.

1.1.4 Types of design based on methods

Rational design

This is based on determining the stresses and strains of components and thereby deciding their dimensions.

Empirical design

This is based on empirical formulae which in turn is based on experience and experiments. For example, when we tighten a nut on a bolt the force exerted or the stresses induced cannot be determined exactly but experience shows that the tightening force may be given by $P=284d$ where, d is the bolt diameter in mm and P is the applied force in kg. There is no mathematical backing of this equation but it is based on observations and experience.

Industrial design

These are based on industrial considerations and norms viz. market survey, external look, production facilities, low cost, use of existing standard products.

1.1.5 Factors to be considered in machine design

There are many factors to be considered while attacking a design problem. In many cases these are a common sense approach to solving a problem. Some of these factors are as follows:

(a) Device or mechanism which to be used. This would decide the relative arrangement of the constituent elements.

(b) Material which to be used

(c) Forces on the elements

(d) Size, shape weight and space requirements.

(e) The method of manufacturing the components and their assembly.

(f) How will it operate?

(g) Reliability and safety aspects of the machin

(h) Inspectibility of the product

(i) Maintenance, cost and aesthetics of the designed product.

Explanations:

(a) Device or mechanism to be used- This is best judged by understanding the problem thoroughly. Sometimes a particular function can be achieved by a number of means or by using different mechanisms and the designer has to decide which one is most effective under the circumstances. A rough design or layout diagram may be made to crystallize the thoughts regarding the relative arrangement of the elements.

(b) Material- This is a very important aspect of any design. A wrong choice of material may lead to failure, over or undersized product or expensive items. The choice of materials is thus dependent on suitable properties of the material for each component, their suitability of fabrication or manufacture and the cost.

(c) Load- The external loads cause internal stresses in the elements and these stresses must be determined accurately since these will be used in determining the component size.

Loading may be classified:

1) Energy transmission by a machine member.
2) Dead weight.
3) Inertial forces.
4) Thermal effects.
5) Frictional forces.

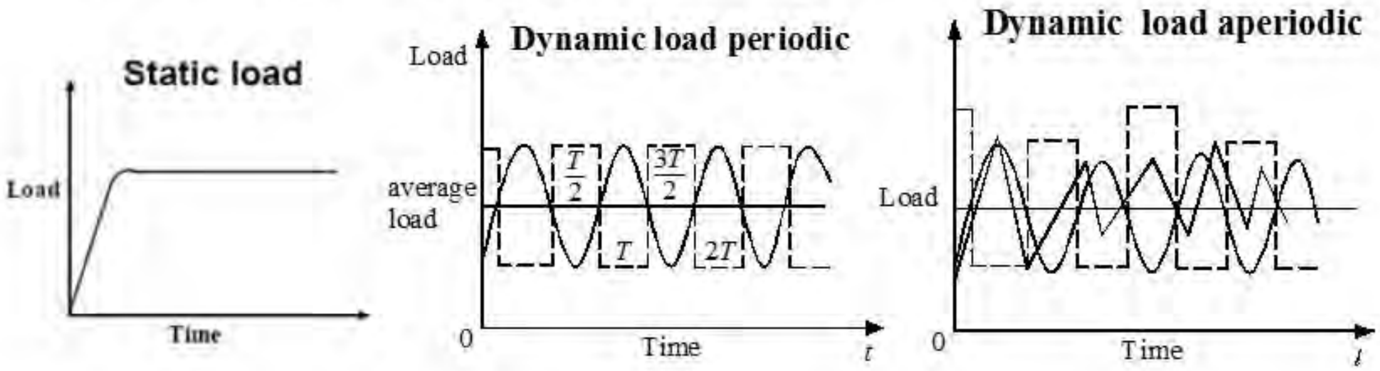
In other ways loads may be classified as:

1) Static load- Does not change in magnitude and direction and normally increases gradually to a steady value.

2) Dynamic load- a) changes in magnitude- for e.g. traffic of varying weight passing a bridge.

b) changes in direction- for e.g. load on piston rod of a double acting cylinder.

The nature of these loads are shown in figure-1.1.3.1.



1.1.5.1 The nature of static and dynamic load

Vibration and shock loading are types of dynamic loading.

(d) Size, shape, space requirements and weight- Preliminary analysis would give an approximate size but if a standard element is to be chosen, the next larger size must be taken. Shapes of standard elements are known but for non-standard element, shapes and space requirements must depend on available space in a particular machine assembly. A scale layout drawing is often useful to arrive at an initial shape and size. Weight is important depending on application. For example, an aircraft must always be made light. This means that the material chosen must have the required strength yet it must be light. Similar arguments apply to choice of material for ships and there too light materials are to be chosen. Portable equipment must be made light.

(e) Manufacture

Care must always be taken to ensure that the designed elements may be manufactured with ease, within the available facilities and at low cost.

(f) How will it operate

In the final stage of the design a designer must ensure that the machine may be operated with ease. In many power operated machines it is simply a matter of pressing a knob or switch to start the machine. However in many other cases, a sequence of operations is to be specified. This sequence must not be complicated and the operations should not require excessive force. Consider the starting, accelerating and stopping a scooter or a car. With time tested design considerations, the sequences have been made user-friendly and as in any other product, these products too go through continuous innovation and development.

(g) Reliability and safety

1. A designed machine should work effectively and reliably. The probability that an element or a machine will not fail in use is called reliability. Reliability lies between $0 \le R < 1$. To ensure this, every detail should be examined. Possible overloading, wear of elements, excessive heat generation and other such detrimental factors must be avoided. There is no single answer for this but an overall safe design approach and care at every stage of design would result in a reliable machine.

2. Safety has become a matter of paramount importance these days in design. Machines must be designed to serve mankind, not to harm it. Industrial regulations ensure that the manufacturer is liable for any damage or harm arising out of a defective product. Use of a factor of safety only in design does not ensure its overall reliability.

(i) Maintenance, cost and aesthetics

Maintenance and safety are often interlinked. Good maintenance ensures good running condition of machinery. Often a regular maintenance schedule is maintained and a thorough check up of moving and loaded parts is carried out to avoid catastrophic failures. Low friction and wear is maintained by proper lubrication. This is a major aspect of design since wherever there are moving parts, friction and wear are inevitable. High friction leads to increased loss of energy. Wear of machine parts leads to loss of material and premature failure.

Cost and aesthetics are essential considerations for product design. Cost is essentially related to the choice of materials which in turn depends on the stresses developed in a given condition. Although in many cases aesthetic considerations are not essential aspects of machine design, ergonomic aspects must be taken into considerations.

1.1.6 Problems with Answers

Q.1: Define machine design.

A.1: A machine is a combination of several machine elements arranged to work together as a whole to accomplish specific purposes. Machine design involves designing the elements and arranging them optimally to obtain some useful work.

Q.2: What is an adaptive design?

A.2: Adaptive design is based on an existing design adapted for a new system or application, for example, design of a new model of passenger car.

Q.3: Suggest briefly the steps to be followed by a designer.

A.3: Machine design requires a thorough knowledge of engineering science in its totality along with a clear decision making capability. Every designer follows his own methodology based on experience and analysis. However, the main steps to be followed in general are :

- Define the problem.
- Make preliminary design decisions.
- Make design sketches.
- Carry out design analysis and optimization.
- Design the elements for strength and durability.
- Prepare documentations to be followed for manufacture.

Q.4: Discuss 'factor of safety ' in view of the reliability in machine design.

A.4: Reliability of a designed machine is concerned with the proper functioning of the elements and the machine as a whole so that the machine does not fail in use within its designed life. There is no single answer to this and an overall safe design approach at every stage of the design is needed. Use of factor of safety in designing the elements is to optimize the design to avoid over-design for reliability.

1.1.7 Summary of this Lecture

The lesson essentially discusses the basic concept of design in general leading to the concept of machine design which involves primarily designing the elements. Different types of design and the factors to be considered have been discussed in detail.
As you well know modern machines are complex systems which consist of mechanical, electrical, electronic, control parts. There **fore dixiplane** machine design in general case is the science about complex systems. Mechatronics is science about modern machins. We will be study only design of mechanical parts of machins.

Lecture
Theme 1

1.2 Classification of materials for machine design

1.2.1 Introduction

Choice of materials for a machine element depends very much on its properties,cost, availability and such other factors. It is therefore important to have some idea of the common engineering materials and their properties before learning the details of design procedure. Common engineering materials are normally classified as **metals** and **nonmetals**. Metals may conveniently be divided into **ferrous** and **non-ferrous** metals. Important ferrous metals for the present purpose are:

(1) cast iron (2) wrought iron (3) steel.

Some of the important **non-ferrous** metals used in engineering design are:

(a) Light metal group such as aluminium and its alloys, magnesium and manganese alloys.

(b) Copper based alloys such as brass (Cu-Zn), bronze (Cu-Sn).

(c) White metal group such as nickel, silver, white bearing metals eg. SnSb7Cu3, Sn60Sb11Pb, zinc etc.

Cast iron, wrought iron and steel will now be discussed under separate headings.

1.2.2 Ferrous materials

1. Cast iron- It is an alloy of iron, carbon and silicon and it is hard and brittle. Carbon content may be within 1.7% to 3% and carbon may be present as free carbon or iron carbide Fe_3C. In general the types of cast iron are (a) grey cast iron and (b) white cast iron (c) malleable cast iron (d) spheroidal or nodular cast iron (e) austenitic cast iron (f) abrasion resistant cast iron.

1.a Grey cast iron- Carbon here is mainly in the form of graphite. This type of cast iron is inexpensive and has high compressive strength. Graphite is an excellent solid lubricant and this makes it easily machinable but brittle.

1.b White cast iron- In these cast irons carbon is present in the form of iron carbide (Fe_3C) which is hard and brittle. The presence of iron carbide increases hardness and makes it difficult to machine. Consequently these cast irons are abrasion resistant.

1.c Malleable cast iron- These are white cast irons rendered malleable by annealing. These are tougher than grey cast iron and they can be twisted or bent without fracture. They have excellent machining properties and are inexpensive.

1.d Spheroidal or nodular graphite cast iron- In these cast irons graphite is present in the form of spheres or nodules. They have high tensile strength and good elongation properties. They are designated as, for example, SG50/7, SG80/2 etc where the first number gives the tensile strength in MPa and the second number indicates percentage elongation.

1.e Austenitic cast iron- Depending on the form of graphite present these cast iron can be classified broadly under two headings

Austenitic spheroidal or nodular graphite iron designated, for example, ASGNi20Cr2. These are alloy cast irons and they contain small percentages of silicon, manganese, sulphur, phosphorus etc. They may be produced by adding alloying elements viz. nickel, chromium, molybdenum, copper and manganese in sufficient quantities. These elements give more strength and improved properties. They are used for making automobile parts such as cylinders, pistons, piston rings, brake drums etc.

1.f Abrasion resistant cast iron- These are alloy cast iron and the alloying elements render abrasion resistance.

2. Wrought iron- This is a very pure iron where the iron content is of the order of 99.5%. It is produced by re-melting pig iron and some small amount of silicon, sulphur, or phosphorus may be present. It is tough, malleable and ductile and can easily be forged or welded. It cannot however take sudden shock. Chains, crane hooks, railway couplings and such other components may be made of this
iron.

3. Steel- This is by far the most important engineering material and there is an enormous variety of steel to meet the wide variety of engineering requirements. The present note is an introductory discussion of a vast topic.

3.a Plain carbon steel- The properties of plain carbon steel depend mainly on the carbon percentages and other alloying elements are not usually present in more than 0.5 to 1% such as 0.5% Si or 1% Mn etc.

Following categorization of these steels is sometimes made for convenience.

Detailed properties of these steels may be found in any standard handbook but in general higher carbon percentage indicates higher strength.

3.b Alloy steel- these are steels in which elements other than carbon are added in sufficient quantities to impart desired properties, such as wear resistance, corrosion resistance, electric or magnetic properties. Chief alloying elements added are usually nickel for strength and toughness, chromium for hardness and strength, tungsten for hardness at elevated temperature, vanadium for tensile strength, manganese for high strength in hot rolled and heat treated condition, silicon for high elastic limit, cobalt for hardness and molybdenum for extra tensile strength.

1.2.3 Specifications

A number of systems for grading steel exist in different countries.

The American system is usually termed as SAE (Society of Automobile Engineers) or AISI (American Iron and Steel Industries) systems. For an example, a steel denoted as SAE 1020 indicates 0.2% carbon and 13% tungsten. In this system the first digit indicates the chief alloying material. Digits 1,2,3,4 and 7 refer to carbon, nickel, nickel/chromium, molybdenum and tungsten respectively. More details may be seen in the standards. The second digit or second and third digits give the percentage of the main alloying element and the last two digits indicate the carbon percentage.

1.2.4 Non-ferrous metals

Metals containing elements other than iron as their chief constituents are usually referred to as non-ferrous metals. There is a wide variety of non-metals in practice. However, only a few exemplary ones are discussed below:

4.a Aluminium- This is the white metal produced from Alumina. In its pure state it is weak and soft but addition of small amounts of Cu, Mn, Si and Magnesium makes it hard and strong. It is also corrosion resistant, low weight and non-toxic.

4.b Duralumin- This is an alloy of 4% Cu, 0.5% Mn, 0.5% Mg and aluminium. It is widely used in automobile and aircraft components.

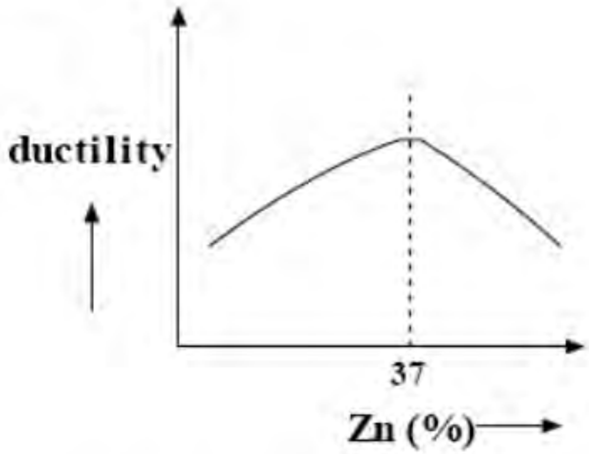
4.c Y-alloy- This is an alloy of 4% Cu, 1.5% Mn, 2% Ni, 6% Si, Mg, Fe and the rest is Al. It gives large strength at high temperature. It is used for aircraft engine parts such as cylinder heads, piston etc.

4.d Magnalium- This is an aluminium alloy with 2 to 10 % magnesium. It also contains 1.75% Cu. Due to its light weight and good strength it is used for aircraft and automobile components.

4.e Copper alloys

Copper is one of the most widely used non-ferrous metals in industry. It is soft, malleable and ductile and is a good conductor of heat and electricity. The following two important copper alloys are widely used in practice:

4.f Brass (Cu-Zn alloy)- It is fundamentally a binary alloy with Zn upto 50% . As Zn percentage increases, ductility increases upto ~37% of Zn beyond which the ductility falls. This is shown in figure-1.2.4.1. Small amount of other elements viz. lead or tin imparts other properties to brass. Lead gives good machining quality and tin imparts strength. Brass is highly corrosion resistant, easily machinable and therefore a good bearing material.



1.2.4.1 *- Variation of ductility of brass with percentage of zinc.*

4.g Bronze (Cu-Sn alloy)-This is mainly a copper-tin alloy where tin percentage may vary between 5 to 25. It provides hardness but tin content also oxidizes resulting in brittleness. Deoxidizers such as Zn may be added. Gun metal is one such alloy where 2% Zn is added as deoxidizing agent and typical compositions are 88% Cu, 10% Sn, 2% Zn. This is suitable for working in cold state. It was originally made for casting guns but used now for boiler fittings, bushes, glands and other such uses.

1.2.5 Non-metals

Non-metallic materials are also used in engineering practice due to principally their low cost, flexibility and resistance to heat and electricity. Though there are many suitable non-metals, the following are important few from design point of view:

5.a Timber- This is a relatively low cost material and a bad conductor of heat and electricity. It has also good elastic and frictional properties and is widely used in foundry patterns and as water lubricated bearings.

5.b Leather- This is widely used in engineering for its flexibility and wear resistance. It is widely used for belt drives, washers and such other applications.

5.c Rubber- It has high bulk modulus and is used for drive elements, sealing, vibration isolation and similar applications.

5.d Plastics

These are synthetic materials which can be moulded into desired shapes under pressure with or without application of heat. These are now extensively used in various industrial applications for their corrosion resistance, dimensional stability and relatively low cost.

There are two main types of plastics:

5.d.1 Thermosetting plastics- Thermosetting plastics are formed under heat and pressure. It initially softens and with increasing heat and pressure, polymerisation takes place. This results in hardening of the material. These plastics cannot be deformed or remoulded again under heat and pressure. Some examples of thermosetting plastics are phenol formaldehyde (Bakelite), phenol-furfural (Durite), epoxy resins, phenolic resins etc.

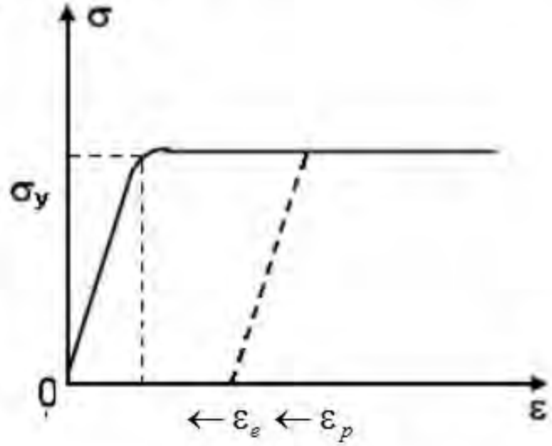
5.d.2 Thermoplastics- Thermoplastics do not become hard with the application of heat and pressure and no chemical change takes place. They remain soft at elevated temperatures until they are hardened by cooling. These can be re-melted and remoulded by application of heat and pressure. Some examples of thermoplastics are cellulose nitrate (celluloid), polythene, polyvinyl acetate, polyvinyl chloride (PVC) etc.

1.2.6 Mechanical properties of common engineering materials

The important properties from design point of view are:

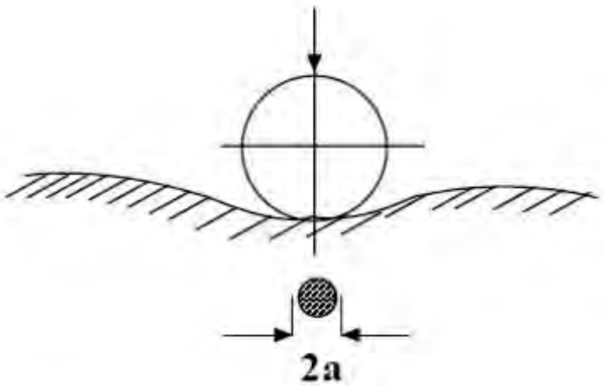
(a) Elasticity- This is the property of a material to regain its original shape after deformation when the external forces are removed. All materials are plastic to some extent but the degree varies, for example, both mild steel and rubber are elastic materials but steel is more elastic than rubber.

(b) Plasticity- This is associated with the permanent deformation of material when the stress level exceeds the yield point. Under plastic conditions materials ideally deform without any increase in stress. A typical stressstrain diagram for an elastic-perfectly plastic material is shown in the figure-1.2.6.1. Mises-Henky criterion gives a good starting point for plasticity analysis. The criterion is given as $\sigma = \sigma_y$ where the stress at the tensile yield point. For $\sigma < \sigma_y$ deformation ε_e of material is elastic, for $\sigma \geq \sigma_y$ deformation ε_p of material is plastic



1.2.6.1 a) *- Stress-strain diagram of an elastic-perfectly plastic material*

A typical example of plastic flow is the indentation test where a spherical ball is pressed in a semi-infinite body where 2a is the contact diameter. In a simplified model we may write that if $\frac{P}{\pi a^2} > p_m$ plastic flow occurs where, p_m is the flow pressure. This is also shown in figure 1.2.6.1 b.



1.2.6.1 b) *- The plastic indentation.*

(c) Hardness- Property of the material that enables it to resist permanent deformation, penetration, indentation etc. Size of indentations by various types of indenters are the measure of hardness e.g. Brinnel hardness test, Rockwell hardness test, Vickers hardness (diamond pyramid) test. These tests give hardness numbers which are related to yield pressure (MPa).

(d) Ductility- This is the property of the material that enables it to be drawn out or elongated to an appreciable extent before rupture occurs. The percentage elongation or percentage reduction in area before rupture of a test specimen is the measure of ductility. Normally if percentage elongation exceeds 15% the material is ductile and if it is less than 5% the material is brittle. Lead, copper, aluminium, mild steel are typical ductile materials.

(e) Malleability- It is a special case of ductility where it can be rolled into thin sheets but it is not necessary to be so strong. Lead, soft steel, wrought iron, copper and aluminium are some materials in order of diminishing malleability.

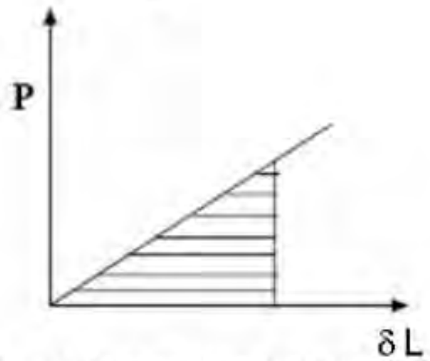
(f) Brittleness- This is opposite to ductility. Brittle materials show little deformation before fracture and failure occur suddenly without any warning. Normally if the elongation is less than 5% the material is considered to be brittle. E.g. cast iron, glass, ceramics are typical brittle materials.

(g) Resilience- This is the property of the material that enables it to resist shock and impact by storing energy. The measure of resilience is the strain energy absorbed per unit volume. For a rod of length L subjected to tensile load P, a linear load-deflection plot is shown in figure-1.2.6.2.

Strain energy (energy stored)

$W = \frac{1}{2}P\delta\delta = \frac{1}{2}\frac{P}{A}\frac{\delta L}{L}AL = \frac{1}{2}\sigma\varepsilon V$, $\sigma = \frac{P}{A}$, $\varepsilon = \frac{\delta L}{L}$, $V = AL$ where σ is stress, ε strain, V is volume, A is area, L is length.

Strain energy/unit volume $W = \frac{1}{2}\sigma\varepsilon$



1.2.6.2 *- A linear load-deflection plot.*

(h) Toughness- This is the property which enables a material to be twisted, bent or stretched under impact load or high stress before rupture. It may be considered to be the ability of the material to absorb energy in the plastic zone. The measure of toughness is the amount of energy absorbed after being stressed upto the point of fracture.

(i) Creep- When a member is subjected to a constant load over a long period of time it undergoes a slow permanent deformation and this is termed as "creep". This is dependent on temperature. Usually at elevated temperatures creep is high.

1.2.7 Problems with Answers

Q.1: Classify common engineering materials.
A.1: Common engineering materials can be broadly classified into metals and non-metals. Metals include ferrous and non-ferrous metal and the nonmetals include timber, leather, rubber and a large variety of polymers. Among the ferrous metals different varieties of cast iron, wrought iron and alloy steels are extensively used in industry. There are also a large variety of timber, leather and polymers that are used in industry.
Q.2: What are the advantages of malleable cast iron over white or grey cast iron?
A.2: Malleable cast iron are tougher than grey or white cast iron and can be twisted or bent without fracture. They also have excellent machining properties and are relatively inexpensive.
Q.3: A standard alloy steel used for making engineering components is 20Cr18 Ni2. State the composition of the steel.

Q.5: Name two important copper alloys and give their typical compositions.

A.5: Two most important copper alloys are bronze and brass. Bronze is a Cu-Sn alloy with the typical composition of 88% Cu, 10% Sn and 2% Zn. Brass is a Cu-Zn alloy with the typical composition of red brass of 85% Cu , 15% Zn.

Q.6: List at least five important non-metals commonly used in machine design.

A.6: Some important non-metals for industrial uses are: Timber, leather, rubber, bakelite, nylon, polythene, polytetraflutoethylene (PTFE).

Q.7: State atleast 5 important mechanical properties of materials to be considered in machine design.

A.7: Some important properties of materials to be considered in design are: Elastic limit, yield and ultimate strength, hardness and toughness.

Q.8: Define resilience and discuss its implication in the choice of materials in machine design.

A.8: Resilience is defined as the property of a material that enables it to resist shock and impact. The property is important in choosing materials for machine parts subjected to shock loading, such as, fasteners, springs etc.

1.2.8 Summary of this Lecture

In this lecture the properties and uses of different types of metals and nonmetals, generally used in machine design, are discussed. Primarily ferrous and non-ferrous metals and some non-metals are discussed. Mechanical properties of some common engineering materials are also discussed briefly.

Lecture
Theme 1

1.3. Limit and Tolerance, Fit system and standard limit

1.3.1 Design and Manufacturing

A machine element, after design, requires to be manufactured to give it a shape of a product. Therefore, in addition to standard design practices like, selection of proper material, ensuring proper strength and dimension to guard against failure, a designer should have knowledge of basic manufacturing aspects.

1.3.2 Limits

Fig. 1.3.1 explains the terminologies used in defining **tolerance** and **limit**. The zero line, shown in the figure, is the basic size or the nominal size. The definition of the terminologies is given below. For the convenience, shaft and hole are chosen to be two mating components.

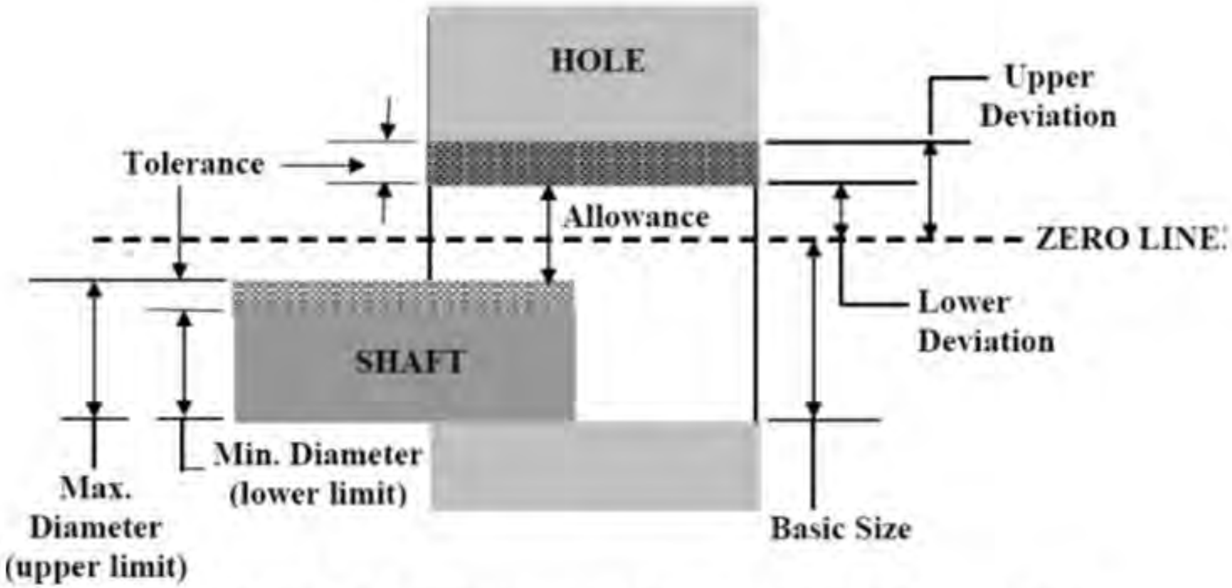


Fig. 1.3.1 Interrelationship between tolerances and limits

Tolerance

Tolerance is the difference between maximum and minimum dimensions of a component, ie, between upper limit and lower limit. Depending on the type of application, the permissible variation of dimension is set as per available standard grades (see Fig. 1.3.1).

Tolerance is of two types, **bilateral and unilateral**. When tolerance is present on both sides of nominal size, it is termed as bilateral; unilateral has tolerance only on one side. The Fig.1.3.2 shows the types of tolerance. 50^{0}_{-y}, 50^{+y}_{0} and 50^{+y}_{-y} is a typical example of specifying tolerance for a shaft of nominal diameter of 50mm. First two values denote unilateral tolerance and the third value denotes bilateral tolerance. Values of the tolerance are given as x and y respectively.

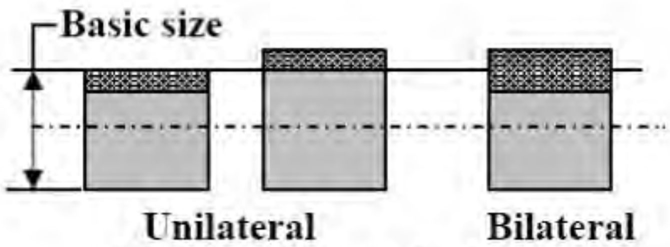


Fig. 1.3.2 Types of tolerance

Allowance

It is the difference of dimension between two mating parts.

Upper deviation

It is the difference of dimension between the maximum possible size of the component and its nominal size.

Lower deviation

Similarly, it is the difference of dimension between the minimum possible size of the component and its nominal size.

Fundamental deviation

It defines the location of the tolerance zone with respect to the nominal size. For that matter, either of the deviations may be considered.

1.3.3 Fit System

We have learnt above that a machine part when manufactured has a specified tolerance. Therefore, when two mating parts fit with each other, the nature of fit is dependent on the limits of tolerances and fundamental deviations of the mating parts. The nature of assembly of two mating parts is defined by three types of fit system, Clearance Fit, Transition Fit and Interference Fit. The fit system is shown schematically in Fig.1.3.3.

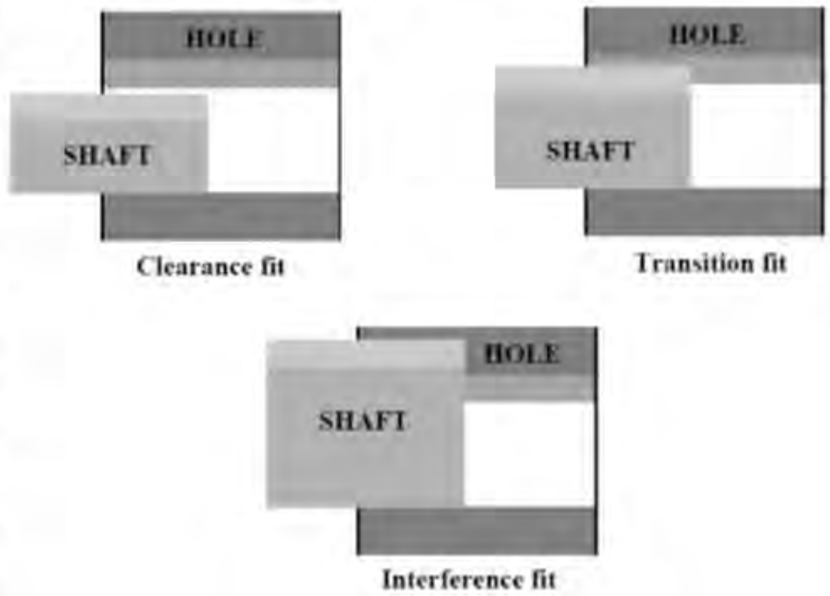


Fig. 1.3.3 Schematic view of Fit system

There are two ways of representing a system. One is the hole basis and the other is the shaft basis. In the hole basis system the dimension of the hole is considered to be the datum, whereas, in the shaft basis system dimension of the shaft is considered to be the datum. The holes are normally made by drilling, followed by reaming. Therefore, the dimension of a hole is fixed due to the nature of the tool used. On the contrary, the dimension of a shaft is easily controllable by standard manufacturing processes. For this reason, the hole basis system is much more popular than the shaft basis system. Here, we shall discuss fit system on hole basis.

Clearance Fit

In this type of fit, the shaft of largest possible diameter can also be fitted easily even in the hole of smallest possible diameter.

Transition Fit

In this case, there will be a clearance between the minimum dimension of the shaft and the minimum dimension of the hole. If we look at the figure carefully, then it is observed that if the shaft dimension is maximum and the hole dimension is minimum then an overlap will result and this creates a certain amount of tightness in the fitting of the shaft inside the hole. Hence, transition fit may have either clearance or overlap in the fit.

Interference Fit

In this case, no matter whatever may be the tolerance level in shaft and the hole, there is always a overlapping of the matting parts. This is known as interference fit. Interference fit is a form of a tight fit.

1.3.4 Standard limit and fit system

Fig. 1.3.4 shows the schematic view of a **standard limit and fit system**. In this figure tolerance is denoted as IT and it has 18 grades; greater the number, more is the tolerance limit. The fundamental deviations for the hole are denoted by capital letters from A and ZC, having altogether 25 divisions. Similarly, the fundamental deviations for the shaft is denoted by small letters from a to zc.

Here H or h is a typical case, where the fundamental deviation is zero having an unilateral tolerance of a specified IT grade.

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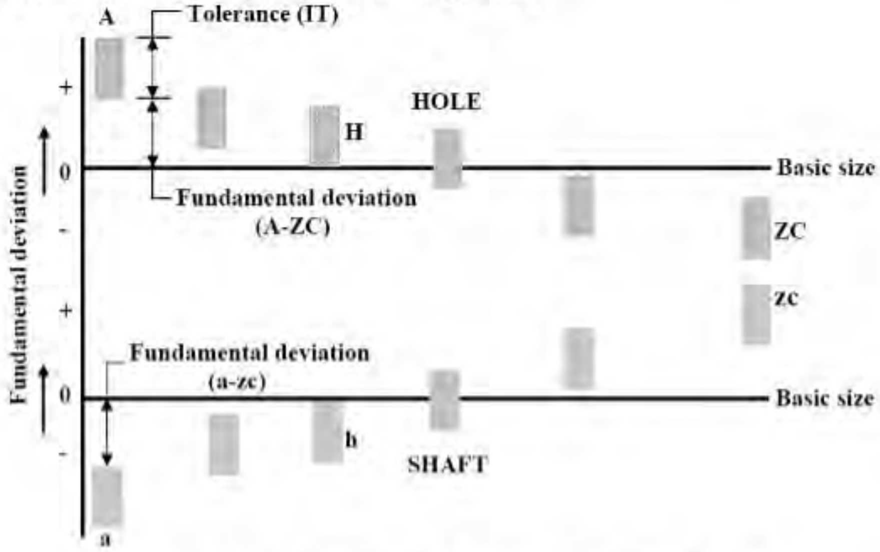


Fig. 1.3.4 Schematic view of standard limit and fit system

Therefore in standard limits and fit system we find that,

Standard tolerances
18 grades: IT01 ,IT0 and IT1-1T16

Fundamental deviations
25 types: A- ZC (For holes)
a- zc (For shafts)

The values of standard tolerances and fundamental deviations can be obtained by consulting design **hand book**. It is to be noted that the choice of tolerance grade is related to the type of manufacturing process; for example, attainable tolerance grade for lapping process is lower compared to plain milling. Similarly, choice of fundamental deviation largely depends on the nature of fit, running fit or tight fit etc. Manufacturing processes involving lower tolerance grade are generally costly. Hence the designer has to keep in view the manufacturing processes to make the design effective and inexpensive.

Sample designation of limit and fit, 50H6/g5.

The designation means that the nominal size of the hole and the shaft is 50 mm. H is the nature of fit for the hole basis system and its fundamental deviation is zero. The tolerance grade for making the hole is IT6. Similarly, the shaft has the fit type g, for which the fundamental deviation is negative, that is, its dimension is lower than the nominal size, and tolerance grade is IT5. The approximate zones for fit are shown in Fig. 1.3.5.

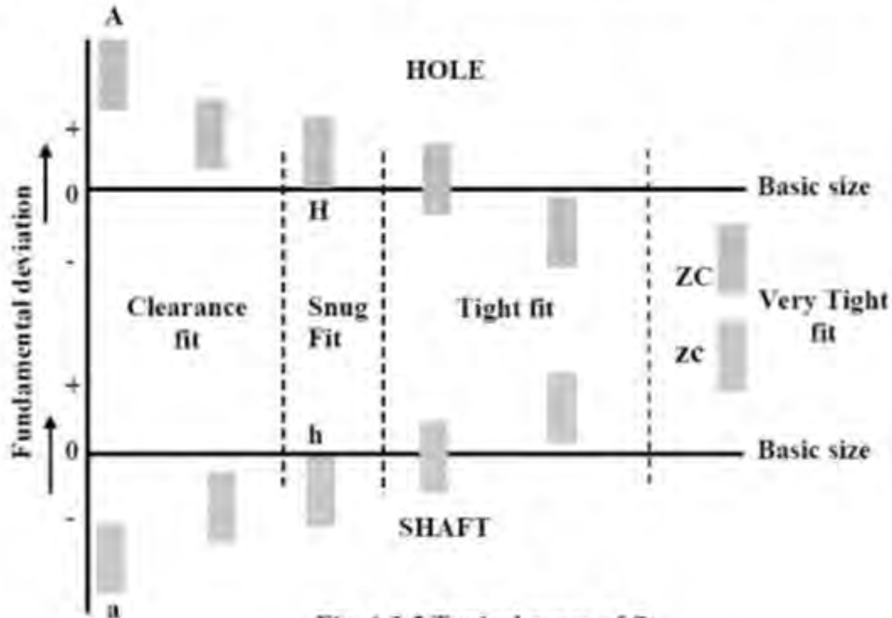


Fig. 1.3.5 Typical zones of fit

Lecture
Theme 1

1.3. The types of common manufacturing processes

1.3.5 Preferred numbers

A designed product needs standardization. It means that some of its important specified parameter should be common in nature. For example, the sizes of the ingots available in the market have standard sizes. A manufacturer does not produce ingots of sizes of his wish, he follows a definite pattern and for that matter designer can choose the dimensions from those standard available sizes. Motor speed, engine power of a tractor, machine tool speed and feed, all follow a definite pattern or series. This also helps in interchangeability of products. It has been observed that if the sizes are put in the form of geometric progression, then wide ranges are covered with a definite sequence.

These numbers are called preferred numbers having common ratios as,

$$\sqrt[5]{10} \approx 1.58,\ \sqrt[10]{10} \approx 1.26,\ \sqrt[20]{10} \approx 1.12 \ \text{ and } \ \sqrt[40]{10} \approx 1.06$$

Depending on the common ratio, four basic series are formed; these are R5, R10 , R20 and R40 . These are named as Renard series. Many other derived series are formed by multiplying or dividing the basic series by 10, 100 etc.

Typical values of the common ratio for four basic G.P. series are given below.

Preferred Numbers

R5:	$\sqrt[5]{10}$	***1.58***: **1.0, 1.6, 2.5, 4.0,…**
R10:	$\sqrt[10]{10}$	***1.26***: **1.0, 1.25, 1.6, 2.0,…**
R20:	$\sqrt[20]{10}$	***1.12***: **1.0, 1.12, 1.25, 1.4,…**
R40:	$\sqrt[40]{10}$	***1.06***: **1.0, 1.06, 1.12, 1.18,..**

Few examples

R_{10} , R_{20} and R_{40} : Thickness of sheet metals, wire diameter

R_5 , R_{10} , R_{20} : Speed layout in a machine tool (R_{10} : 1000, 1250,1600, 2000)

R_{20} or R_{40} : Machine tool feed

R_5 : Capacities of hydraulic cylinder

1.3.6 Common manufacturing processes

The types of common manufacturing processes are given below in the Fig.1.3.6.

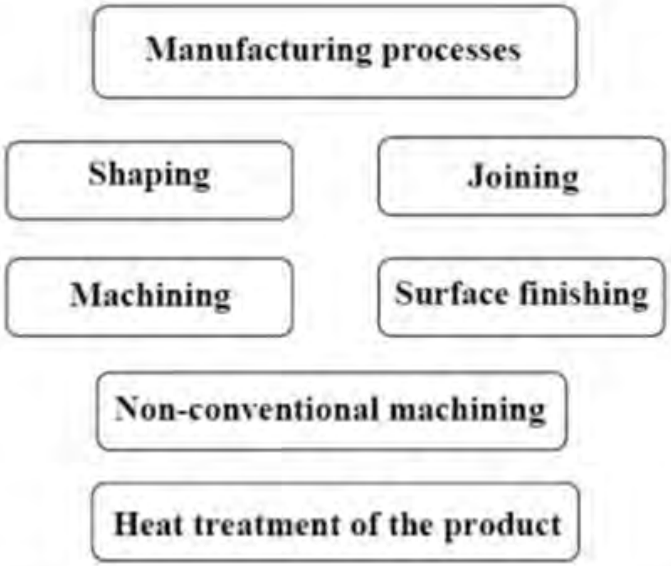


Fig. 1.3.6 Common manufacturing processes

The types of shaping processes are given below in the Fig.1.3.7.

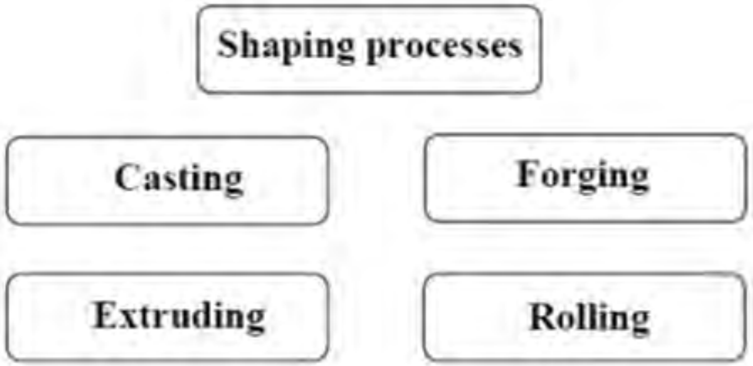


Fig. 1.3.7 Shaping processes

Following are the type of machining processes, shown in Fig.1.3.8.

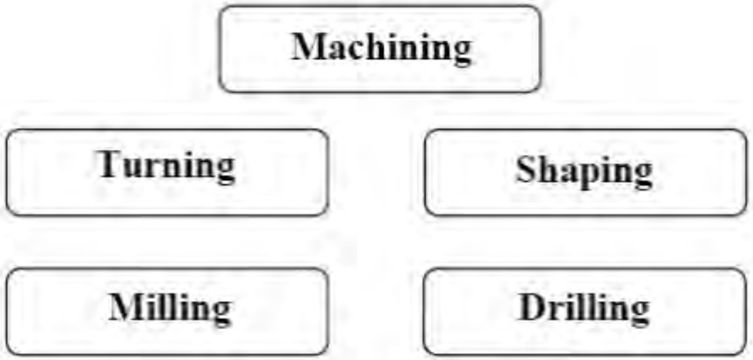


Fig. 1.3.8 Machining processes

Various joining processes are shown in Fig.1.3.9.

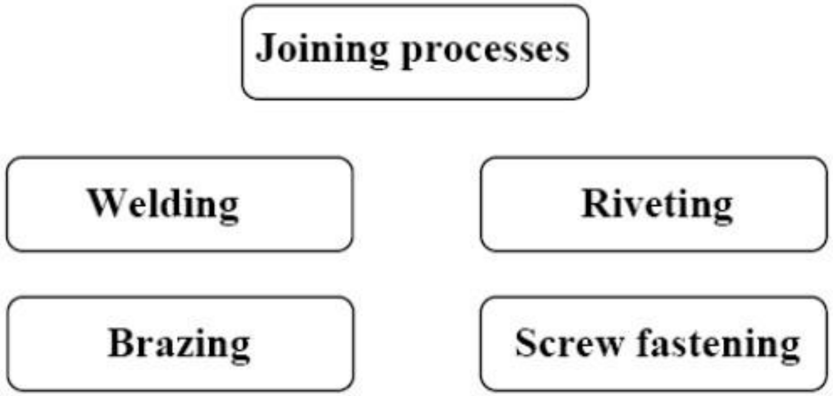


Fig. 1.3.9 Joining processes

The surface finishing processes are given below (Fig.1.3.10),

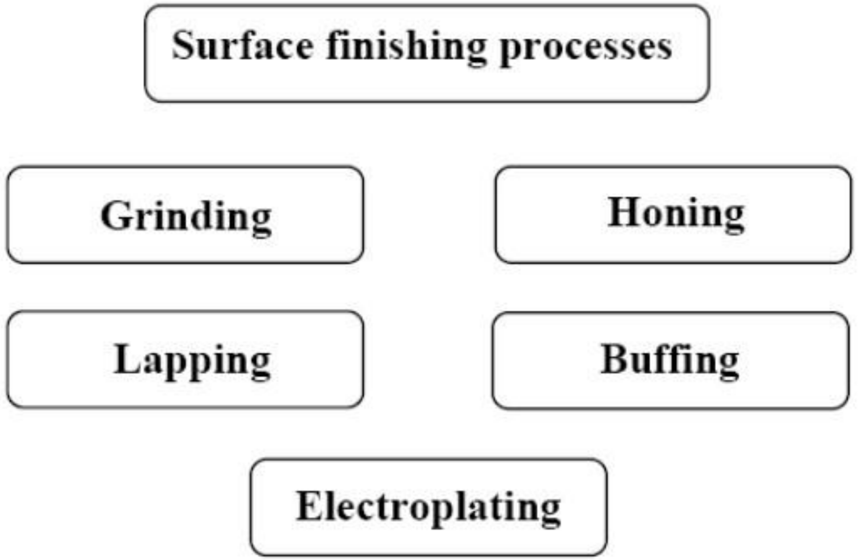


Fig. 1.3.10 Surface finishing processes

The non-conventional machining processes are as follows (Fig.1.3.11),

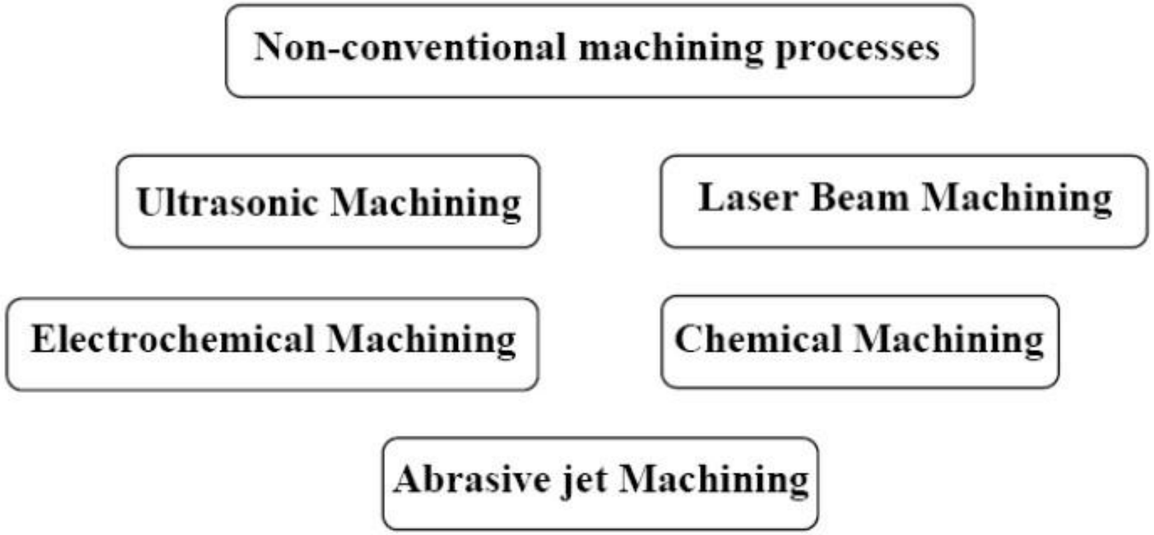


Fig. 1.3.11 Non conventional machining processes

Problems and answers

Q1. What is meant by tolerance? How many types of tolerance is there?

A1. Tolerance is the difference between maximum and minimum dimensions of a component, ie, between upper limit and lower limit. Depending on the type of application, the permissible variation of dimension is set as per available standard grades. Tolerance is of two types, bilateral and unilateral. When tolerance is present on both sides of nominal size, it is termed as bilateral; unilateral has tolerance only on one side.

Q2. What are the types fit? Describe the differences.

A2. The nature of assembly of two mating parts is defined by three types of fit system, Clearance Fit, Transition Fit and Interference Fit.

Clearance Fit: In this type of fit, the shaft of largest possible diameter can be fitted easily in the hole of smallest possible diameter.

Interference Fit : In this type of fit, irrespective of tolerance grade there is always a overlapping of the matting parts.

Transition Fit: In this case, a clearance is present between the minimum dimension of the shaft and the minimum dimension of the hole. However, the fit is tight, if the shaft dimension is maximum and the hole dimension is minimum. Hence, transition fit have both the characteristics of clearance fit and interference fit.

Q3. What are preferred numbers?

A3. Preferred numbers are the numbers belonging to four categories of geometric progression series, called basic series, having common ratio of,

$$\sqrt[5]{10} \approx 1.58,\ \sqrt[10]{10} \approx 1.26,\ \sqrt[20]{10} \approx 1.12 \ \text{ and } \ \sqrt[40]{10} \approx 1.06$$

Preferred numbers of derived series are formed by multiplying or dividing the basic series by 10, 100 etc. These numbers are used to build-up or manufacture a product range. The range of operational speeds of a machine or the range of powers of a typical machine may be also as per a series of preferred numbers.

Lecture

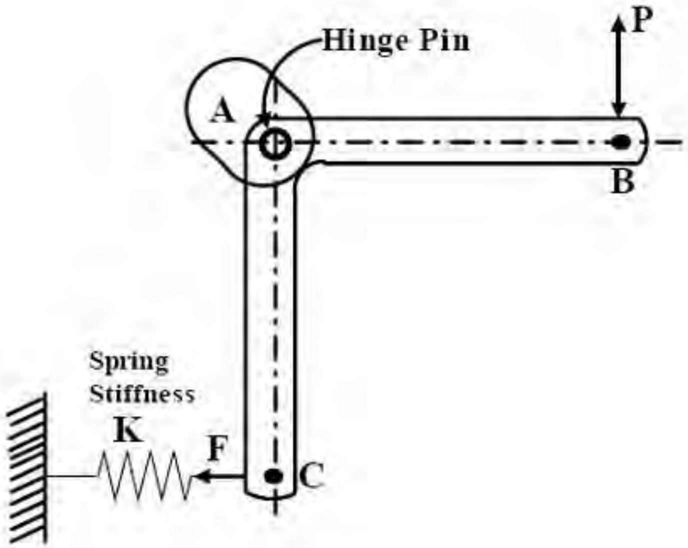
Theme 2

Stresses in machine elements

2.1. Simple stresses

2.1.1 Introduction

Stresses are developed in machine elements due to applied load and machine design involves ensuring that the elements can sustain the induced stresses without yielding. Consider a simple lever as shown in figure-2.1.1.1:



2.1.1.1 *- A simple lever subjected to forces at the ends.*

A proper design of the spring would ensure the necessary force P at the lever end B. The stresses developed in sections AB and AC would decide the optimum cross-section of the lever provided that the material has been chosen correctly.

The design of the hinge depends on the stresses developed due to the reaction forces at A. A closer look at the arrangement would reveal that the following types of stresses are developed in different elements:

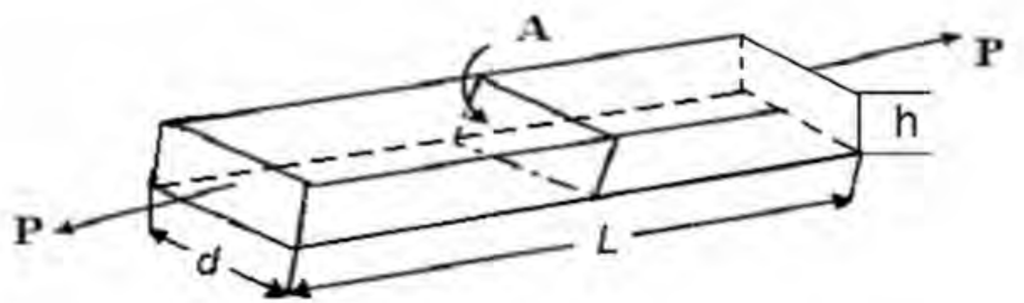
Lever arms AB and AC	-	Bending stresses
Hinge pin	-	Shear and bearing stresses.
Spring	-	Shear stress.

It is therefore important to understand the implications of these and other simple stresses. Although it is more fundamental to consider the state of stress at a point and stress distribution, in elementary design analysis simple average stresses at critical cross-sections are considered to be sufficient. More fundamental issues of stress distribution in design analysis will be discussed later in this lecture.

2.1.2 Some basic issues of simple stresses

Tensile stress

The stress developed in the bar (figure-2.1.2.1) subjected to tensile loading is given by $\sigma_t = \frac{P}{A}$, P is force, A is area of section A = h · d

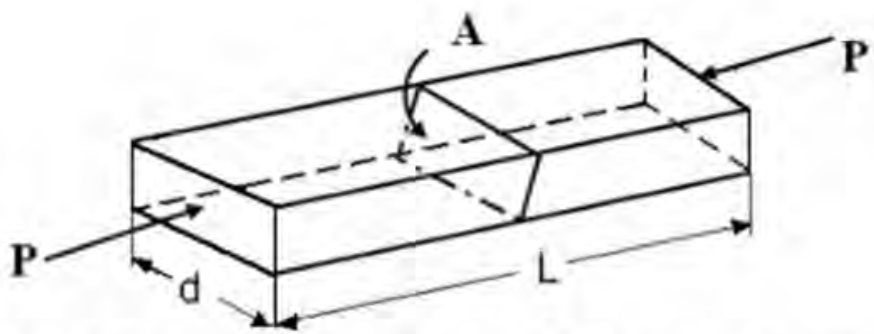


2.1.2.1 - *A prismatic bar subjected to tensile loading.*

Compressive stress

The stress developed in the bar (figure-2.1.2.2) subjected to compressive loading is given by

$$\sigma_c = \frac{P}{A}$$



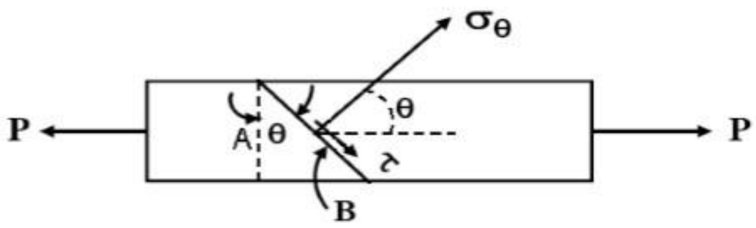
2.1.2.2 - *A prismatic bar subjected to compressive loading.*

However, if we consider the stresses on an inclined cross-section B (figure-2.1.2.3) then the normal stress perpendicular to the section is

$$\sigma_\theta = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{B}\cos\theta, \; B = A/\cos\theta$$

and shear stress parallel to the section

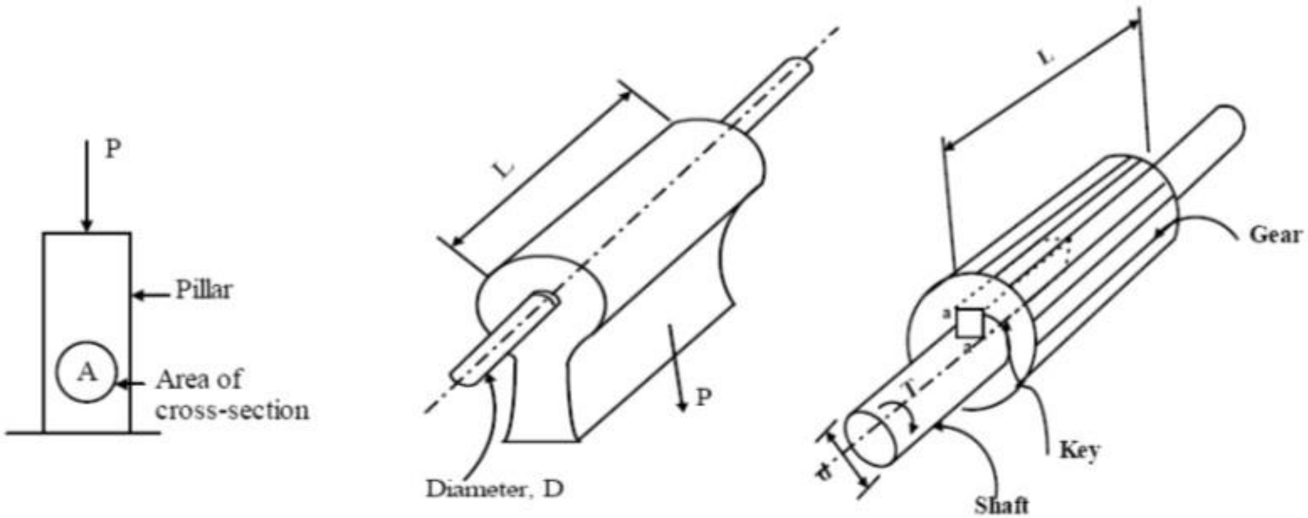
$$\tau = \frac{P\sin\theta}{A/\cos\theta}$$



2.1.2.3 *- Stresses developed at an inclined section of a bar subjected to tensile loading.*

Bearing stress

When a body is pressed against another, the compressive stress developed is termed bearing stress. For example, bearing stress developed at the contact between a pillar and ground (figure- 2.1.2.4a) is $\sigma_{br} = \frac{P}{A}$, at the contact surface between a pin and a member with a circular hole (figure- 2.1.2.4b) is $\sigma_{br} = \frac{P}{Ld}$ and at the faces of a rectangular key fixing a gear hub on a shaft (figure- 2.1.2.4c) is $\sigma_{br} = \frac{4T}{aLd}$.



(a) (b) (c)

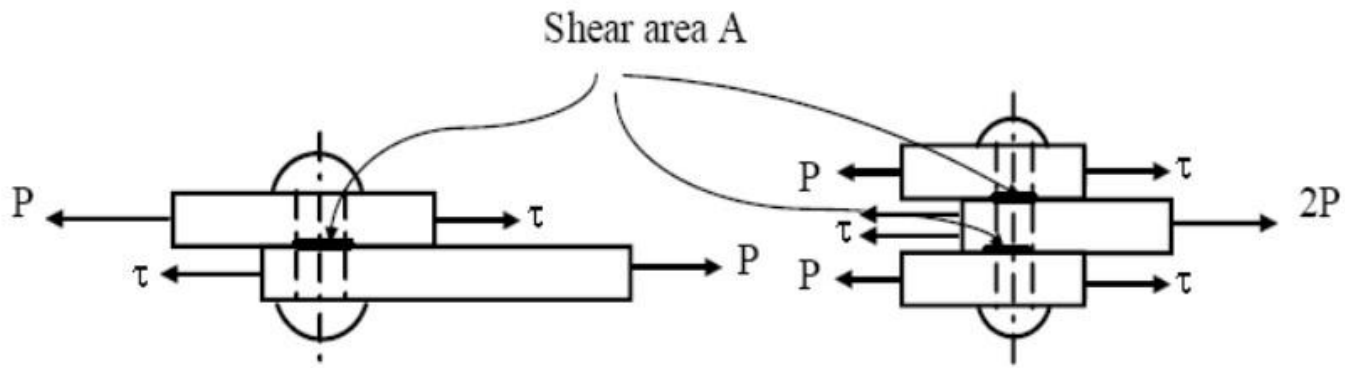
2.1.2.4 - The bearing stresses developed in pillar and machine parts.

The pressure developed may be irregular in the above examples but the expressions give the average values of the stresses.

Shear stress

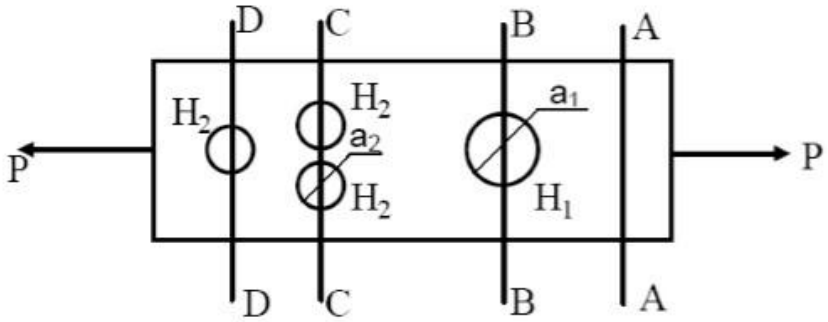
When forces are transmitted from one part of a body to other, the stresses developed in a plane parallel to the applied force are the shear stresses (figure-2.1.2.5) and the average values of the shear stresses are given by

$$\tau = \frac{P}{A} \quad \text{in single shear} \qquad \tau = \frac{P}{2A} \quad \text{in double shear}$$



2.1.2.5 *- Stresses developed in single and double shear modes*

In design problems, critical sections must be considered to find normal or shear stresses. We consider a plate with holes under a tensile load (figure-2.1.2.6) to explain the concept of critical sections.



2.1.2.6 *- The concept of critical sections explained with the help of a loaded plate with holes at selected locations.*

Let the cross-sectional area of the plate, the larger hole H_1 and the smaller holes H_2 be A, a_1, a_2 respectively. If $2a_2 > a_1$ the critical section in the above example is CC and the average normal stress at the critical section is

$$\sigma_2 = \frac{P}{A - 2a_2} > \sigma_1 = \frac{P}{A - a_1}; \text{ as such } A - 2a_2 < A - a_1$$

2.1.3 Bending of beams

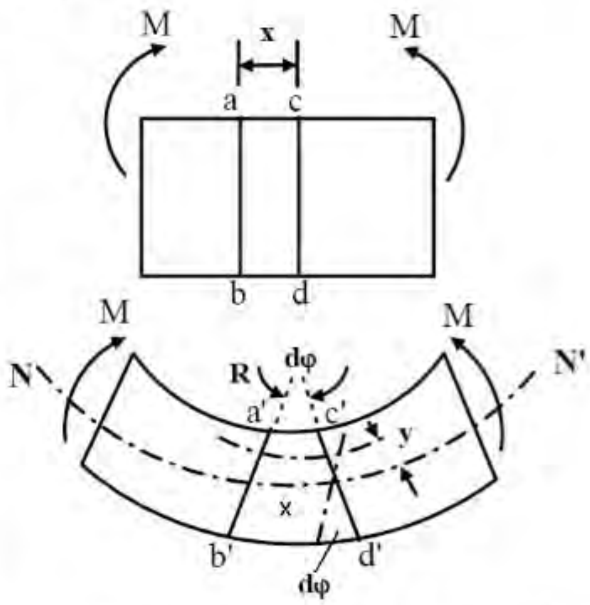
2.1.3.1 Bending stresses

Consider two sections ab and cd in a beam subjected to a pure bending. Due to bending the top layer is under compression and the bottom layer is under tension. This is shown in figure-2.1.3.1.1. This means that in between the two extreme layers there must be a layer which remains un-stretched and this layer is known as neutral layer. Let this be denoted by NN′.

We consider that a plane section remains plane after bending- a basic assumption in pure bending theory. If the rotation of cd with respect to ab is dφ the contraction of a layer y distance away from the neutral axis is given by ds=y dφ and original length of the layer is x=R dφ, R being the radius of curvature of the beam. This gives the strain ε in the layer as $\varepsilon = \frac{y}{R}$

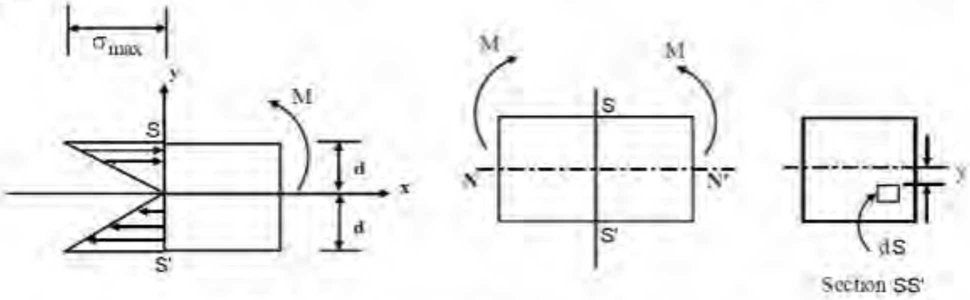
We also consider that the material obeys Hooke's law σ = Eε. This is another basic assumption in pure bending theory and substituting the expression for ε we have

$$\frac{\sigma}{y} = \frac{E}{R}$$



2.1.3.1.1- Pure bending of beams

Consider now a small element dS which distance y away from the neutral axis. This is shown in the figure 2.1.3.1.2



2.1.3.1.2 *- Bending stress developed at any cross-section*

Axial force on the element $dF_x = \sigma_x dS$ and considering the linearity in stress variation across the section $\frac{\sigma_x}{\sigma_{max}} = \frac{y}{d}$ we have where σ_x and σ_{max} are the stresses at distances y and d respectively from the neutral axis.

The axial force on the element is thus given by $dF_x = \frac{\sigma_{max} y}{d} dS$. **Where dS is area at section SS'.**

For static equilibrium total force at any cross-section $F_x = \int_{SS'} \frac{\sigma_{max}}{d} y d S = 0$

This gives $\int_{SS'} y dS = \bar{y} S = 0$ and since $S \neq 0, \bar{y} = 0$. This means that the neutral axis passes through the centroid.

Again for static equilibrium total moment about NA must the applied moment M. This is given by

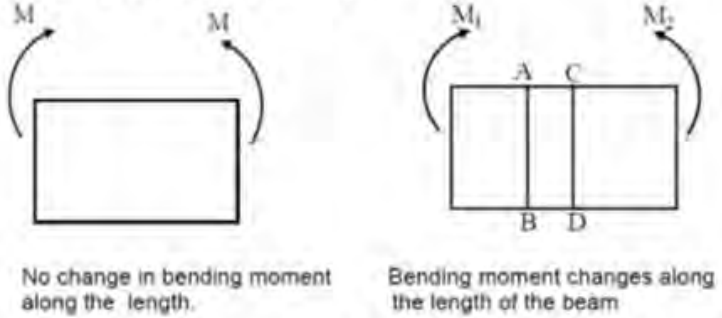
$$I = \int y^2 dS, \quad \int_{SS'} \frac{\sigma_{max} y}{d} y dS = M \quad \text{and this gives} \quad \sigma_{max} = \frac{Md}{I},$$

For any fibre at a distance of y from the centre line we may therefore write $\boldsymbol{\sigma = \frac{My}{I}}$

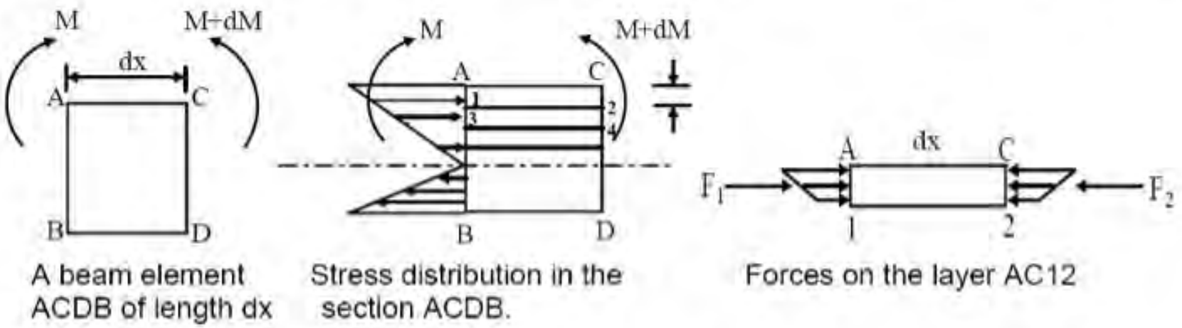
We therefore have the general equation for pure bending as $\boxed{\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}}$

2.1.3.2 Shear stress in bending

In an idealized situation of pure bending of beams, no shear stress occurs across the section. However, in most realistic conditions shear stresses do occur in beams under bending. This can be visualized if we consider the arguments depicted in figure-2.1.3.2.1 and 2.1.3.2.2.



2.1.3.2.1- Bending of beams with a steady and varying moment along its length.



2.1.3.2.2 - Shear stress developed in a beam subjected to a moment varying along the length

When bending moment changes along the beam length, layer AC12 for example, would tend to slide against section 1243 and this is repeated in subsequent layers. This would cause interplanar shear forces F_1 and F_2 at the faces A1 and C2 and since the $F = \int_A \sigma_x dS$ force at any cross-section is given by , we may write

$$F_1 = \frac{M}{I}Q \quad \text{and} \quad F_2 = \frac{(M + dM)}{I}Q$$

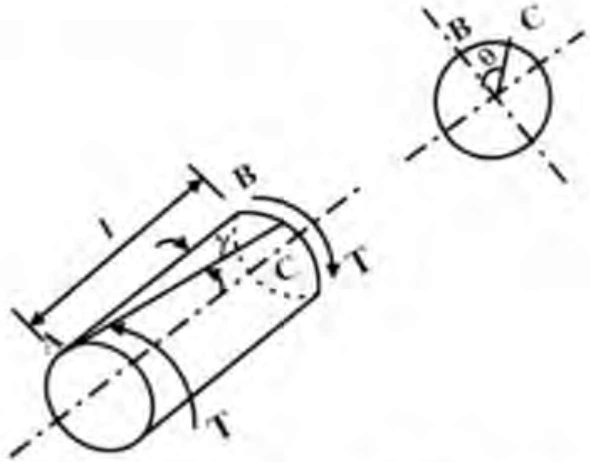
Here M and dM are the bending moment and its increment over the length dx and Q is the 1st moment of area about the neutral axis. Since shear stress across the layers can be given by $\frac{dM}{dx}$ and $\tau = \frac{VQ}{It}$ shear force is given by V = $\tau = \frac{dF}{tdx}$

2.1.4 Torsion of circular members

A torque applied to a member causes shear stress. In order to establish a relation between the torque and shear stress developed in a circular member, the following assumptions are needed:

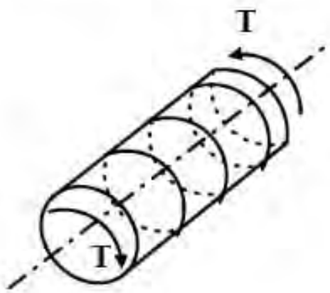
1. Material is homogeneous and isotropic
2. A plane section perpendicular to the axis of the circular member remains plane even after twisting i.e. no warping.
3. Materials obey Hooke's law.

Consider now a circular member subjected to a torque T as shown in figure 2.1.4.1

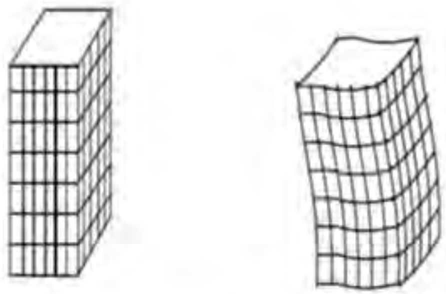


2.1.4.1 - A circular member of radius r and length L subjected to torque T.

The assumption of plane section remaining plane assumes no warping in a circular member as shown in figure- 2.1.4.2



2.1.4.2 - Plane section remains plane- No warping.



2.1.4.3 -Warping during torsion of a non-circular member.

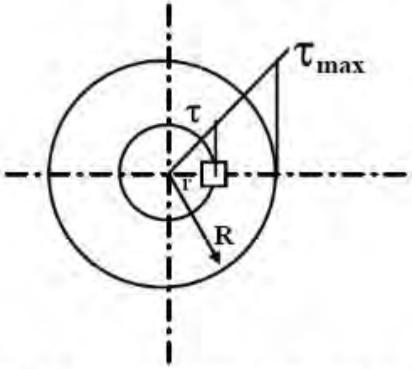
Let the point B on the circumference of the member move to point C during twisting and let the angle of twist be θ. We may also assume that strain γ varies linearly from the central axis. This gives

$$\gamma l = r\theta \text{ and from Hooke's law } \gamma = \frac{\tau}{G}$$

where τ is the shear stress developed and G is the modulus of rigidity. This gives

$$\frac{\tau}{r} = \frac{G\theta}{l}$$

Consider now, an element of area dA at a radius r as shown in figure-2.1.4.4. The torque on the element is given by $T = \int \tau r dA$



2.1.4.4 - Shear stress variation in a circular cross-section during torsion.

For linear variation of shear stress we have $\frac{\tau}{\tau_{max}} = \frac{r}{R}$

Combining this with the torque equation we may write $T = \frac{\tau_{max}}{R} \int r^2 dA$.

Now $\int r^2 dA$ may be identified as the polar moment of inertia J .

And this gives $T = \frac{\tau_{max}}{R} J$.

Therefore for any radius r we may write in general $T/J=\tau/r$.

We have thus the general torsion equation for circular shafts as

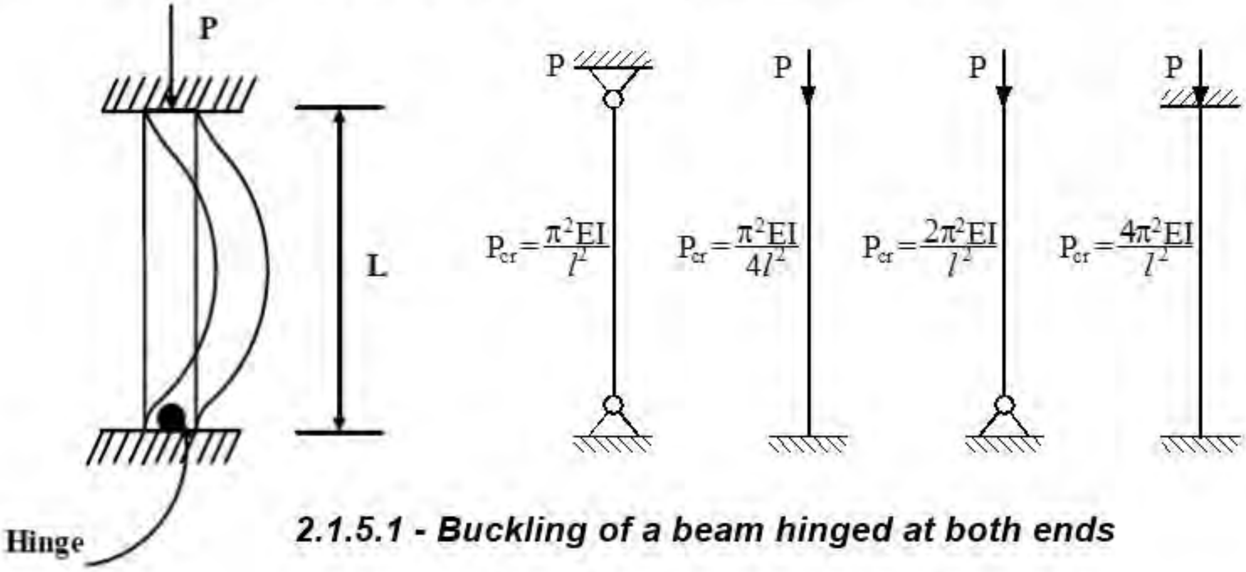
$$\boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}}$$

2.1.5 Buckling

The compressive stress of P/A is applicable only to short members but for long compression members there may be buckling, which is due to elastic instability. The critical load for buckling of a column with different end fixing conditions is given by Euler's formula (figure-2.1.5.1)

$$P_{cr} = n\frac{\pi^2 EI}{l^2}$$

where E is the elastic modulus, I the second moment of area, l the column length and n is a constant that depends on the end condition. For columns with both ends hinged n=1, columns with one end free and other end fixed n=0.25, columns with one end fixed and other end hinged n=2, and for columns with both ends fixed n=4.

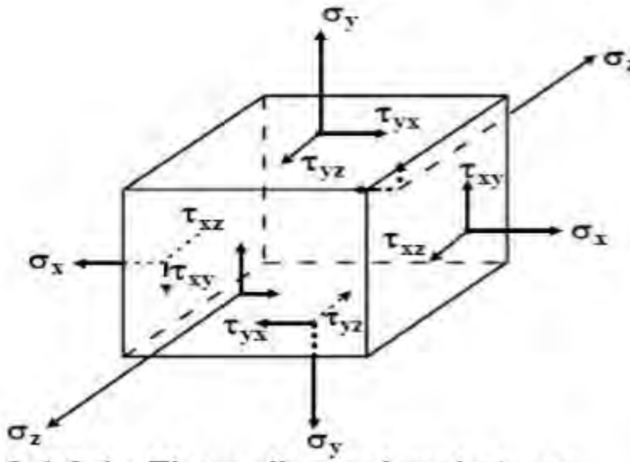


2.1.5.1 - Buckling of a beam hinged at both ends

2.1.6 Stress at a point—its implication in design

The state of stress at a point is given by nine stress components as shown in figure 2.1.6.1 and this is represented by the general matrix as shown below σ_{ij} is tensor or matrix of stress.

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \qquad \begin{array}{l} i = 1,2,3, \quad j = 1,2,3, \\ 1 = x, \; 2 = y, \; 3 = z. \end{array}$$



2.1.6.1 - Three dimensional stress field on an infinitesimal element.

$$\left.\begin{array}{l} \sigma_x = \sigma_{11} = \sigma_{xx} \\ \sigma_y = \sigma_{22} = \sigma_{yy} \\ \sigma_z = \sigma_{33} = \sigma_{zz} \end{array}\right\} \text{tensile stresses.}$$

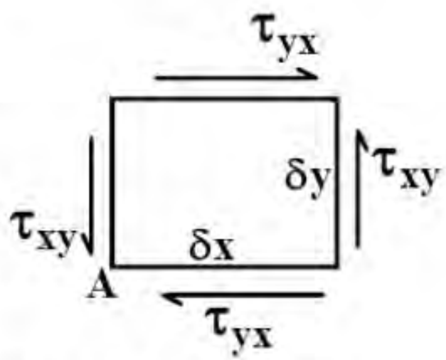
$$\tau_{yx}, \; \tau_{zx}, \; \tau_{zy} - \text{shear stresses.}$$

On crossection perpendiecu larity to axis y, z, x.

Consider now a two dimensional stress element subjected only to shear stresses.

For equilibrium of a 2-D element we take moment of all the forces about point A (figure-2.1.6.2) and equate to zero as follows:

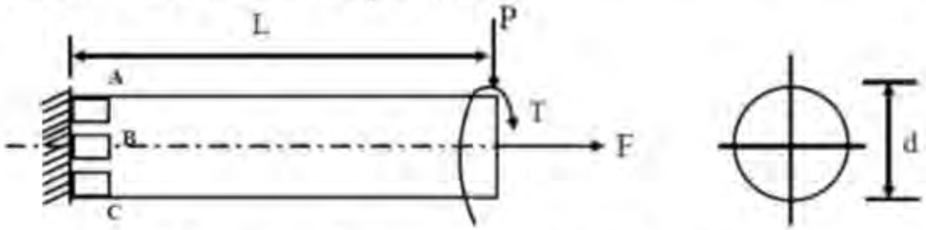
$$\left(\tau_{xy}\delta y\delta z\right)\delta x-\left(\tau_{yx}\delta x\delta z\right)\delta y=0$$



2.1.6.2 - Complimentary shear stresses on a 2-D element.

This gives $\tau_{xy}=\tau_{yx}$ indicating that τ_{xy} and τ_{yx} are complimentary. On similar arguments we may write $\tau_{yz}=\tau_{zy}$ and $\tau_{zx}=\tau_{xz}$. This means that the state of stress at a point can be given by six stress components only. It is important to understand the implication of this state of stress at a point in the design of machine elements where all or some of the stresses discussed above may act.

For example, let us consider a cantilever beam of circular cross-section subjected to a vertical loading P at the free end and an axial loading F in addition to a torque T as shown in figure 2.1.6.3. Let the diameter of cross-section and the length of the beam be d and L respectively.



2.1.6.3 - A cantilever beam subjected to bending, torsion and an axial loading.

The maximum stresses developed in the beam are :

Bending stress, $\sigma_A = \frac{32PL}{\pi d^3}$

Axial stress, $\sigma_B = \frac{4F}{\pi d^2}$

Torsional shear stress $\tau = \frac{16T}{\pi d^3}$

The formulae were obtained before, here F – is force along axial, T – is moment.

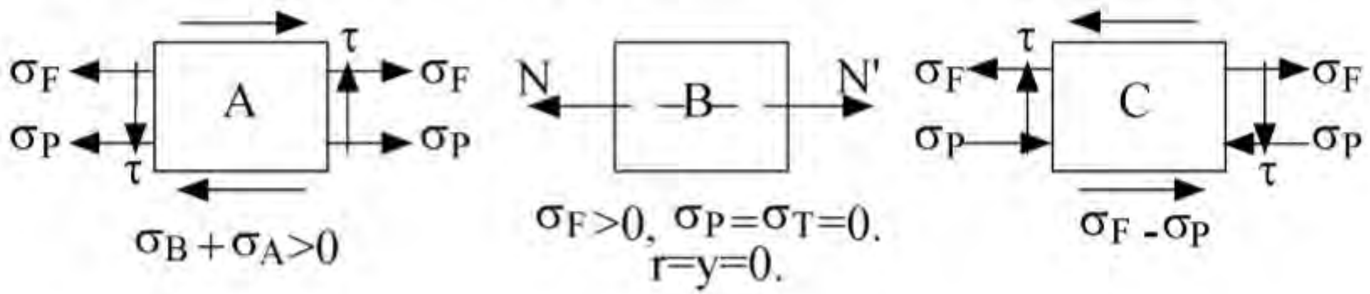
It is now necessary to consider the most vulnerable section and element. Since the axial and torsional shear stresses are constant through out the length, the most vulnerable section is the built-up end.

We now consider the three elements A, B and C. There is no bending stress on the element B and the bending and axial stresses on the element C act in the opposite direction. Therefore, for the safe design of the beam we consider the stresses on the element A which is shown in figure 2.1.6.4.

On element B located on neutral axis NN'.

On element C bending stresses is contract, axial stresses ore tensile.

On element A bending stresses are tensile and axial stress are tensile.

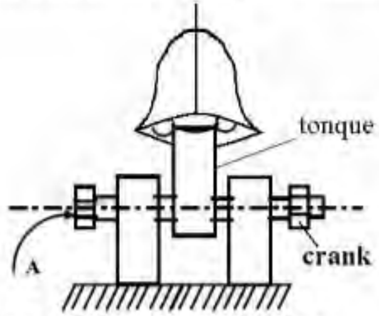


2.1.6.4 - Stresses developed on elements A, B, C in figure-2.1.6.3

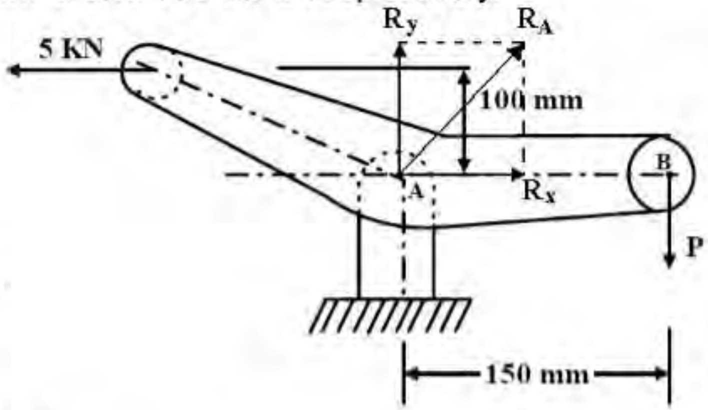
Principal stresses and maximum shear stresses can now be obtained and using a suitable failure theory a suitable diameter of the bar may be obtained. In this case in cantilever beam we have combination of simple stresses: tensile, bending, torsinal stresses.

2.1.7 Problems with Answers

Q.1: What stresses are developed in the pin A for the bell crank mechanism shown in the figure-2.1.7.1? Find the safe diameter of the pin if the allowable tensile and shear stresses for the pin material are 350 MPa and 170 MPa respectively.



2.1.7.1



Equation of force equilibium

A.1: y: $R_y+P_B=0$,

x: $R_x-P_C=0$. Hene R – constrain face.

Force at B = $\frac{5x0.1}{0.15}=3.33KN$

Resultant force at A= $\sqrt{5^2+3.33^2}$ kN = 6 kN.

Stresses developed in pin A: (a) shear stress (b) bearing stress

Considering double shear at A, pin diameter d = $\sqrt{\frac{2x6x10^3}{\pi x170x10^6}}$ m = 4.7 mm

Considering bearing stress at A, pin diameter d = $\frac{6x10^3}{0.01x7.5x10^6}$ m = 8mm

A safe pin diameter is 10 mm.

Q.2: What are the basic assumptions in deriving the bending equation?

A.2: The basic assumptions in deriving bending equation are:

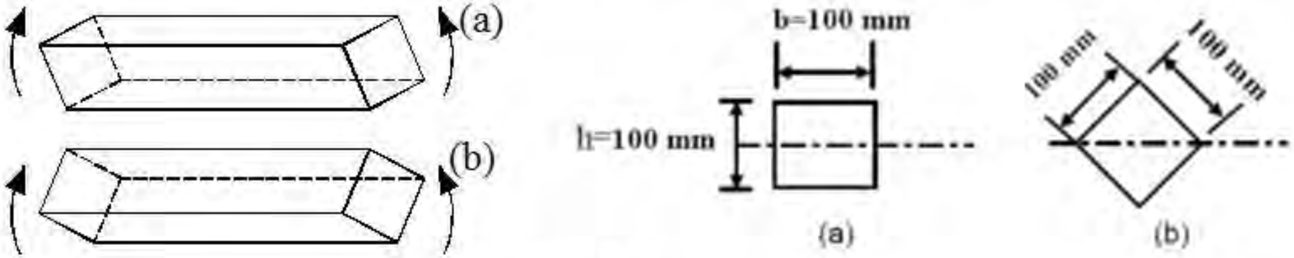
a) The beam is straight with a constant area of cross-section and is symmetrical about the plane of bending.

b) Material is homogeneous and isotropic.

c) Plane sections normal to the beam axis remain plane even after bending.

d) Material obeys Hooke's law

Q.3: Two cast iron machine parts of cross-sections shown in figure-2.1.7.2 are subjected to bending moments. Which of the two sections can carry a higher moment and determine the magnitude of the applied moments?



2.1.7.2

A.3:

Assuming that bending takes place about the horizontal axis, the 2nd moment of areas of the two sections are:

$$I_a = \frac{b.b^3}{12} \qquad I_b = 2\frac{\left(\sqrt{2}b\right)\left(\frac{b}{\sqrt{2}}\right)^3}{36} + 2\frac{\left(\sqrt{2}b\right)\left(\frac{b}{\sqrt{2}}\right)}{2}\left(\frac{b/\sqrt{2}}{3}\right)^2 = \frac{b^4}{12}$$

$$\therefore I_a = I_b$$

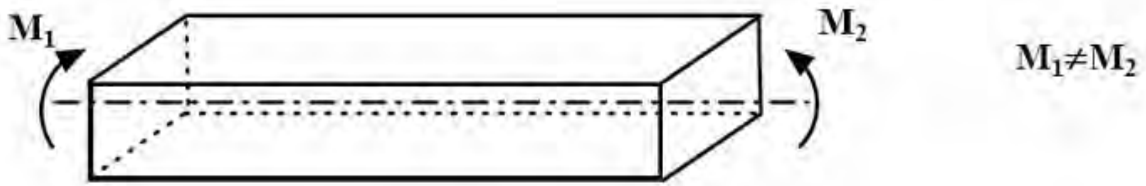
Considering that the bending stress σ_B is same for both the beams and moments applied M_a and M_b, we have

$$\sigma_B = \frac{M_a y_a}{I_a} = \frac{M_b y_b}{I_b}$$

Here, $y_a = 0.5b$, $y_b = b/\sqrt{2}$. Then $M_a = \sqrt{2}M_b$, $M_a > M_b$.

Q.4: Under what condition transverse shear stresses are developed in a beam subjected to a bending moment?

A.4: Pure bending of beams is an idealized condition and in the most realistic situation,bending moment would vary along the bending axis (figure-2.1.7.3).

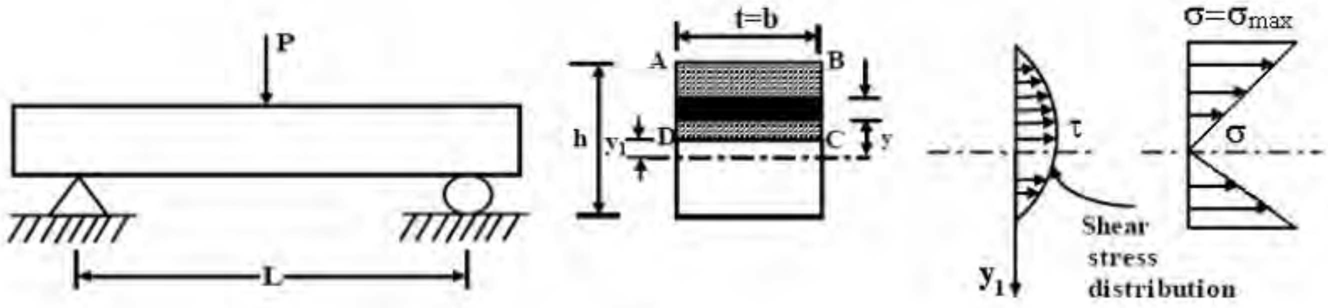


2.1.7.3

Under this condition transverse shear stresses would be developed in a beam.

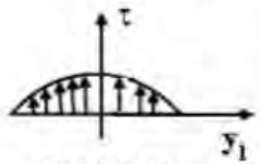
Q.5: Show how the transverse shear stress is distributed in a beam of solid rectangular cross-section transmitting a vertical shear force.

A.5: Consider a beam with a rectangular cross-section (figure-2.1.7.4). Consider now a longitudinal cut through the beam at a distance of y1 from the neutral axis isolating an area ABCD. An infinitesimal area within the isolated area at a distance y from the neutral axis is then considered to find the first moment of area Q.



2.1.7.4

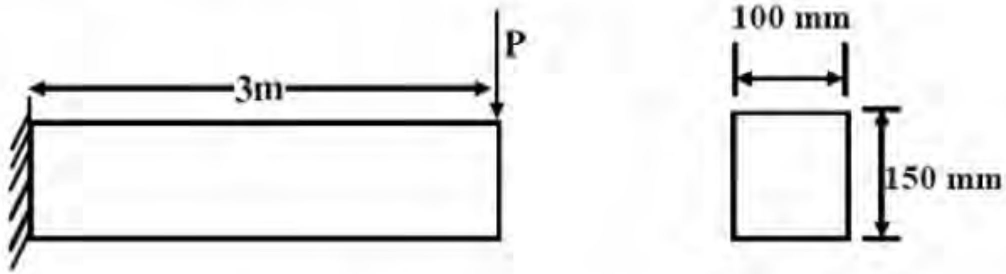
A simply supported beam with a Enlarged view of the rectangular cross-section Concentrated load at the centre.



Horizontal shear stress at y, $\tau = \frac{VQ}{It} = \frac{V}{It}\int_{y_1}^{h} b y \, dy$

This gives $\tau = \frac{V}{2I}\left[\frac{h^2}{4} - y_1^2\right]$ indicating a parabolic distribution of shear stress across the cross-section. Here, V is shear force, I is the second moment of area of the beam cross-section, t is the beam width which is b in this case.

Q.6: A 3m long cantilever beam of solid rectangular cross-section of 100mm width and 150mm depth is subjected to an end loading P as shown in the figure-2.1.7.5. If the allowable shear stress in the beam is 150 MPa, find the safe value of P based on shear alone.



2.1.7.5

A.6: Maximum shear stress in a rectangular cross-section $\tau_{max} = \frac{3}{2}\frac{V}{A}$ where, A is the cross-section area of the beam.

Substituting values we have τmax= 100P and for an allowable shear stress of 150 MPa the safe value of P works out to be 1.5 MN.

Q.7: What are the basic assumptions in deriving the torsion equation for a circular member?

A.7: Basic assumptions in deriving the torsion formula are:

a) Material is homogenous and isotropic.

b) A plane section perpendicular to the axis remains plane even after the torque is applied. This means there is no warpage.

c) In a circular member subjected to a torque, shear strain varies linearly from the central axis.

d) Material obeys Hooke's law.

Q.8: In a design problem it is necessary to replace a 2m long aluminium shaft of 100mm diameter by a tubular steel shaft of the same outside diameter transmitting the same torque and having the same angle of twist. Find the inner radius of the steel bar if G_{Al} = 28GPa and G_{St} = 84GPa.

A.8:

Since the torque transmitted and angle of twist are the same for both the solid and hollow shafts, we may write from torsion formula

$$\tau_{Al} J_{Al} = \tau_{St} J_{St} \quad \text{and} \quad \frac{\tau_{Al}}{\tau_{St}} = \frac{G_{Al}}{G_{St}}$$

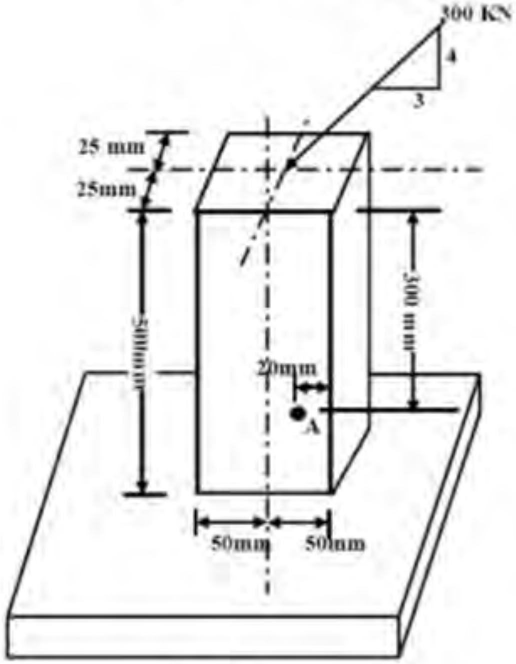
where τ, J and G are shear stress, polar moment of inertia and modulus of rigidity respectively. This gives

$$\frac{d_0^4 - d_i^4}{d_0^4} = \frac{28}{84} \quad \text{and with } d_0 = 100\,\text{mm} \;\; d_i = 90.36\,\text{mm}$$

$$1 - \frac{d_1^4}{d_0^4} = \frac{29}{84}: \quad \frac{d_1^4}{d_0^4} = 1 - \frac{28}{84} = \frac{56}{84} = \frac{14}{21} = \frac{2}{3};$$

$$d_{Al} = 100 = d_0: \quad d_{St} = d_1 = 90{,}36.$$

Q.10: Show the stresses on the element at A in figure-2.1.7.6.



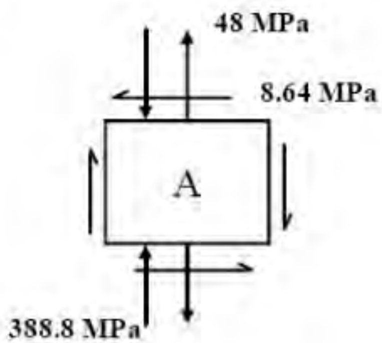
2.1.7.6

A.10: The element A is subjected to a compressive stress due to the vertical component 240 KN and a bending stress due to a moment caused by the horizontalcomponent 180 KN.

Compressive stress, $\sigma_c = \dfrac{240}{0.05\text{x}0.1} = 48\,\text{MPa}$

Bending (tensile) stress, $\sigma_B = \dfrac{(180\text{x}0.3)\text{x}0.03}{\left(\dfrac{0.05\text{x}0.1^3}{12}\right)} = 388.8\,\text{MPa}$

Shear stress due to bending = $\dfrac{VQ}{It} = 8.64\,\text{MPa}$



2.1.8 Summary of this Lecture

It is important to analyse the stresses developed in machine parts and design the components accordingly. In this lecture simple stresses such as tensile, compressive, bearing, shear, bending and torsional shear stress and buckling of beams have been discussed along with necessary formulations. Methods of combining normal and shear stresses are also discussed.

Lecture
Theme 2

Stresses in machine elements

2.2. Compound stresses in machine parts

2.2.1 Introduction

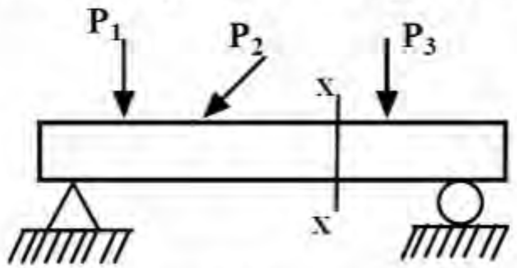
The elements of a force system acting at a section of a member are axial force, shear force and bending moment and the formulae for these force systems were derived based on the assumption that only a single force element is acting at the section. Figure-2.2.1.1 shows a simply supported beam while figure-2.2.1.2 shows the forces and the moment acting at any cross-section X-X of the beam. The force system can be given as:

Axial force : $\sigma = \frac{P}{A}$

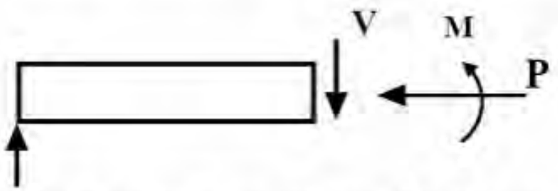
Bending moment : $\sigma = \frac{My}{I}$

Shearforce : $\tau = \frac{VQ}{It}$

Torque $T = \frac{\tau J}{r}$



2.2.1.1 *- A simply supported beam with concentrated loads*

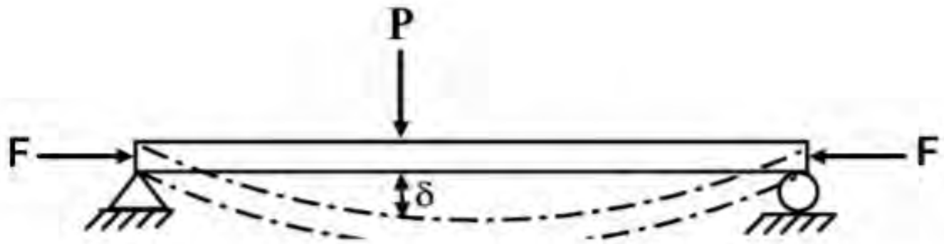


2.2.1.2 *- Force systems on section XX of figure-2.2.1.1*

where, σ is the normal stress, τ the shear stress, P the normal load, A is the crosssectional area, M is the moment acting at section X-X, V the shear stress acting at section X-X, Q the first moment of area, I is the moment of inertia, t the width at which transverse shear is calculated, J is the polar moment of inertia and r is the radius of the circular cross-section.

Combined effect of these elements at a section may be obtained by the method of superposition provided that the following limitations are tolerated:

(a) Deformation is small (figure-2.2.1.3)



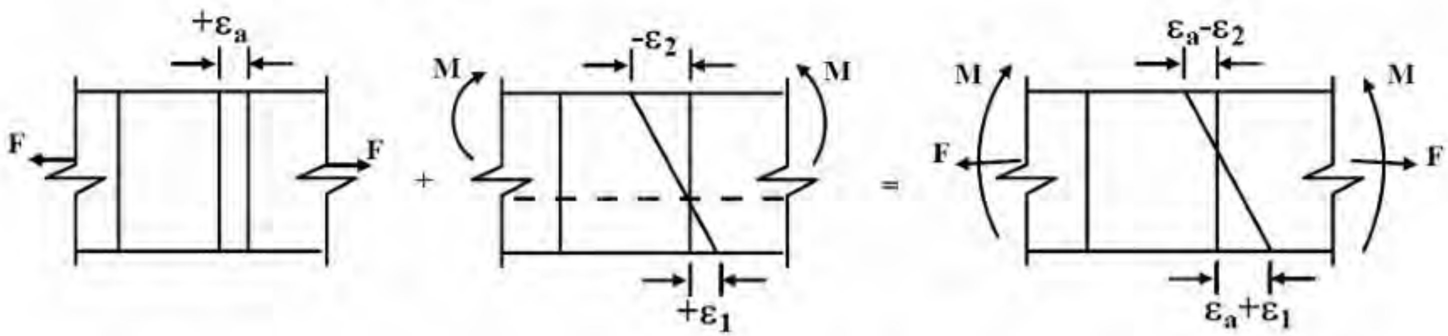
2.2.1.3 - Small deflection of a simply supported beam with a concentrated load

If the deflection is large, another additional moment of Pδ would be developed.

(b) Superposition of strains are more fundamental than stress superposition and the principle applies to both elastic and inelastic cases.

2.2.2 Strain superposition due to combined effect of axial force P and bending moment M.

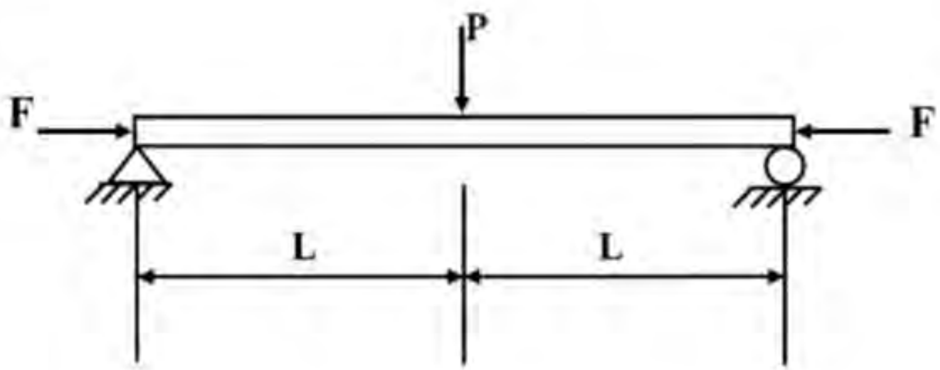
Figure-2.2.2.1 shows the combined action of a tensile axial force and bending moment on a beam with a circular cross-section. At any cross-section of the beam, the axial force produces an axial strain ε_a while the moment M causes a bending strain. If the applied moment causes upward bending such that the strain at the upper most layer is compressive ($-\varepsilon_2$) and that at the lower most layer is tensile ($+\varepsilon_1$), consequently the strains at the lowermost fibre are additive ($\varepsilon_a+\varepsilon_1$) and the strains at the uppermost fibre are subtractive ($\varepsilon_a-\varepsilon_2$). This is demonstrated in figure-2.2.2.1.



2.2.2.1 - Superposition of strain due to axial loading and bending moment.

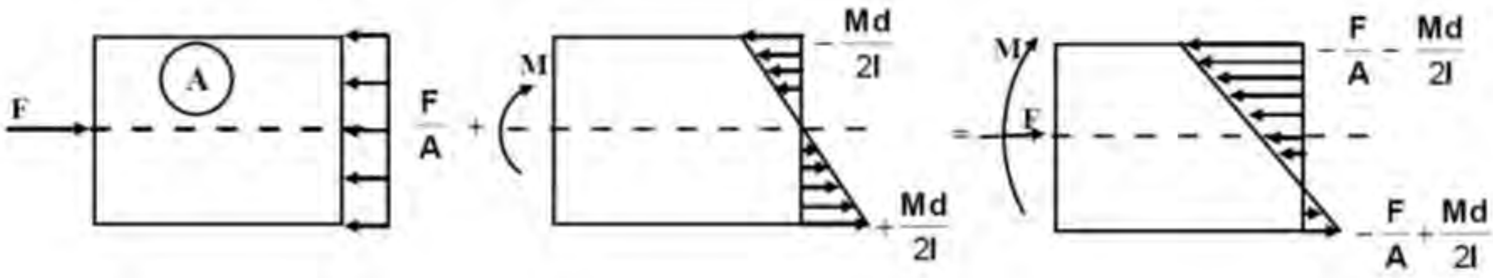
2.2.3 Superposition of stresses due to axial force and bending moment

In linear elasticity, stresses of same kind may be superposed in homogeneous and isotropic materials. One such example (figure-2.2.3.1) is a simply supported beam with a central vertical load P and an axial compressive load F.



2.2.3.1 - A simply supported beam with an axial and transverse loading.

At any section a compressive stress of $4F/\pi d^2$ and a bending stress of My/I are produced. Here d is the diameter of the circular bar, I is the second moment of area and the moment is PL/2 where the beam length is 2L. Total stresses at the upper and lower most fibres in any beam cross-section are $-\left(\frac{32M}{2\pi d^3}+\frac{4F}{\pi d^2}\right)$ and $\left(\frac{32M}{2\pi d^3}-\frac{4F}{\pi d^2}\right)$ respectively. This is illustrated in figure-2.2.3.2



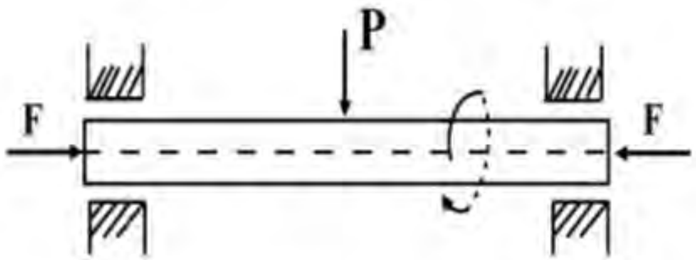
2.2.3.2 - Combined stresses due to axial loading and bending moment.

2.2.4 Superposition of stresses due to axial force, bending moment and torsion

Until now, we have been discussing the methods of compounding stresses of same kind for example, axial and bending stresses both of which are normal stresses. However, in many cases members on machine elements are subjected to both normal and shear stresses, for example, a shaft subjected to torsion, bending and axial force. This is shown in figure-2.2.4.1.

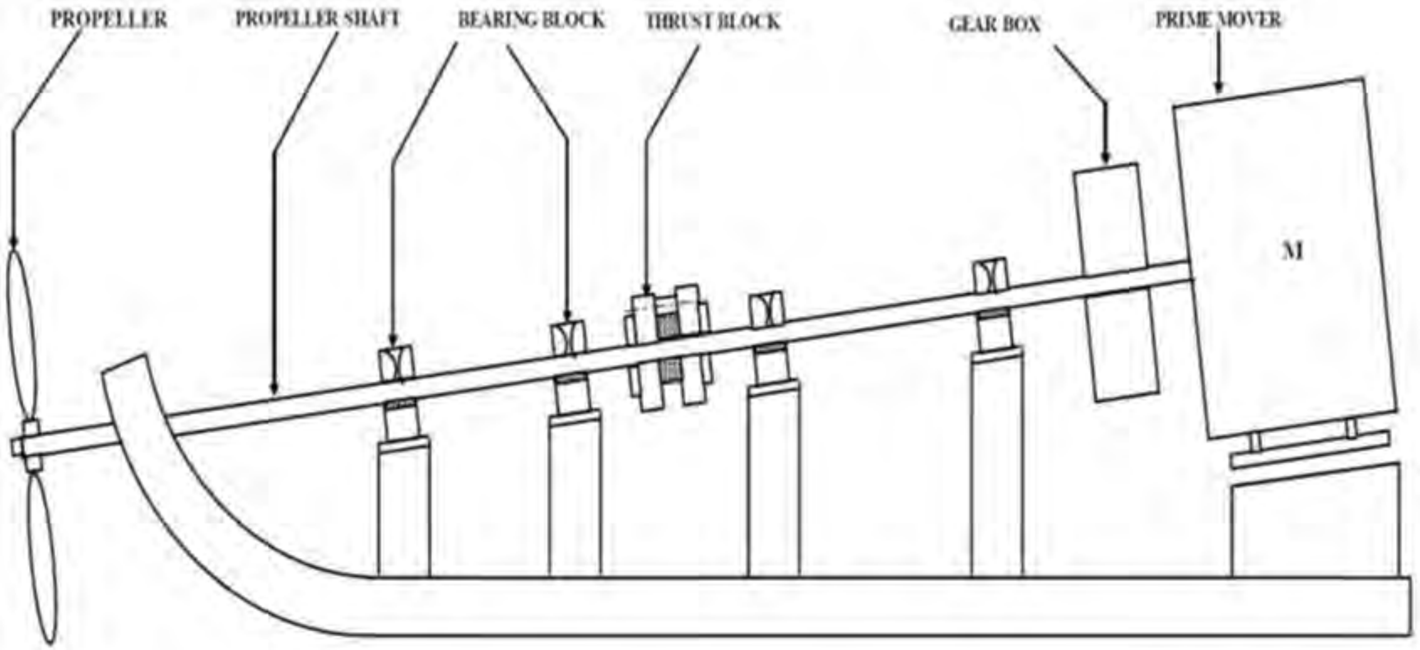
In this case we have

$$\sigma_F = \frac{F}{A};\ \sigma_P = \frac{M_y}{I};\ T = \frac{\tau J}{r}$$



2.2.4.1 - A simply supported shaft subjected to axial force bending moment and torsion.

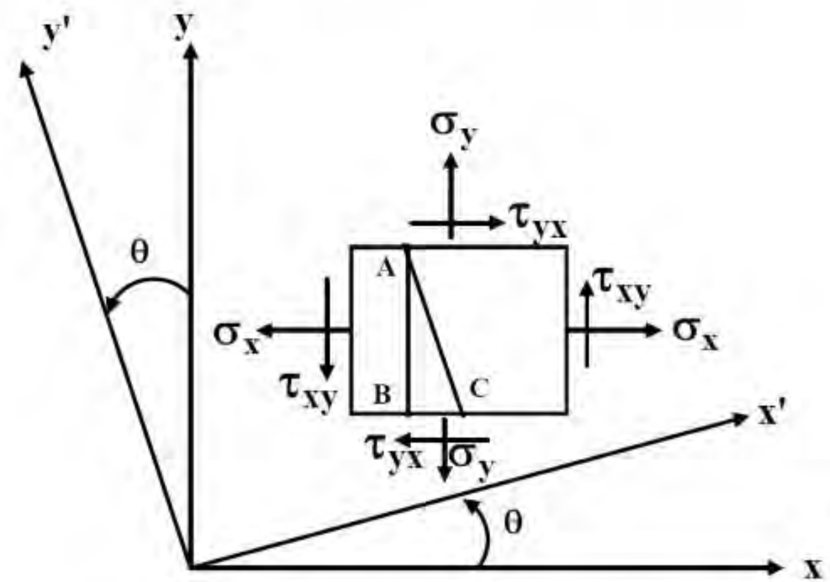
A typical example of this type of loading is seen in a ship's propeller shafts. Figure-2.2.4.2 gives a schematic view of a propulsion system. In such cases normal and shearing stresses need to be compounded.



2.2.4.2 - A schematic diagram of a typical marine propulsion shafting

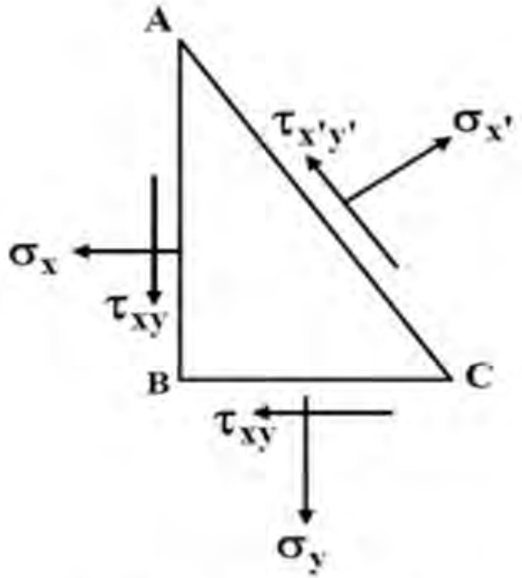
2.2.5 Transformation of plane stresses

Consider a state of general plane stress in x-y co-ordinate system. We now wish to transform this to another stress system in, say, x′- y′ co-ordinates, which is inclined at an angle θ. This is shown in figure-2.2.5.1.



2.2.5.1 - Transformation of stresses from x-y to x'-y' co-ordinate system.

A two dimensional stress field acting on the faces of a cubic element is shown in figure-2.2.5.2. In plane stress assumptions, the non-zero stresses are σx, σy and $\tau_{xy}=\tau_{yx}$. We may now isolate an element ABC such that the plane AC is inclined at an angle θ and the stresses on the inclined face are σ'_x and τ'_{xy} .



2.2.5.2 - Stresses on an isolated triangular element

Considering the force equilibrium in x-direction we may write

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

This may be reduced to

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (1)$$

Similarly, force equilibrium in y-direction gives

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (2)$$

Since plane AC can assume any arbitrary inclination, a stationary value of σx′ is given by

$$\frac{d\sigma_{x'}}{d\theta} = 0$$

This gives

$$\tan 2\theta = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} \qquad (3)$$

This equation has two roots and let the two values of θ be θ_1 and (θ_1+90°). Therefore these two planes are the planes of maximum and minimum normal stresses. Now if we set $\tau_{x'y'}$=0 we get the values of θ corresponding to planes of zero shear stress.

This also gives

$$\tan 2\theta = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2}$$

And this is same as equation (3) indicating that at the planes of maximum and minimum stresses no shearing stress occurs. These planes are known as **Principal planes** and stresses acting on these planes are known as **Principal stresses.** From equation (1) and (3) the principal stresses are given as

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (4)$$

In the same way, condition for maximum shear stress is obtained from

$$\frac{d}{d\theta}(\tau_{x'y'}) = 0$$

$$\tan 2\theta = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} \tag{5}$$

This also gives two values of θ say θ_2 and (θ_2+90°), at which shear stress is maximum or minimum. Combining equations (2) and (5) the two values of maximum shear stresses are given by

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{6}$$

One important thing to note here is that values of tan2θ2 is negative reciprocal of $\tan 2\theta_1$ and thus θ_1 and θ_2 are 45° apart. This means that principal planes and planes of maximum shear stresses are 45° apart. It also follows that although no shear stress exists at the principal planes, normal stresses may act at the planes of maximum shear stresses.

2.2.6 An example

Consider an element with the following stress system (figure-2.2.6.1)

σ_x=-10 MPa, σ_y = +20 MPa, τ = -20 MPa.

We need to find the principal stresses and show their senses on a properly oriented element.

Solution:

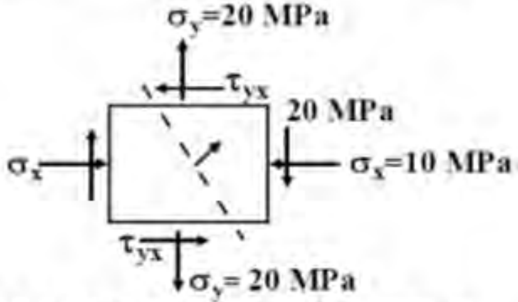
The principal stresses are

$$\sigma_{1,2} = \frac{-10+20}{2} \pm \sqrt{\left(\frac{-10-20}{2}\right)^2 + (-20)^2}$$

This gives -20 MPa and 30 MPa

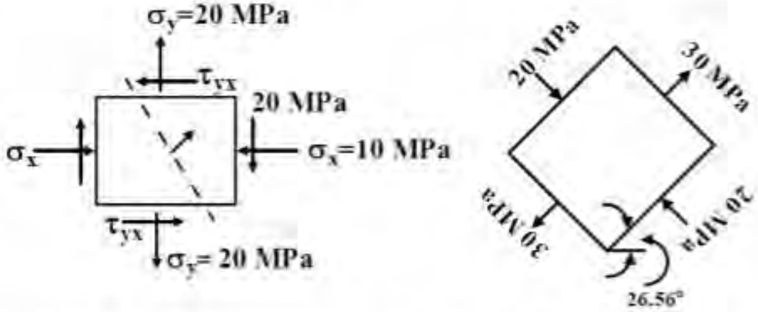
The principal planes are given by

$$\tan 2\theta_1 = \frac{-20}{(-10-20)/2} = 1.33$$



2.2.6.1 - A 2-D element with normal and shear stresses.

The two values are 26.56 and 116.56°. The oriented element to show the principal stresses is shown in figure-2.2.6.2.



2.2.6.2 - Orientation of the loaded element in the left to show the principal stresses.

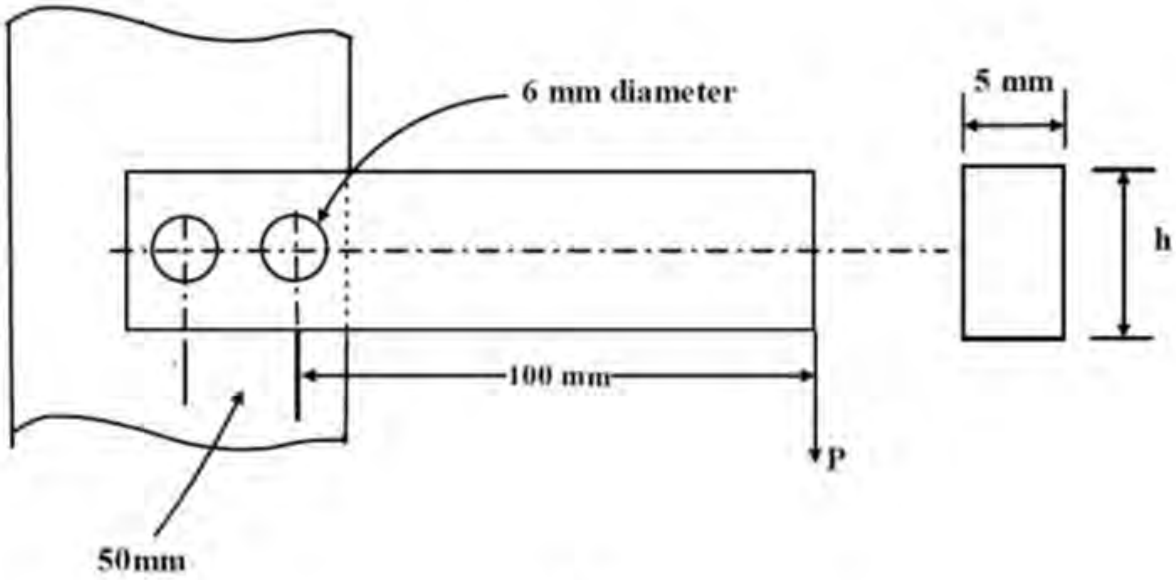
In this case tensor of stress has simple form $\sigma_{ij} = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix}$, $\tau_{xy} = 0$. If we turn the element on angle 26.56° then we have simple state of element in principal axises.

2.2.7 Problems with Answers

Q.1: A 5mm thick steel bar is fastened to a ground plate by two 6 mm diameter pins as shown in figure-2.2.7.1. If the load P at the free end of the steel bar is 5 KN, find

(a) The shear stress in each pin

(b) The direct bearing stress in each pin.



2.2.7.1

A.1:

Due to the application of force P the bar will tend to rotate about point 'O' causing shear and bearing stresses in the pins A and B. This is shown in figure-2.2.7.2F. Let the forces at pins A and B be F_A and F_B and equating moments about 'O',

$$5x10^3 x0.125 = (F_A+F_B)x\ 0.025 \quad (1)$$

Also, from force balance, $F_A+P = F_B$ (2)

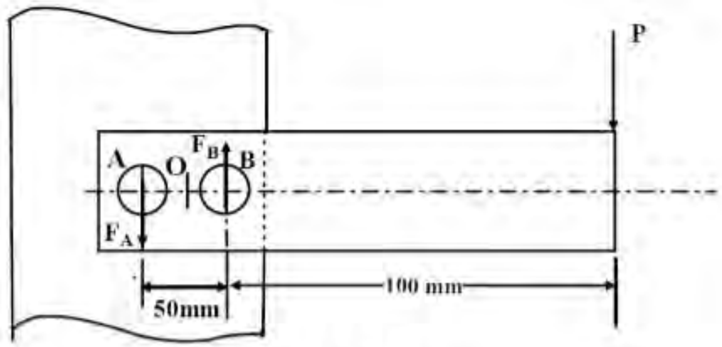
Solving equations-1 and 2 we have, F_A =10 KN and F_B = 15 KN.

(a) Shear stress in pin A = $\dfrac{10x10^3}{\left(\dfrac{\pi x0.006^2}{4}\right)} = 354\,MPa$

Shear stress in pin B = $\dfrac{15x10^3}{\left(\dfrac{\pi x0.006^2}{4}\right)} = 530.5\,MPa$

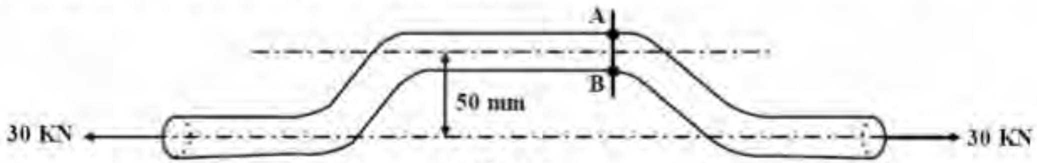
(b) Bearing stress in pin A = $\dfrac{10x10^3}{(0.006x0.005)} = 333\,MPa$

Bearing stress in pin B = $\dfrac{15x10^3}{(0.006x0.005)} = 500\,MPa$



2.2.7.2

Q.2: A 100 mm diameter off-set link is transmitting an axial pull of 30 KN as shown in the figure- 2.2.7.3. Find the stresses at points A and B.



2.2.7.3

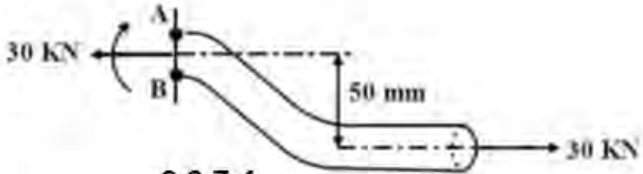
A.2: The force system at section AB is shown in figure-2.2.7.4.

$$\sigma_A = \sigma_A^{(b)} + \sigma_A^{(t)} = -\frac{30\times10^3\times0.05\times0.05}{\frac{\pi}{64}(0.1)^4} + \frac{30\times10^3}{\frac{\pi}{4}(0.1)^2} = -11.46\text{MPa}$$

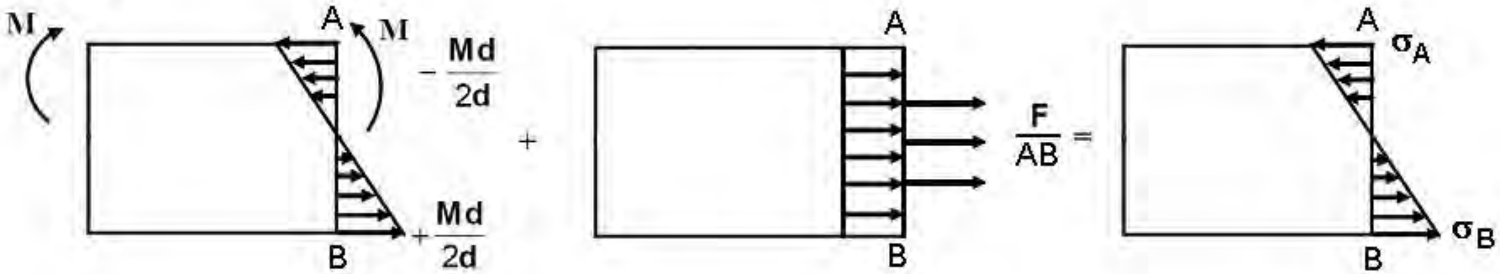
$$\sigma_B = \sigma_B^{(b)} + \sigma_B^{(t)} = \frac{30\times10^3\times0.05\times0.05}{\frac{\pi}{64}(0.1)^4} + \frac{30\times10^3}{\frac{\pi}{4}(0.1)^2} = 19.1\text{MPa}$$

At section AB in this case we have pure bending plus stretch at paint B or minus carpesscs at paint A.

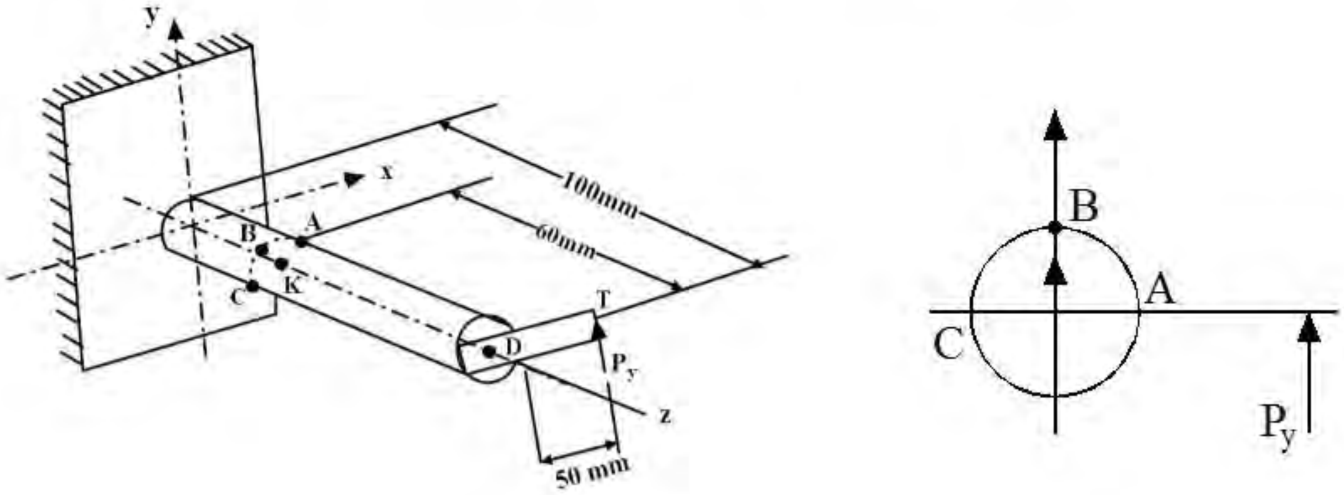
We applicate princinl of superposition for solving.



2.2.7.4



Q.3: A vertical load P_y = 20 KN is applied at the free end of a cylindrical bar of radius 50 mm as shown in figure-2.2.7.5. Determine the principal and maximum shear stresses at the points A, B and C.



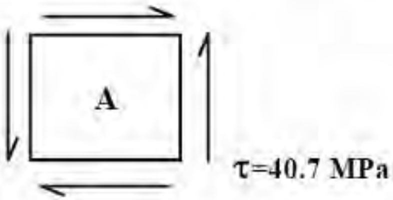
2.2.7.5

A.3: At section ABC a bending moment of 1.2 KN-m and a torque of 1KN-m act. On elements A and C there is no bending stress because they are in plane perpendicularity to force P_y. Only torsional shear stress acts and

$$M^{(b)} = P_y DK = 1.2KN,$$

$$M^{(T)} = P_y DT = 1KN$$

$$\tau = \frac{16T}{\pi d^3} = 40.7 \text{ MPa}$$

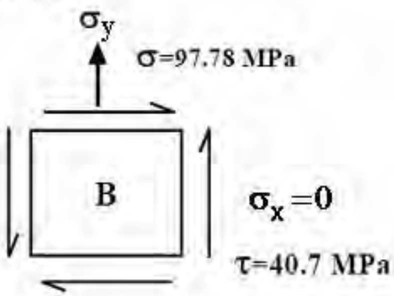


$$\tau_A = \tau_C$$

On element B both bending (compressive) and torsional shear stress act.

$$\sigma_B = \frac{32M}{\pi d^3} = 97.78\,\text{MPa} = \frac{Mr}{I}, \; r = 50$$

$$\tau = 40.7\,\text{MPa}$$



Principal stresses at B = $\left(\frac{97.78}{2} \pm \sqrt{\left(\frac{97.78}{2}\right)^2 + (40.7)^2} \right)$

$$\sigma_{B1} = 112.5\text{MPa}; \quad \sigma_{B2} = -14.72\text{MPa}$$

Q.4: A propeller shaft for a launch transmits 75 KW at 150 rpm and is subjected to a maximum bending moment of 1KN-m and an axial thrust of 70 KN. Find the shaft diameter based on maximum principal stress if the shear strength of the shaft material is limited to 100 MPa.

A.4:

$$\text{Torque, } T = \frac{75x10^3}{\left(\frac{2\pi x150}{60}\right)} = 4775\,\text{Nm; then, } \tau = \frac{24.3}{d^3}\,\text{KPa}$$

$$\text{Maximum bending moment} = 1\,\text{KNm; then, } \sigma_b = \frac{10.19}{d^3}\,\text{KPa}$$

$$\text{Axial force} = 70\,\text{KN; then, } \sigma = \frac{70}{\frac{\pi d^2}{4}}\,\text{KPa} = \frac{89.12}{d^2}\,\text{KPa}$$

$$\text{Maximum shear stress} = \sqrt{\left(\frac{89.12}{2d^2} - \frac{10.19}{2d^3}\right)^2 + \left(\frac{24.3}{d^3}\right)^2} = 100x10^3$$

Solving we get the value of shaft diameter d = 63.4 mm.

2.2.8 Summary of this Lecture

The stresses developed at a section within a loaded body and methods of superposing similar stresses have been discussed. Methods of combining normal and shear stresses using transformation of plane stresses have been illustrated. Formulations for principal stresses and maximum shear stresses have been derived and typical examples are solved.

Lecture
Theme 2
Stresses in machine elements

2.3. Strain analysis

2.3.1 Introduction

No matter what stresses are imposed on an elastic body, provided the material does not rupture, displacement at any point can have only one value. Therefore the displacement at any point can be completely given by the three single valued components u, v and w along the three co-ordinate axes x, y and z respectively. The normal and shear strains may be derived in terms of these displacements.

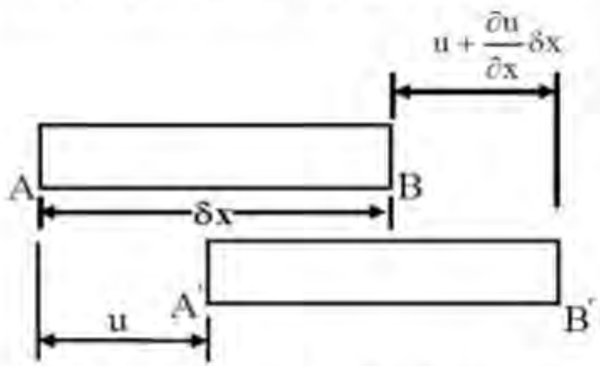
2.3.2 Normal strais

Consider an element AB of length δx (figure-2.3.2.1). If displacement of end A is u, that of end B is $u+\frac{\partial u}{\partial x}\delta x$. This gives an increase in length of ($u+\frac{\partial u}{\partial x}\delta x$ -u) and therefore the strain in x-direction is $\frac{\partial u}{\partial x}$.Similarly, strains in y and z directions are $\frac{\partial v}{\partial y}$ and $\frac{\partial w}{\partial z}$.Therefore, we may write the three normal strain components as

$$\varepsilon_x=\frac{\partial u}{\partial x},\ \varepsilon_y=\frac{\partial v}{\partial y}\ \text{and}\ \varepsilon_z=\frac{\partial w}{\partial z}.$$

$$u_A=u(x),\ u_B=u(x+\delta x)=u(x)+\frac{\partial u}{\partial x}\delta x+\ldots$$

AB is initial length till deformation, A'B' is length after deforme.



2.3.2.1 - Change in length of an infinitesimal element.

2.3.3 Shear strain

In the same way we may define the shear strains. For this purpose consider an element ABCD in x-y plane and let the displaced position of the element be A′B′C′D′ (Figure-2.3.3.1). This gives shear strain in xy plane as $\varepsilon_{xy}=\alpha+\beta$ where α is the angle made by the displaced line B′C′ with the vertical and β is the angle made by the displaced line A′D′ with the horizontal. This gives

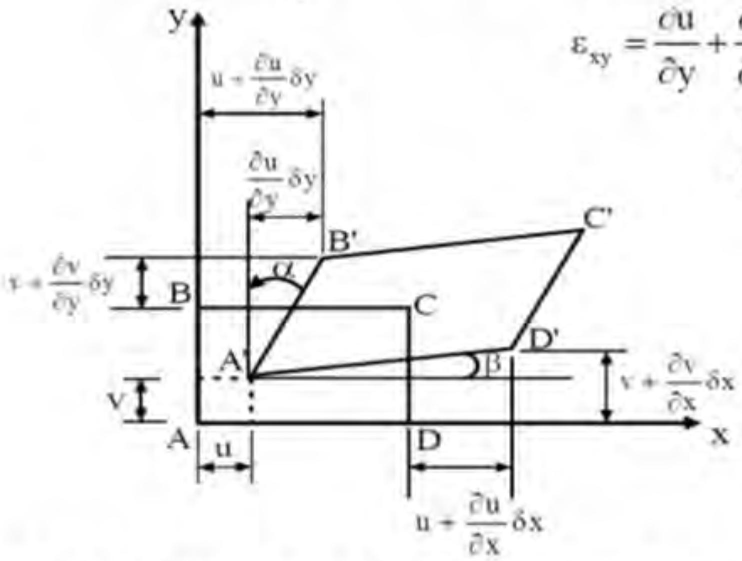
$$\mathrm{tg}\,\alpha \approx \alpha = \frac{\frac{\partial u}{\partial y}\delta y}{\delta y} = \frac{\partial u}{\partial y} \quad \text{and} \quad \mathrm{tg}\,\beta \approx \beta = \frac{\frac{\partial v}{\partial x}\delta x}{\delta x} = \frac{\partial v}{\partial x}$$

We may therefore write the three shear strain components as

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x},\ \varepsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \ \text{ and } \ \varepsilon_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

Therefore, the complete strain matrix can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$$



2.3.3.1 - Shear strain associated with the distortion of an infinitesimal element.

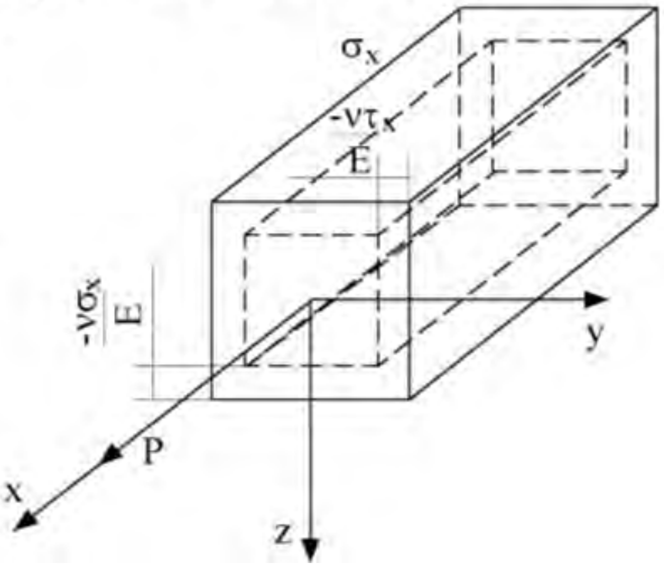
2.3.4 Constitutive equation

The state of strain at a point can be completely described by the six strain components and the strain components in their turns can be completely defined by the displacement components u, v, and w. **The constitutive equations relate stresses and strains and in linear elasticity we simply have $\sigma=E\varepsilon$ where E is modulus of elasticity.** It is also known that σx produces a strain of σ_x/E where E is direction, $-\nu\sigma_x/E$ in y-direction and $-\nu\sigma x/E$ in z-direction. The bar is stretched in direction x and is contracted in directions y and z. Therefore we may write the generalized **Hooke's law** as

$$\varepsilon_x=\frac{1}{E}\left[\sigma_x-\nu(\sigma_y+\sigma_z)\right],\quad \varepsilon_y=\frac{1}{E}\left[\sigma_y-\nu(\sigma_z+\sigma_x)\right]$$

$$\varepsilon_z=\frac{1}{E}\left[\sigma_z-\nu(\sigma_x+\sigma_y)\right]$$

$$\varepsilon_{xy}=\gamma_{xy}=\frac{\tau_{xy}}{G},\quad \varepsilon_{yz}=\gamma_{yz}=\frac{\tau_{yz}}{G}\ \text{and}\ \varepsilon_{zx}=\gamma_{zx}=\frac{\tau_{zx}}{G}$$



In general each strain is dependent on each stress and we may write. Hooke's law in the form

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

For isotropic material mechanical and physical properties are indepent on direction. In this case we get

$$K_{11} = K_{22} = K_{33} = \frac{1}{E}$$

$$K_{12} = K_{13} = K_{21} = K_{23} = K_{31} = K_{32} = -\frac{\nu}{E}$$

$$K_{44} = K_{55} = K_{66} = \frac{1}{G}$$

Rest of the elements in K matrix are zero.

On substitution, this reduces the general constitutive equation to equations for isotropic materials as given by the generalized Hooke's law. Since the principal stress and strains axes coincide, we may write the principal strains in terms of principal stresses as

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right]$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \nu(\sigma_3 + \sigma_1)\right]$$

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu(\sigma_1 + \sigma_2)\right]$$

From the point of view of volume change or dilatation resulting from hydrostatic pressure we also have

$$\bar{\sigma} = K\Delta$$

where $\bar{\sigma} = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right) = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ and $\Delta = (\varepsilon_x + \varepsilon_y + \varepsilon_z) = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$

These equations allow the principal strain components to be defined in terms of principal stresses. For isotropic and homogeneous materials only two constants viz. E and ν are sufficient to relate the stresses and strains.

The strain transformation follows the same set of rules as those used in stress transformation except that the shear strains are halved wherever they appear.

2.3.5 Relations between E, G and K

The largest maximum shear strain and shear stress can be given by

$\gamma_{max} = \varepsilon_2 - \varepsilon_3$ and $\tau_{max} = \dfrac{\sigma_2 - \sigma_3}{2}$ and since $\gamma_{max} = \dfrac{\tau_{max}}{G}$ we have

$\dfrac{1}{E}\left[\sigma_2 - \nu(\sigma_1 + \sigma_3)\right] - \dfrac{1}{E}\left[\sigma_3 - \nu(\sigma_1 + \sigma_2)\right] = \dfrac{1}{G}\left(\dfrac{\sigma_2 - \sigma_3}{2}\right)$ and this gives

$$\boxed{G = E/2(1+\nu)}$$

Where G is modulus of rigidity or shear modulus, $0 \le \nu \le 0.5$ is Puasson's rafio.

Considering now the hydrostatic state of stress and strain we may write

$\dfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = K(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$. Substituting ε_1, ε_2 and ε_3 in terms of σ_1, σ_2 and σ_3

we may write

$\dfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = K\left[(\sigma_1 + \sigma_2 + \sigma_3) - 2\nu(\sigma_1 + \sigma_2 + \sigma_3)\right]$ and this gives

$$\boxed{K = E/3(1-2\nu).}$$

Where K is modulus or modulus of compession.

2.3.6 Elementary thermoelasticity

So far the state of strain at a point was considered entirely due to applied forces. Changes in temperature may also cause stresses if a thermal gradient or some external constraints exist. Provided that the materials remain linearly elastic, stress pattern due to thermal effect may be superimposed upon that due to applied forces and we may write

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha T, \qquad \varepsilon_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right] + \alpha T, \qquad \varepsilon_{yz} = \frac{\tau_{yz}}{G}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] + \alpha T, \qquad \varepsilon_{zx} = \frac{\tau_{zx}}{G}$$

Where T is temperature, α is cocfficient of thermal expension.

It is important to note that the shear strains are not affected directly by temperature changes.

It is sometimes convenient to express stresses in terms of strains. This may be done using the relation $\Delta(\varepsilon)=\varepsilon_x+\varepsilon_y+\varepsilon_z$. Substituting the above expressions for ε_x, ε_y and ε_z we have,

$$\Delta(\varepsilon)=\frac{1}{E}\left[(1-2\nu)\left(\sigma_x+\sigma_y+\sigma_z\right)\right]+3\alpha T$$

and substituting $K = E/3(1-2\nu)$ we have

$$\Delta(\sigma)=\frac{1}{3K}\left(\sigma_x+\sigma_y+\sigma_z\right)+3\alpha T\,.$$

Combining this with $\varepsilon_x=\frac{1}{E}\left[\sigma_x-\nu(\sigma_y+\sigma_z)\right]+\alpha T$ we have

$$\sigma_x=\frac{E\varepsilon_x}{1+\nu}+\frac{3\nu K(\Delta(\varepsilon)-3\alpha T)}{1+\nu}-\frac{E\alpha T}{1+\nu}$$

Substituting $G = E/2(1+\nu)$ and $\lambda=\frac{3\nu K}{1+\nu}$ we may write the normal and shear stresses as

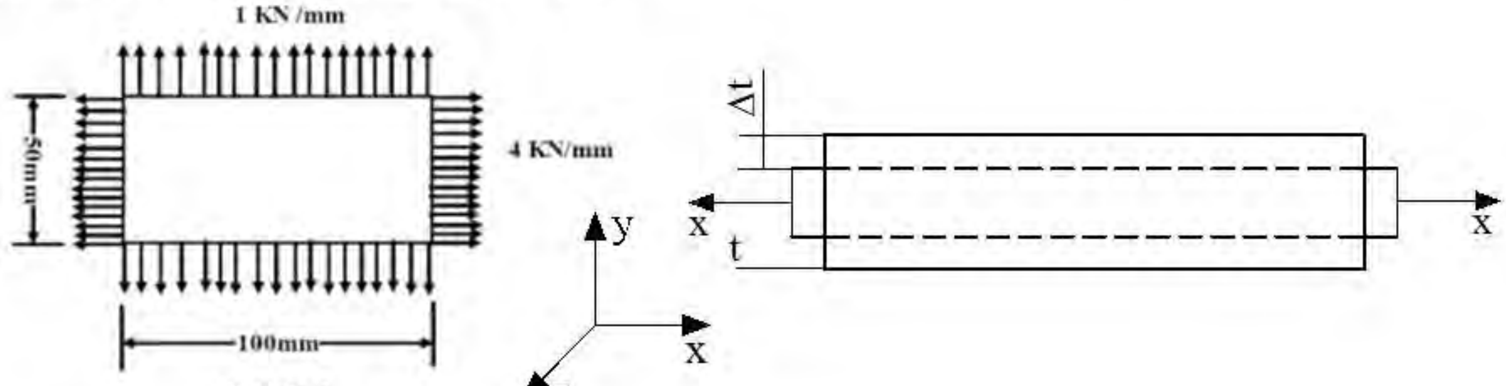
$$\sigma_x = 2G\varepsilon_x+\lambda\Delta(\varepsilon)-3K\alpha T \qquad \tau_{xy}=G\varepsilon_{xy}$$
$$\sigma_y = 2G\varepsilon_y+\lambda\Delta(\varepsilon)-3K\alpha T \qquad \tau_{yz}=G\varepsilon_{yz}$$
$$\sigma_z = 2G\varepsilon_z+\lambda\Delta(\varepsilon)-3K\alpha T \qquad \tau_{zx}=G\varepsilon_{zx}$$

Where λ Lames coefficient, for T=0 we get Hooke's law.

These equations are considered to be suitable in thermoelastic situations.

2.3.7 Problems with Answers

Q.1: A rectangular plate of 10mm thickness is subjected to uniformly distributed load along its edges as shown in figure-2.3.7.1. Find the change in thickness due to the loading. E=200 GPa, v = 0.3



2.3.7.1

A.1: Here σ_x = 400 MPa, σ_y = 100 MPa and σ_z = 0

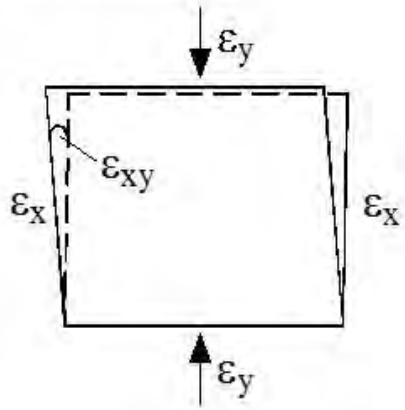
This gives $\varepsilon_z = -\frac{v}{E}\left(\sigma_x + \sigma_y\right) = -7.5\text{x}10^{-4}$

Now, $\varepsilon_z = \frac{\Delta t}{t}$ where, t is the thickness and Δt is the change in thickness.

Therefore, the change in thickness = 7.5 μm.

Q.2: At a point in a loaded member, a state of plane stress exists and the strains are ε_x=-90x10^{-6}, ε_y=-30x10^{-6} and ε_{xy}=120x10^{-6}. If the elastic constants E , v and G are 200 GPa , 0.3 and 84 GPa respectively, determine the normal stresses σ_x and σ_y and the shear stress τ_{xy} at the point.

A.2:



$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\sigma_x\right]$$

$$\varepsilon_{xy} = \frac{\tau_{xy}}{G}$$

This gives $\sigma_x = \frac{E}{1-\nu^2}\left[\varepsilon_x + \nu\varepsilon_y\right]$

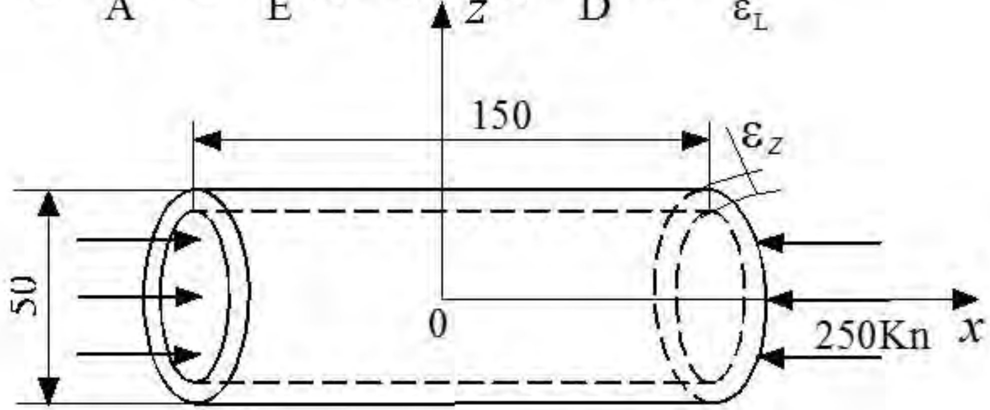
$$\sigma_y = \frac{E}{1-\nu^2}\left[\varepsilon_y + \nu\varepsilon_x\right]$$

Substituting values, we get

σ_x = -21.75 MPa, σ_y = -12.53 MPa and τ_{xy} = 9.23 MPa.

Q.3: A rod 50 mm in diameter and 150 mm long is compressed axially by an uniformly distributed load of 250 KN. Find the change in diameter of the rod if E = 200 GPa and v=0.3.

A.3: $\tau_x = \frac{F}{A},\ \varepsilon_x = \frac{\lambda}{E}\sigma_x,\ \varepsilon_L = \nu\varepsilon_x = \frac{\Delta}{D},\ D = \frac{\Delta}{\varepsilon_L}$



Axial stress $\sigma_x = \dfrac{250}{\frac{\pi}{4}(0.05)^2} = 127.3\,\text{MPa}$

Axial strain, $\varepsilon_x = 0.636\text{x}10^{-3}$

Lateral strain = $\nu\varepsilon_x = 1.9\text{x}10^{-4}$

Now, lateral strain, $\varepsilon_L = \frac{\Delta}{D}$ and this gives
$\Delta(\varepsilon) = 9.5\mu\text{m}.$

Q.4: If a steel rod of 50 mm diameter and 1m long is constrained at the ends and heated to 200°C from an initial temperature of 20°C, what would be the axial load developed? Will the rod buckle? Take the coefficient of thermal expansion, α=12x10^{-6} per °C and E=200 GPa.

A.4: Thermal strain, $\varepsilon_t = \alpha\Delta T = 2.16x10^{-3}$, $\Delta T = T_1 - T_0$

In the absence of any applied load, the force developed due to thermal expansion, $F = E\varepsilon_t A = 848KN$

For buckling to occur the critical load is given by

$$F_{cr} = \frac{\pi^2 EI}{l^2} = 605.59\,KN.$$

So F>F_{cr} Therefore, the rod will buckle when heated to 200°C.

2.3.8 Summary of this Lecture

Normal and shear strains along with the 3-D strain matrix have been defined. Generalized Hooke's law and elementary thermo-elasticity are discussed.

Lecture

Theme 3

Design for Strength

3.1. Design for static loading

3.1.1 Introduction

Machine parts fail when the stresses induced by external forces exceed their strength. The external loads cause internal stresses in the elements and the component size depends on the stresses developed. Stresses developed in a link subjected to uniaxial loading is shown in **figure-3.1.1.1**. Loading may be due to:

a) The energy transmitted by a machine element.

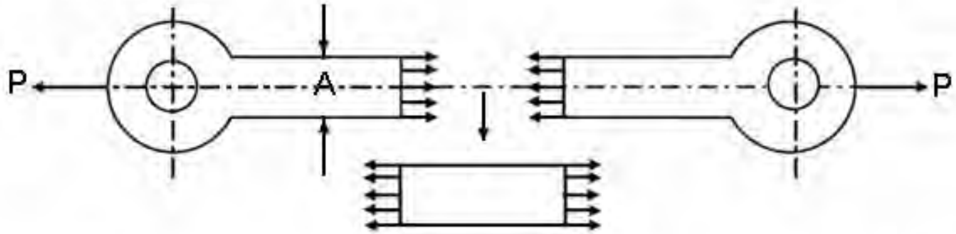
b) Dead weight.

c) Inertial forces.

d) Thermal loading.

e) Frictional forces.

$$\sigma = \frac{P}{A}$$



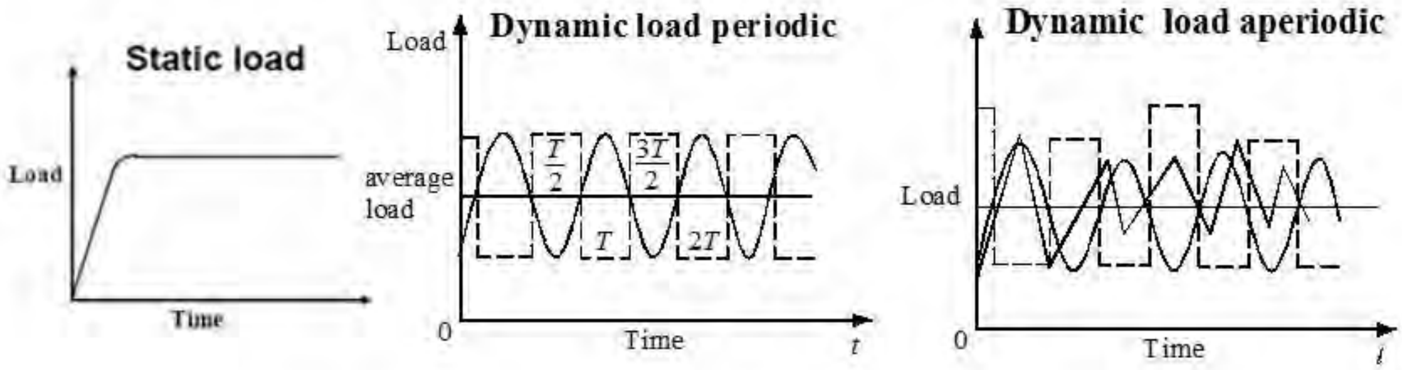
3.1.1.1 - Stresses developed in a link subjected to uniaxial loading

In another way, load may be classified as:

a) Static load- Load does not change in magnitude and direction and normally increases gradually to a steady value.

b) Dynamic load- Load may change in magnitude for example, traffic of varying weight passing a bridge.Load may change in direction, for example, load on piston rod of a double acting cylinder.

Vibration and shock are types of dynamic loading. **Figure-3.1.1.2** shows load vs time characteristics for both static and dynamic loading of machine elements.



3.1.1.2 - Types of loading on machine elements.

3.1.2 Allowable Stresses: Factor of Safety

Determination of stresses in structural or machine components would be meaningless unless they are compared with the material strength. If the induced stress is less than or equal to the limiting material strength then the designed component may be considered to be safe and an indication about the size of the component is obtained. The strength of various materials for engineering applications is determined in the laboratory with standard specimens. For example, for tension and compression tests a round rod of specified dimension is used in a tensile test machine where load is applied until fracture occurs. This test is usually carried out in a **Universal testing machine**. The load at which the specimen finally ruptures is known as **Ultimate load P_U and the ratio of load to original cross-sectional area A is the Ultimate stress σ_U.**

$$\frac{P_U}{A} = \sigma_U$$

Similar tests are carried out for bending, shear and torsion and the results for different materials are available in handbooks. For design purpose an allowable stress is used in place of the critical stress to take into account the uncertainties including the following:

1) Uncertainty in loading.
2) Inhomogeneity of materials.
3) Various material behaviors. e.g. corrosion, plastic flow, creep.
4) Residual stresses due to different manufacturing process.
5) Fluctuating load (fatigue loading): Experimental results and plot-ultimate strength depends on number of cycles.
6) Safety and reliability.

For ductile materials, the yield strength and for brittle materials the ultimate strength are taken as the critical stress.

An allowable stress is set considerably lower than the ultimate strength. The ratio of ultimate to allowable load or stress is known as factor of safety i.e.

$$\frac{\textbf{Ultimate Stress}}{\textbf{Allowable Stress}} = \textbf{F.S.} \qquad \frac{\sigma_U}{\sigma_a} = F.S.$$

The ratio must always be greater than unity. It is easier to refer to the ratio of stresses since this applies to material properties.

3.1.3 Theories of failure

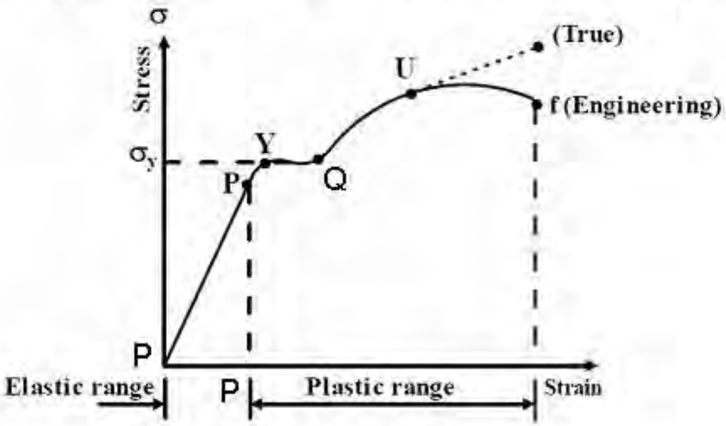
When a machine element is subjected to a system of complex stress system, it is important to predict the mode of failure so that the design methodology may be based on a particular failure criterion. Theories of failure are essentially a set of failure criteria developed for the ease of design. In machine design an element is said to have failed if it ceases to perform its function. There are basically two types of mechanical failure:

(a) **Yielding**- This is due to excessive inelastic deformation rendering the machine part unsuitable to perform its function. This mostly occurs in ductile materials.

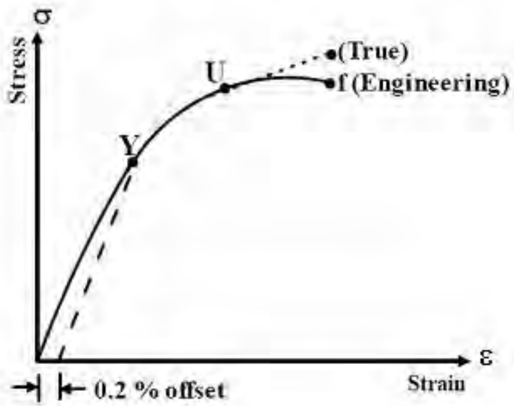
(b) **Fracture**- in this case the component tears apart in two or more parts. This mostly occurs in brittle materials. There is no sharp line of demarcation between ductile and brittle materials. However a rough guideline is that if percentage elongation is less than 5% then the material may be treated as brittle and if it is more than 15% then the material is ductile. However, there are many instances when a ductile material may fail by fracture. This may occur if a material is subjected to

(a) Cyclic loading.
(b) Long term static loading at elevated temperature.
(c) Impact loading.
(d) Work hardening.
(e) Severe quenching.

Yielding and fracture can be visualized in a typical tensile test as shown in the clipping- Typical engineering stress-strain relationship from simple tension ests for same engineering materials are shown in **figure- 3.1.3.1**.

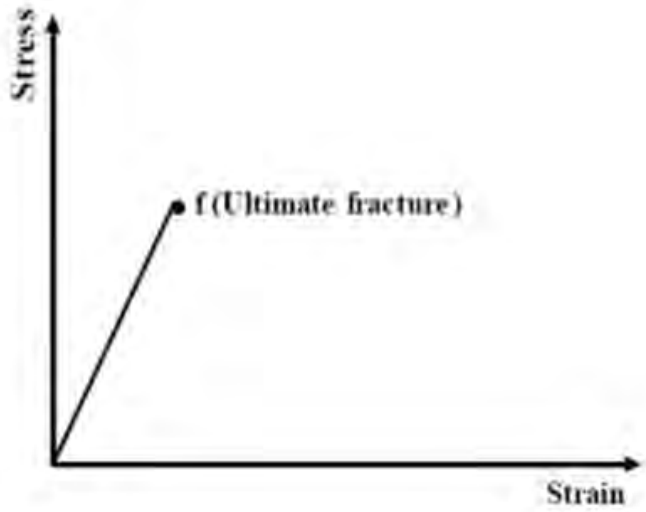


3.1.3.1 - (a) Stress-strain diagram for a ductile material e.g. low carbon steel.

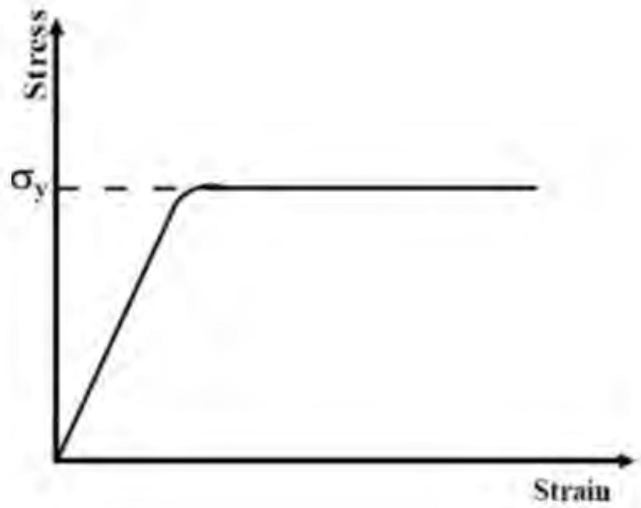


3.1.3.1 - (b) Stress-strain diagram for low ductility.

YQ on diagram is area of yeld, QU is area of plastic hardening, Uf is of plastic dishardening or fracture.



3.1.3.1 - (c) Stress-strain diagram for a brittle material.



3.1.3.1 - (d) Stress-strain diagram for an elastic – perfectly plastic material.

For a typical ductile material as shown in **figure-3.1.3.1 (a)** there is a definite yield point where material begins to yield more rapidly without any change in stress level. Corresponding stress is σ_y . Close to yield point is the proportional limit which marks the transition from elastic to plastic range. Beyond elastic limit for an elastic- perfectly plastic material yielding would continue without further rise in stress i.e. stress-strain diagram would be parallel to parallel to strain axis beyond the yield point. However, for most ductile materials, such as, low-carbon steel beyond yield point the stress in the specimens rises upto a peak value known as ultimate tensile stress σ_o . Beyond this point the specimen starts to neck-down i.e. the reduction in cross-sectional area. However, the stress-strain curve falls till a point where fracture occurs. The drop in stress is apparent since original cross-sectional area is used to calculate the stress. If instantaneous cross-sectional area is used the curve would rise as shown in **figure- 3.1.3.1 (a)** . For a material with low ductility there is no definite yield point and usually off-set yield points are defined for convenience. This is shown in **figure-3.1.3.1**. For a brittle material stress increases linearly with strain till fracture occurs. These are demonstrated in the **clipping- 3.1.3.2** .

3.1.4 Yield criteria

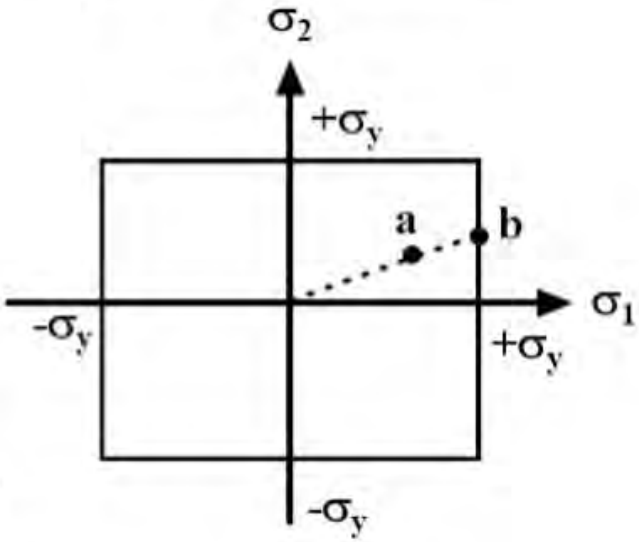
There are numerous yield criteria, going as far back as Coulomb (1773). Many of these were originally developed for brittle materials but were later applied to ductile materials. Some of the more common ones will be discussed briefly here.

3.1.4.1 Maximum principal stress theory (Rankine theory)

According to this, if one of the principal stresses σ_1 (maximum principal stress), σ_2 (minimum principal stress) or σ_3 exceeds the yield stress, yielding would occur. In a two dimensional loading situation for a ductile material where tensile and compressive yield stress are nearly of same magnitude

$$\sigma_1 = \pm \sigma_y \qquad \sigma_2 = \pm\sigma_y$$

Using this, a yield surface may be drawn, as shown in **figure- 3.1.4.1.1**. Yielding occurs when the state of stress is at the boundary of the rectangle. Consider, for example, the state of stress of a thin walled pressure vessel. Here $\sigma_1 = 2\sigma_2$, σ_1 being the circumferential or hoop stress and σ2 the axial stress. As the pressure in the vessel increases the stress follows the dotted line. At a point (say) a, the stresses are still within the elastic limit but at b, σ_1 reaches σy although σ_2 is still less than σy. Yielding will then begin at point b. This theory of yielding has very poor agreement with experiment. **However, the theory has been used successfully for brittle materials**



3.1.4.1.1 - Yield surface orresponding to maximum principal stress theory

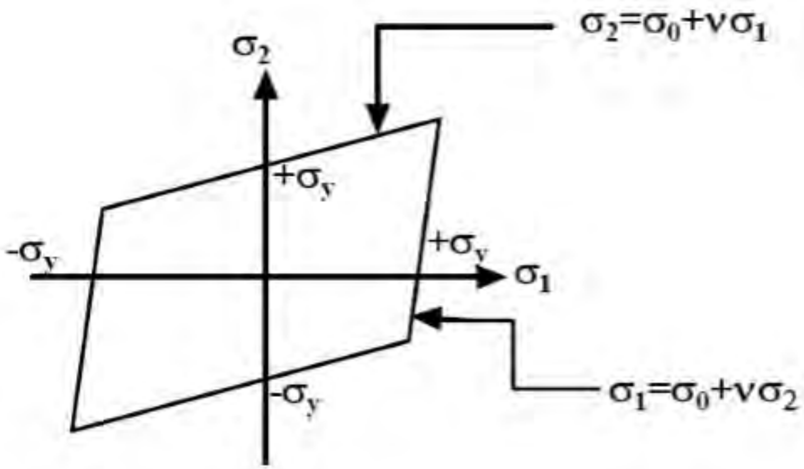
3.1.4.2 Maximum principal strain theory (St. Venant's theory)

According to this theory, yielding will occur when the maximum principal strain just exceeds the strain at the tensile yield point in either simple tension or compression. If ε_1 and ε_2 are maximum and minimum principal strains corresponding to σ_1 and σ_2, in the limiting case

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad |\sigma_1| \geq |\sigma_2| \qquad\qquad \varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad |\sigma_2| \geq |\sigma_1|$$

This gives, $E\varepsilon_1 = \sigma_1 - \nu\sigma_2 = \pm\sigma_0 \qquad E\varepsilon_2 = \sigma_2 - \nu\sigma_1 = \pm\sigma_0$

The boundary of a yield surface in this case is thus given as shown in **figure-3.1.4.2.1**



3.1.4.2.1- Yield surface corresponding to maximum principal strain theory

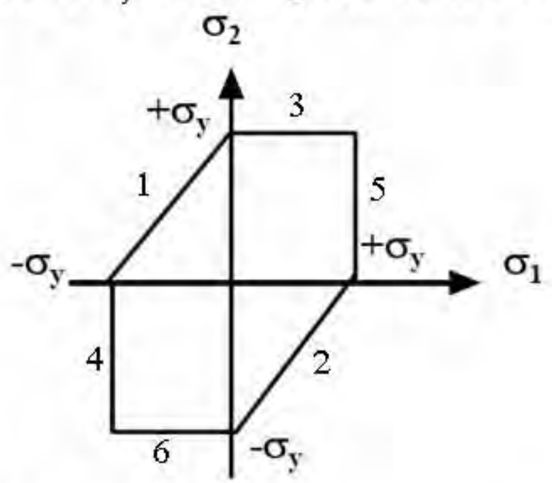
3.1.4.3 Maximum shear stress theory (Tresca theory)

According to this theory, yielding would occur when the maximum shear stress just exceeds the shear stress at the tensile yield point. At the tensile yield point $\sigma_2 = \sigma_3 = 0$ and thus maximum shear stress is $\sigma_y/2$. This gives us six conditions for a three-dimensional stress situation:

$$\sigma_1 - \sigma_2 = \pm\sigma_y$$

$$\sigma_2 - \sigma_3 = \pm\sigma_y$$

$$\sigma_3 - \sigma_1 = \pm\sigma_y$$



3.1.4.3.1 - Yield surface corresponding to maximum shear stress theory

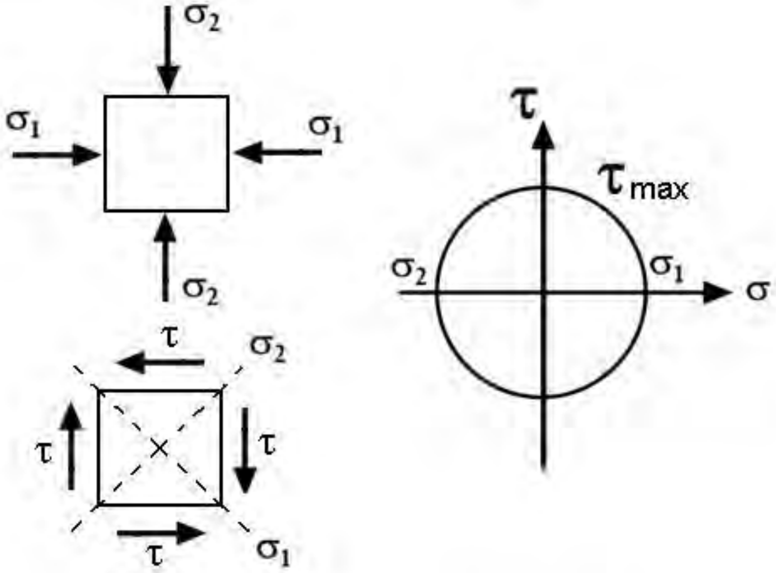
In a biaxial stress situation (**figure-3.1.4.3.1**) case, $\sigma_3 = 0$ and this gives

$1.\ \sigma_1 - \sigma_2 = \sigma_y$	if $\sigma_1 > 0, \sigma_2 < 0$	$4.\ \sigma_1 = -\sigma_y$	if $\sigma_1 < \sigma_2 < 0$
$2.\ \sigma_1 - \sigma_2 = -\sigma_y$	if $\sigma_1 < 0, \sigma_2 > 0$	$5.\ \sigma_1 = -\sigma_y$	if $\sigma_1 > \sigma_2 > 0$
$3.\ \sigma_2 = \sigma_y$	if $\sigma_2 > \sigma_1 > 0$	$6.\ \sigma_2 = -\sigma_y$	if $\sigma_2 < \sigma_1 < 0$

This criterion agrees well with experiment.

In the case of pure shear, $\sigma_1=-\sigma_2=k$ (say), $\sigma_3 = 0$ and this gives $\sigma_1-\sigma_2=2k=\sigma_y$

This indicates that yield stress in pure shear is half the tensile yield stress and this is also seen in the Mohr's circle (figure- 3.1.4.3.2) for pure shear.



$$k = \frac{\sigma_y}{2} = \frac{\sigma_1 - \sigma_2}{2}$$

3.1.4.3.2 - Mohr's circle for pure shear

3.1.4.4 Maximum strain energy theory (Beltrami's theory)

According to this theory failure would occur when the total strain energy absorbed at a point per unit volume exceeds the strain energy absorbed per unit volume at the tensile yield point. This may be given

$$\frac{1}{2}(\sigma_1\varepsilon_1+\sigma_2\varepsilon_2+\sigma_3\varepsilon_3)=\frac{1}{2}\sigma_y\varepsilon_y \quad \text{by} \quad \frac{1}{2}(\sigma_1\varepsilon_1+\sigma_2\varepsilon_2+\sigma_3\varepsilon_3)=\frac{1}{2}\sigma_y\varepsilon_y$$

Substituting, ε_1, ε_2 , ε_3 and ε_y in terms of stresses we have

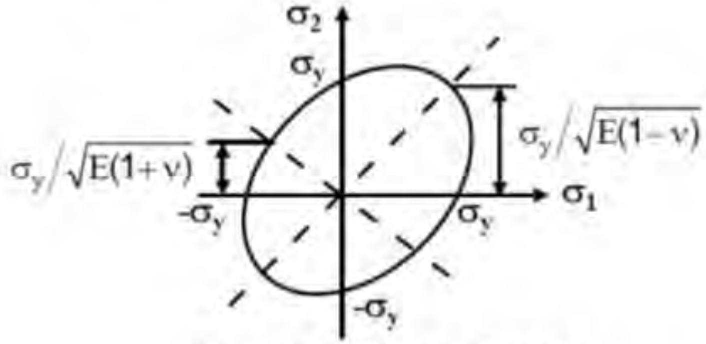
$$\sigma_1^{\ 2}+\sigma_2^{\ 2}+\sigma_3^{\ 2}-2\upsilon(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)=\sigma_y^{\ 2}$$

$$\begin{cases}\varepsilon_1=\dfrac{1}{E}[\sigma_1-U(\sigma_2+\sigma_1)]\\ \varepsilon_2=\dfrac{1}{E}[\sigma_2-U(\sigma_3+\sigma_1)]\\ \varepsilon_3=\dfrac{1}{E}[\sigma_3-U(\sigma_1+\sigma_2)]\end{cases}$$

For plan state of stresses, $\sigma_3=0$. This may be written as

For 2D: $$\left(\frac{\sigma_1}{\sigma_y}\right)^2+\left(\frac{\sigma_2}{\sigma_y}\right)^2-2\nu\left(\frac{\sigma_1\sigma_2}{\sigma_y{}^2}\right)=1\ ,\ \varepsilon_2-\varepsilon_1=\frac{1}{E}\left[\sigma_2-\sigma_1\right]$$

This is the equation of an ellipse and the yield surface is shown in **figure-3.1.4.4.1** .



3.1.4.4.1 - Yield surface corresponding to Maximum strain energy theory.

It has been shown earlier that only distortion energy can cause yielding but in the above expression at sufficiently high hydrostatic pressure $\sigma_1=\sigma_2=\sigma_3=\sigma$ (say), yielding may also occur.

3.1.4.5 Distortion energy theory(von Mises yield criterion)

According to this theory yielding would occur when total distortion energy absorbed per unit volume due to applied loads exceeds the distortion energy absorbed per unit volume at the tensile yield point. Total strain energy ET and strain energy for volume change E_V can be given as

$$E_T = \frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3) \text{ and } E_V = \frac{3}{2}\sigma_{av}\varepsilon_{av}$$

Substituting strains in terms of stresses the distortion energy can be given as

$$E_d = E_T - E_V = \frac{2(1+\nu)}{6E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1\right)$$

At the tensile yield point, $\sigma_1 = \sigma_y$, for 1-D $\sigma_2 = \sigma_3 = 0$ which gives

$$E_{dy} = \frac{2(1+\nu)}{3E}\sigma_y^2$$

The failure criterion is thus obtained by equating E_d and E_{dy} , which gives

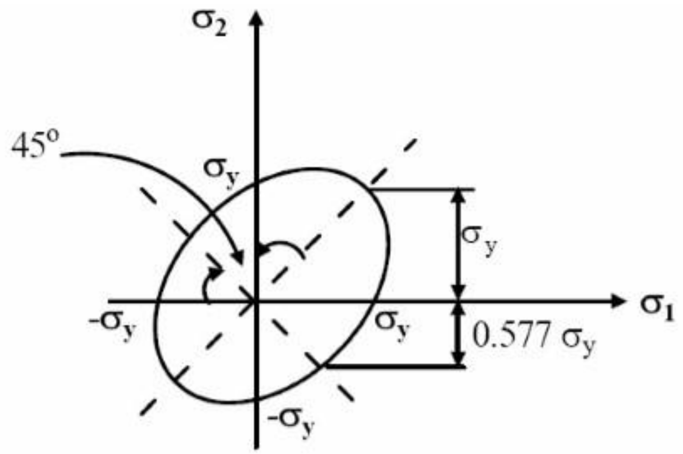
$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2$$

In a 2-D situation if $\sigma_3 = 0$, the criterion reduces to

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2$$

$$\text{i.e.} \quad \left(\frac{\sigma_1}{\sigma_y}\right)^2 + \left(\frac{\sigma_2}{\sigma_y}\right)^2 - \left(\frac{\sigma_1}{\sigma_y}\right)\left(\frac{\sigma_2}{\sigma_y}\right) = 1$$

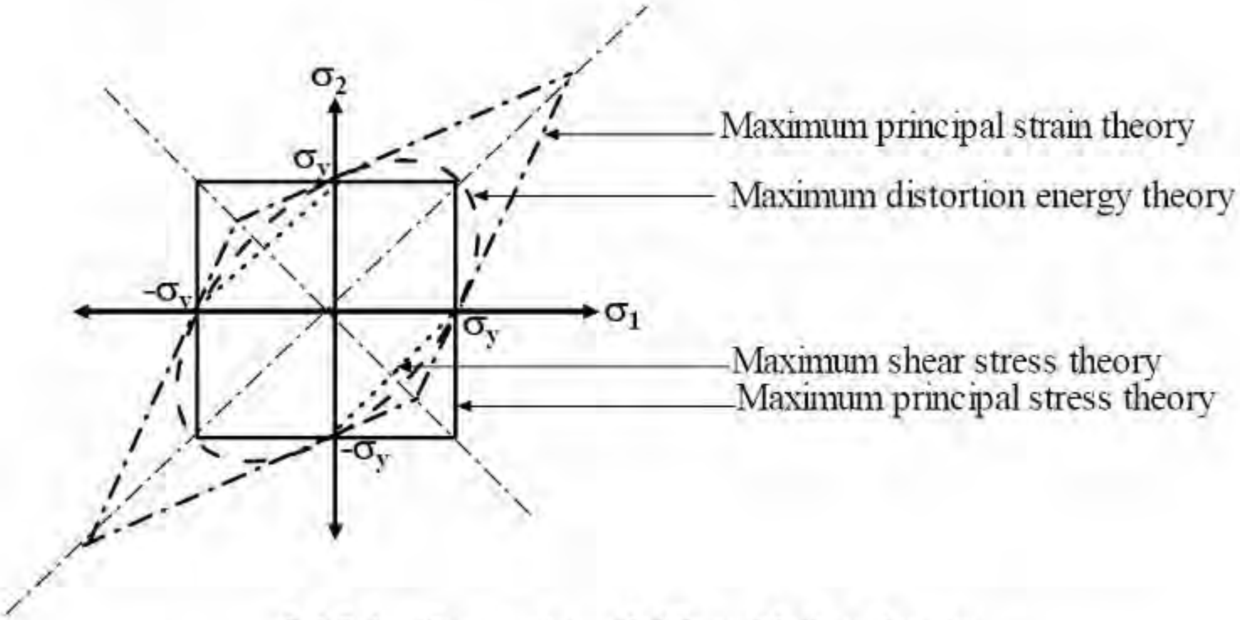
This is an equation of ellipse and the yield surface is shown in **figure-3.1.4.5.1** . This theory agrees very well with experimental results and is widely used for ductile materials.



3.1.4.5.1 *- Yield surface corresponding to von Mises yield criterion.*

3.1.5 Superposition of yield surface

A comparison among the different failure theories can be made by superposing the yield surfaces as shown in **figure- 3.1.5.1**.



3.1.5.1 *- Comparison of different failure theories.*

It is clear that an immediate assessment of failure probability can be made just by plotting any experimental in the combined yield surface. Failure of ductile materials is most accurately governed by the distortion energy theory where as the maximum principal strain theory is used for brittle materials.

3.1.6 Problems with Answers

Q.1: A shaft is loaded by a torque of 5 KN-m. The material has a yield point of 350 MPa. Find the required diameter using

(a) Maximum shear stress theory

(b) Maximum distortion energy theory

Take a factor of safety of 2.5.

$$\tau = \frac{TJ}{r}$$

τ is shear stress, T is torque, r is radius, J is polar moment of inertia.

A.1: Torsional shear stress induced in the shaft due to 5 KN-m torque is

$$\tau = \frac{16x(5x10^3)}{\pi d^3}$$ where d is the shaft diameter in m.

(b) Maximum shear stress theory,

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$ Since $\sigma_x = \sigma_y = 0$, $\tau_{max}=25.46x10^3/d^3 = \frac{\sigma_Y}{2xF.S.} = \frac{350x10^6}{2x2.5}$

This gives d=71.3 mm.

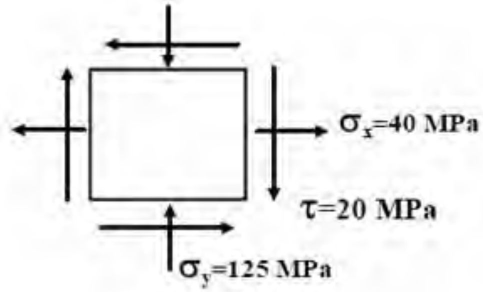
(b) Maximum distortion energy theory

In this case $\sigma_1 = 25.46x10^3/d^3$ $\quad \sigma_2 = -25.46x10^3/d^3$

According to this theory, $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 2(\sigma_Y/F.S)^2$

Since $\sigma_3 = 0$, substituting values of σ_1, σ_2 and σ_Y we get d=68 mm.

Q.2: The state of stress at a point for a material is shown in the **figure-3.1.6.1**. Find the factor of safety using (a) Maximum shear stress theory (b) Maximum distortion energy theory. Take the tensile yield strength of the material as 400 MPa.



3.1.6.1

A.2:

From the Mohr's circle, shown in **figure-3.1.6.2**

σ_1 = 42.38 MPa

σ_2 = -127.38 MPa

(a) Maximum shear stress theory

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_Y}{2xF.S}$$

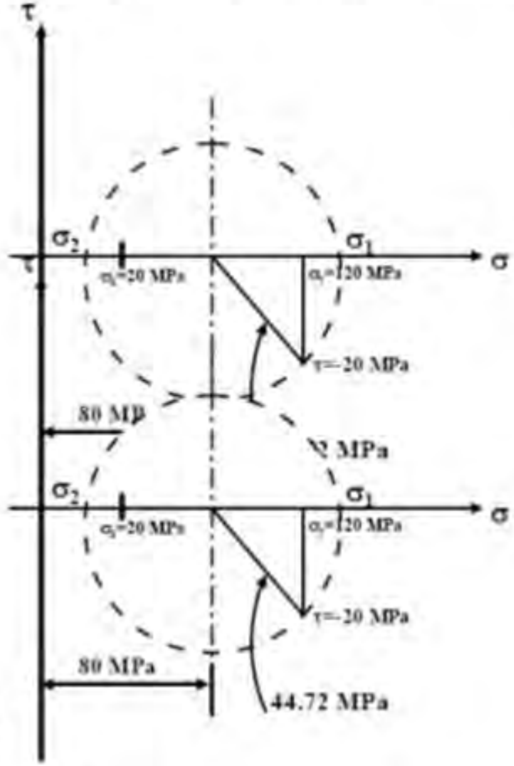
This gives F.S = 2.356.

(b) Maximum distortion energy theory

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 2(\sigma_Y / F.S)^2$$

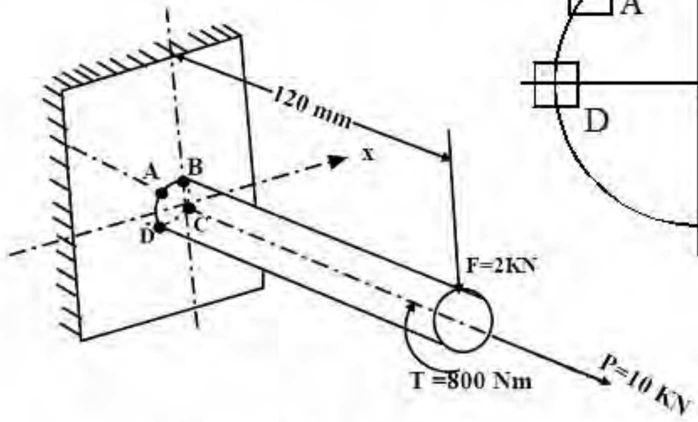
If σ_3 = 0 this gives F.S = 2.613.

Compare a) and b) we get $(F.S.)_M > (F.S.)_T$

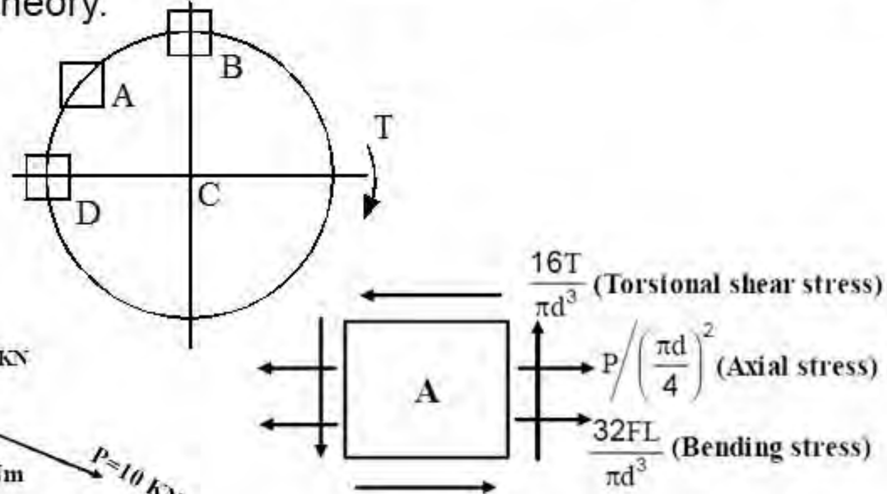


3.1.6.2

Q.3: A cantilever rod is loaded as shown in the **figure- 3.1.6.3**. If the tensile yield strength of the material is 300 MPa determine the rod diameter using (a) Maximum principal stress theory (b) Maximum shear stress theory (c) Maximum distortion energy theory.



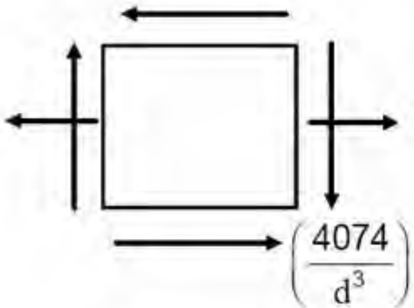
3.1.6.3



3.1.6.4

A.3: At the outset it is necessary to identify the mostly stressed element. Torsional shear stress as well as axial normal stress is the same throughout the length of the rod but the bearing stress is largest at the welded end. Now among the four corner elements on the rod, the element A is mostly loaded as shown in **figure-3.1.6.4**

Shear stress due to bending VQ/It is also developed but this is neglected due to its small value compared to the other stresses. Substituting values of T, P, F and L, the elemental stresses may be shown as in **figure-3.1.6.5**:



$$(\sigma_A + \tau_B) = \left(\frac{12732}{d^2} + \frac{2445}{d^3}\right)$$

$$\sigma_x = \tau - \text{torsional}$$

$$\sigma_y = \tau_B - \text{bending}$$

3.1.6.5

This gives the principal stress as

$$\sigma_{12} = \frac{1}{2}\left(\frac{12732}{d^2} + \frac{2445}{d^3}\right) \pm \sqrt{\frac{1}{4}\left(\frac{12732}{d^2} + \frac{2445}{d^3}\right)^2 + \left(\frac{4074}{d^3}\right)^2}$$

(a) Maximum principal stress theory, Setting $\sigma_1 = \sigma_Y$ we get d = 26.67 mm.

(b) Maximum shear stress theory,

Setting $\frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_Y}{2}$, we get d = 30.63 mm.

(c) Maximum distortion energy theory,

Setting $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 2(\sigma_Y)^2$

We get d = 29.36 mm.

3.1.7 Summary of this Lecture

Different types of loading and criterion for design of machine parts subjected to static loading based on different failure theories have been demonstrated. Development of yield surface and optimization of design criterion for ductile and brittle materials were illustrated.

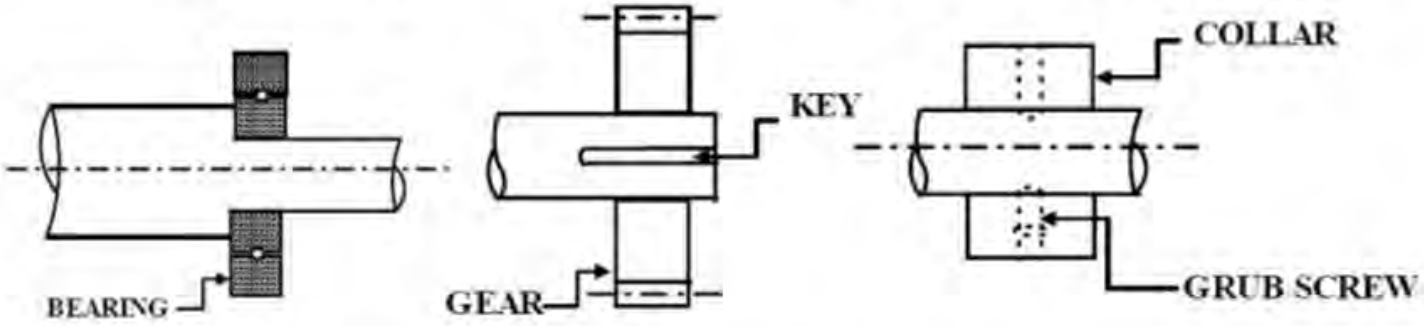
Lecture
Theme 3
Design for Strength

3.2. Stress Concentration

3.2.1 Introduction

Before we studed in what way the choice of material for machine parts affects (ufluences) to distribution of stresses and strain in elements of machines. Now we cansider in what way the schope of the machine element influences (affects) to distribution of stresses and strain in machine parts. Euch designer must be able to give out instructions what material to take for creation of element and what elements hape is optimum.

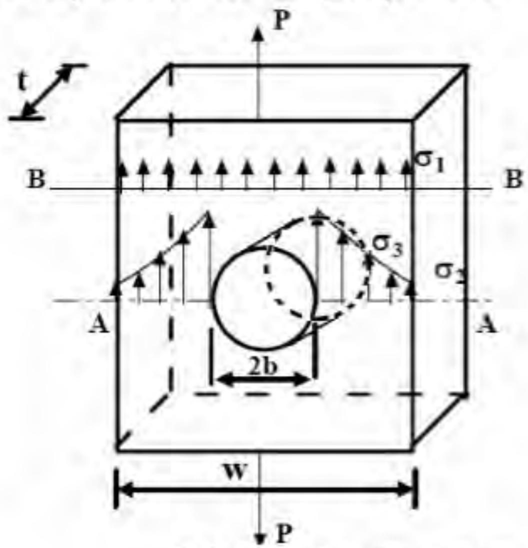
In developing a machine it is impossible to avoid changes in cross-section, holes,



3.2.1.1 - Some typical illustrations leading to stress concentration.

Any such discontinuity in a member affects the stress distribution in the neighbourhood and the discontinuity acts as a stress raiser.

Consider a plate with a centrally located hole and the plate is subjected to uniform tensile load at the ends. Stress distribution at a section A-A passing through the hole and another section BB away from the hole are shown in **figure- 3.2.1.2**. Stress distribution away from the hole is uniform but at AA there is a sharp rise in stress in the vicinity of the hole. Stress concentration factor k_t is defined as $k_t = \sigma_3/\sigma_{av}$, where σ_{av} at section AA is simply $P/t(w-2b)=P/tw$. This is the theoretical or geometric stress concentration factor and the factor is not affected by the material properties.



$$k_t = \frac{\sigma_3}{\sigma_{av}}$$

$$\sigma_{av} = \frac{P}{t(w-2b)} = \frac{P}{tw}$$

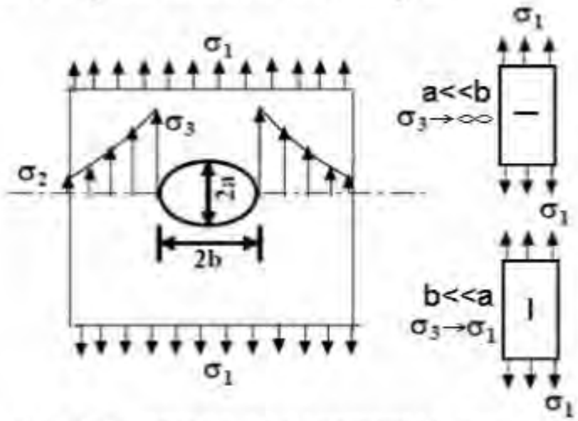
$$\boxed{\sigma_3 = \sigma_{max} = \sigma_{av} \cdot k_t}$$

3.2.1.2 - Stress concentration due to a central hole in a plate subjected to an uni-axial loading.

It is possible to predict the stress concentration factors for certain geometric shapes using theory of elasticity approach. For example, for an elliptical hole in an infinite plate, subjected to a uniform tensile stress σ_1 (**figure- 3.2.1.3**), stress z distribution around the discontinuity is disturbed and at points remote from the discontinuity the effect is insignificant. According to such an analysis

$$\sigma_3 = \sigma_1\left(1+\frac{2b}{a}\right)$$

If a=b the hole reduces to a circular one and therefore $\sigma_3 = 3\sigma_1$ which gives k_t=3. If, however 'b' is large compared to 'a' then the stress at the edge of transversecrack is very large and consequently k is also very large. If 'b' is small compared to a then the stress at the edge of a longitudinal crack does not rise and k_t =1.



3.2.1.3 - Stress concentration due to a central elliptical hole in a plate subjected to a uni-axial loading.

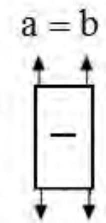
Stress concentration factors may also be obtained using any one of the following experimental techniques:

1. Strain gage method
2. Photoelasticity method
3. Brittle coating technique
4. Grid method

$$\sigma_{max} = \sigma_{av} \cdot k_t$$

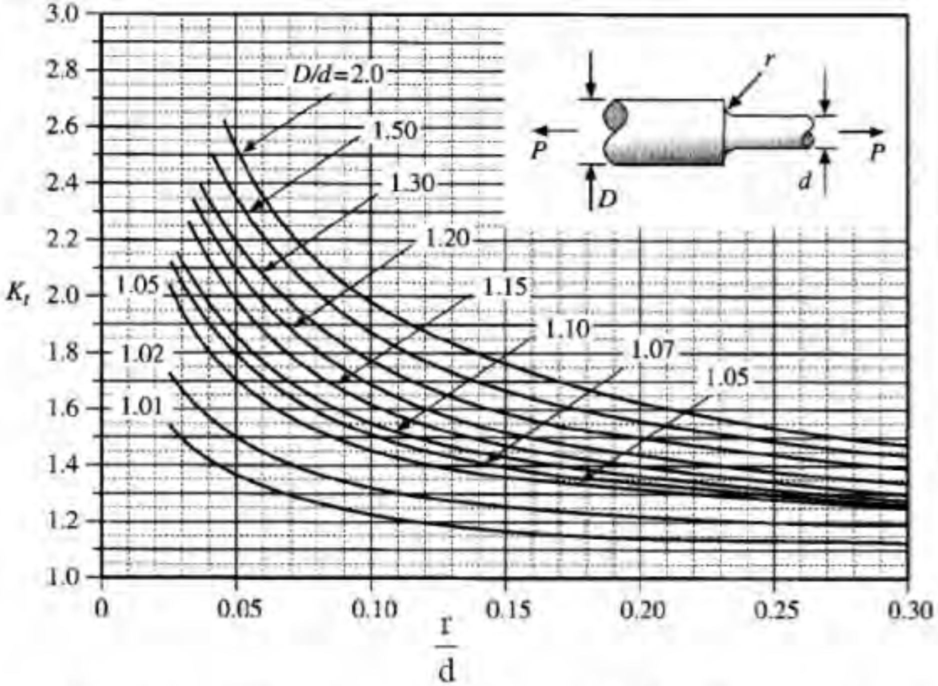
$$k_t = 1 \text{ without hole, } k_t \to \infty,\ b >> a$$

$$k_t = 3 \text{ round hole, } k_t \to 1,\ b << a$$



For more accurate estimation numerical methods like Finite element analysis may be employed.

Theoretical stress concentration factors for different configurations are available in handbooks. Some typical plots of theoretical stress concentration factors and r/d ratio for a stepped shaft are shown in **figure-3.2.1.4.**



3.2.1.4 - Variation of theoretical stress concentration factor with r/d of a stepped shaft for different values of D/d subjected to uni-axial loading.

$$\sigma_a = k_t \sigma_c$$

In design under fatigue loading, stress concentration factor is used in modifying the values of endurance limit while in design under static loading it simply acts as stress modifier. This means Actual stress σ_a= k_t × calculated stress σ_c. For ductile materials under static loading effect of stress concentration is not very serious but for brittle materials even for static loading it is important.

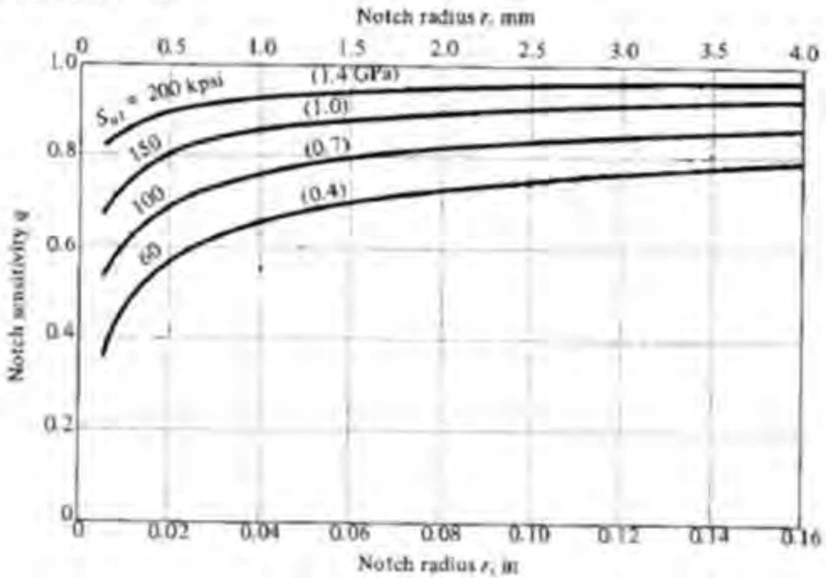
It is found that some materials are not very sensitive to the existence of notches or discontinuity. In such cases it is not necessary to use the full value of t k and instead a reduced value is needed. This is given by a factor known as fatigue strength reduction factor k_f and this is defined as

$$k_f = \frac{\text{Endurance limit of notch free specimens}}{\text{Endurance limit of notched specimens}} \qquad \boxed{k_f = \frac{\sigma_f}{\sigma_t}}$$

Another term called Notch sensitivity factor, q is often used in design and this is defined as

$$\boxed{q = \frac{k_f - 1}{k_t - 1}}$$

The value of 'q' usually lies between 0 and 1. If q=0, k_f =1 and this indicates no notch sensitivity. If however q=1, then f k_f = k_t and this indicates full notch sensitivity. Design charts for 'q' can be found in design hand-books and knowing k_t, k_f may be obtained. A typical set of notch sensitivity curves for steel is



3.2.1.5 - Variation of notch sensitivity with notch radius for steels of different ultimate tensile strength.

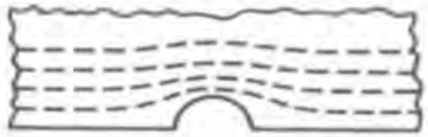
3.2.2 Methods of reducing stress concentration

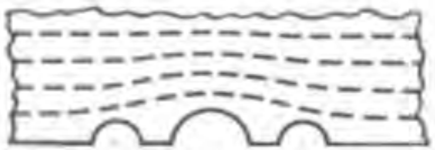
A number of methods are available to reduce stress concentration in machine parts. Some of them are as follows:

1. Provide a fillet radius so that the cross-section may change gradually.
2. Sometimes an elliptical fillet is also used.
3. If a notch is unavoidable it is better to provide a number of small notches rather than a long one. This reduces the stress concentration to a large extent.
4. If a projection is unavoidable from design considerations it is preferable to provide a narrow notch than a wide notch.
5. Stress relieving groove are sometimes provided.

These are demonstrated in **figure- 3.2.2.1.**

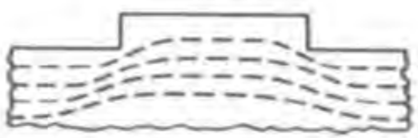
(a) Force flow around a sharp corner   Force flow around a corner with fillet: Low stress concentration.





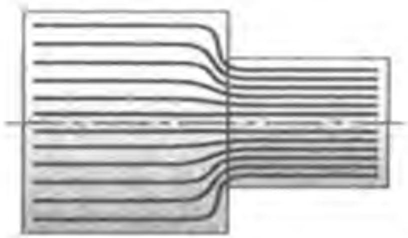
(b) Force flow around a large notch

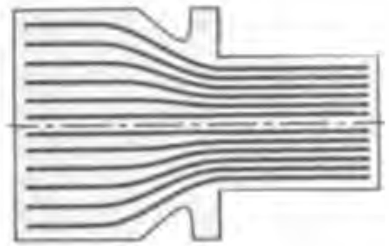
Force flow around a number of small notches: Low stress concentration.



(c) Force flow around a wide projection

Force flow around a narrow projection: Low stress concentration.





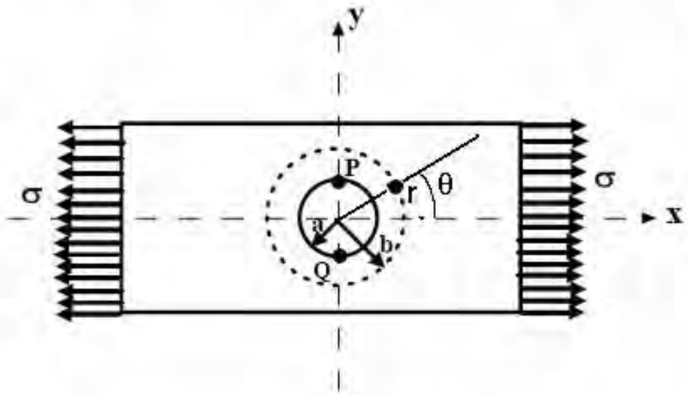
(d) Force flow around a sudden change in diameter in a shaft

Force flow around a stress relieving groove..

3.2.2.1 - Illustrations of different methods to reduce stress Concentration.

3.2.3 Theoretical basis of stress concentration

Consider a plate with a hole acted upon by a stress σ . St. Verant's principle states that if a system of forces is replaced by another statically equivalent system of forces then the stresses and displacements at points remote from the region concerned are unaffected. In **figure-3.2.3.1** 'a' is the radius of the hole and at r=b, b>>a the stresses are not affected by the presence of the hole.



3.2.3.1 - A plate with a central hole subjected to a uni-axial stress

Here, $\sigma_x = \sigma$, $\sigma_y = 0$, $\tau_{xy} = 0$

For plane stress conditions:
in polar coordinates system we get

$$\sigma_r = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta$$

$$\sigma_\theta = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$

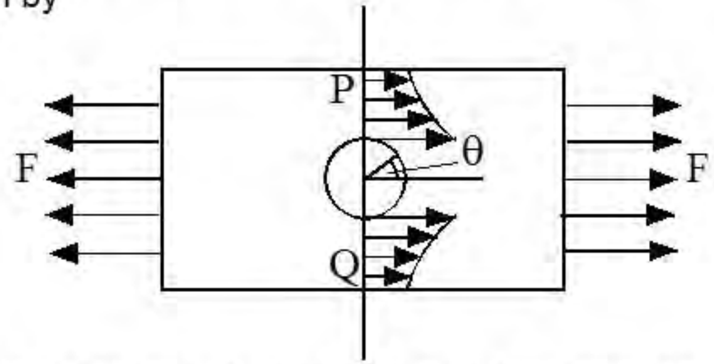
$$\tau_{r\theta} = (\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

We can this reduce to $\sigma_r = \sigma\cos^2\theta = \frac{\sigma}{2}(\cos 2\theta + 1) = \frac{\sigma}{2} + \frac{\sigma}{2}\cos 2\theta$

$$\sigma_\theta = \sigma\sin^2\theta = \frac{\sigma}{2}(1 - \cos 2\theta) = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta \qquad \tau_{r\theta} = -\frac{\sigma}{2}\sin 2\theta$$

such that 1st component in σ_r and σ_θ is constant and the second component varies with θ . Similar argument holds for $\tau_{r\theta}$ if we write $\tau_{r\theta} = (\sigma/2)\sin 2\theta$. The stress distribution within the ring with inner radius r_i=a and outer radius or r_o=b

due to 1st component can be analyzed using the solutions of thick cylinders and the effect due to the 2nd component can be analyzed following the Stress-function approach. Using a stress function of the form φ=R(r)cos2θ the stress distribution due to the 2nd component can be found and it was noted that the dominant stress is the Hoop Stress, given by



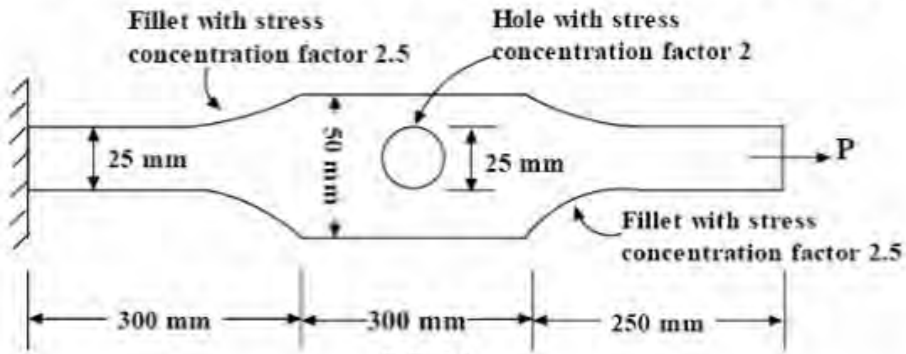
$$\sigma_\theta = \frac{\sigma}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta$$

This is maximum at $\theta = \pm\pi/2$ and the maximum value of $\sigma_\theta = \frac{\sigma}{2}\left(2 + \frac{a^2}{r^2} + \frac{3a^4}{r^4}\right)$

Therefore at points P and Q where r = a σ_θ is maximum and is given by $\sigma_\theta = 3\sigma$
i.e. stress concentration factor is 3. This result we had before when we got answer which is same to problem for a plate with a hole.

3.2.4 Problems with Answers

Q.1: The flat bar shown in **figure- 3.2.4.1** is 10 mm thick and is pulled by a force P producing a total change in length of 0.2 mm. Determine the maximum stress developed in the bar. Take E= 200 GPa.



3.2.4.1

$$t = 10mm$$
$$l = 0.2mm$$
$$E = 200GPa$$
$$\sigma_{max} = ?$$

A.1: Total change in length of the bar is made up of three components and this is given by

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$$

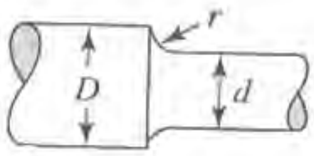
$$\Delta l = \frac{l_1 P_1}{S_1 E} + \frac{l_2 P_2}{S_2 E} + \frac{l_3 P_3}{S_3 E} = \left[\frac{0.3}{0.025x0.01} + \frac{0.3}{0.05x0.01} + \frac{0.25}{0.025x0.01}\right]\frac{P}{200x10^9} = 0.2x10^{-3}$$

This gives P=14.285 KN.

Stress at the shoulder $\sigma_s = k\frac{16666}{(0.05-0.025)x0.01}$ where k=2.

This gives σh = 114.28 MPa.

Q.2: Find the maximum stress developed in a stepped shaft subjected to a twisting moment of 100 Nm as shown in **figure- 3.2.4.2**. What would be the maximum stress developed if a bending moment of 150 Nm is applied.



r = 6 mm

d = 30 mm

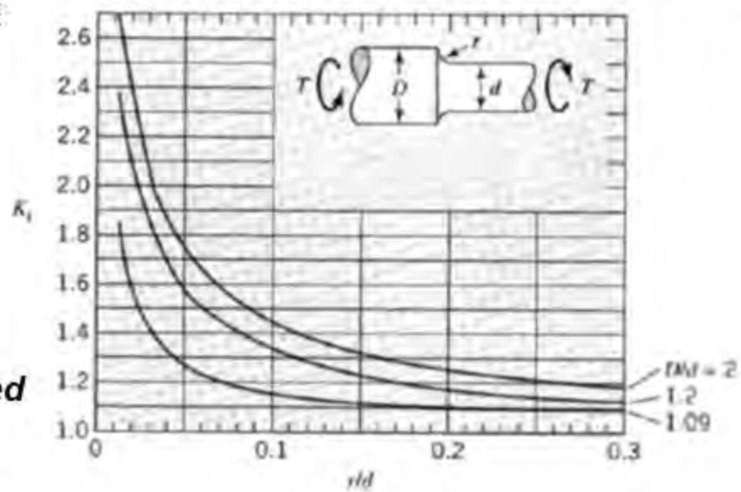
D = 40 mm.

3.2.4.2

A.2: Referring to the stress- concentration plots in **figure- 3.2.4.3** which we take from handbook, for stepped shafts subjected to torsion for r/d = 0.2 and D/d = 1.33, $K_t \approx 1.23$. Tby $\tau = \frac{16T}{\pi d^3}$. ear stress is given

Considering the smaller diameter and the stress concentration effect at the step, we ha $\tau_{max} = K_t \frac{16x100}{\pi(0.03)^3}$ hear stre

This gives τ_{max} = 23.201 MPa.



3.2.4.3 - Variation of theoretical stress concentration factor with r/d for a stepped shaft subjected to torsion.

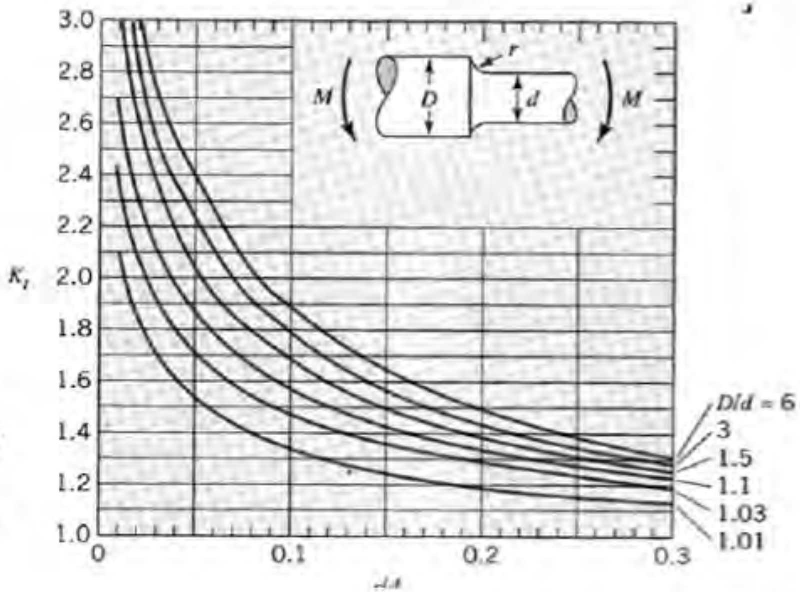
Similarly referring to stress-concentration plots in **figure- 3.2.4.4** which we take from handbook for stepped shaft subjected to bending , for r/d = 0.2 and D/d = 1.33, $K_t \approx 1.48$

Bending stress is given by $\sigma = \frac{32M}{\pi d^3}$

Considering the smaller diameter and the effect of stress concentration at the step, we have the maximum bending stress as

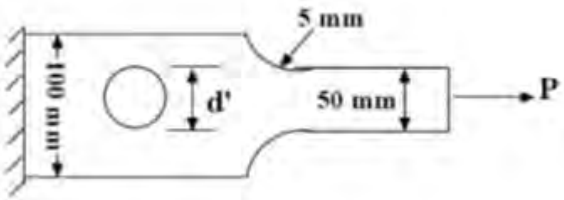
$$\sigma_{max} = K_t \frac{32x150}{\pi(0.03)^3}$$

This gives σ_{max} = 83.75 MPa.



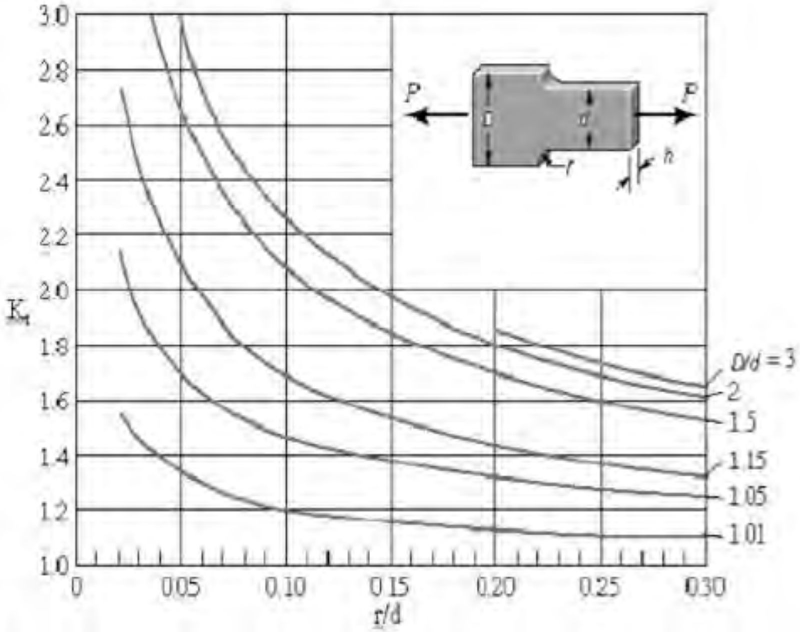
3.2.4.4 - Variation of theoretical stress concentration factor with r/d for a stepped shaft subjected to a bending moment.

Q.3: In the plate shown in **figure- 3.2.4.5** it is required that the stress concentration at Hole does not exceed that at the fillet. Determine the hole diameter.



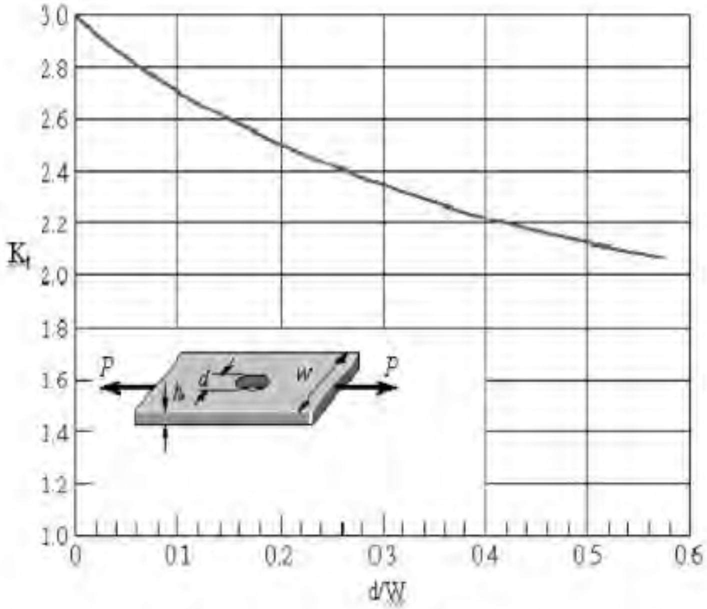
3.2.4.5

A.3: Referring to stress-concentration plots for plates with fillets under axial loading (figure- **3.2.4.6**) for r/d = 0.1 and D/d = 2, stress concentration factor, $K_t \approx 2.3$.



3.2.4.6 - Variation of theoretical stress concentration factor with r/d for a plate with fillets subjected to a uni-axial loading.

From stress concentration plots for plates with a hole of diameter 'd' under axial loading (**figure- 3.2.4.7**) we have for $K_t = 2.3$, $d'/D = 0.35$. This gives the hole diameter d' = 35 mm.



3.2.4.7 - Variation of theoretical stress concentration factor with d/W for a plate with a transverse hole subjected to a uni-axial loading.

3.2.5 Summary of this Lecture

Stress concentration for different geometric configurations and its relation to fatigue strength reduction factor and notch sensitivity have been discussed. Methods of reducing stress concentration have been demonstrated and a theoretical basis for stress concentration was considered.

Lecture

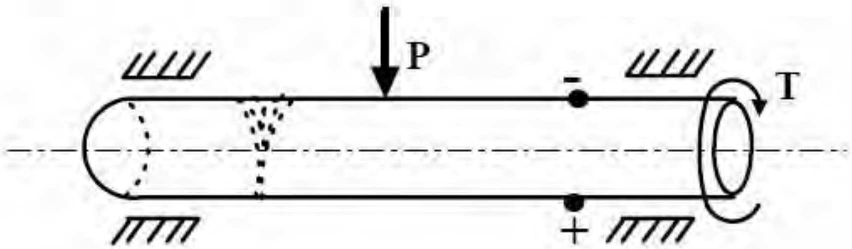
Theme 3

Design for Strength

3.3. Design for dynamic loading

3.3.1 Introduction

Conditions often arise in machines and mechanisms when stresses fluctuate between a upper and a lower limit. For example in **figure-3.3.1.1**, the fiber on the surface of a rotating shaft subjected to a bending load, undergoes both tension and compression for each revolution of the shaft.



3.3.1.1 *- Stresses developed in a rotating shaft subjected to a bending load.*

Any fiber on the shaft is therefore subjected to fluctuating stresses. Machine elements subjected to fluctuating stresses usually fail at stress levels much below their ultimate strength and in many cases below the yield point of the material too. These failures occur due to very large number of stress cycle and are known as fatigue failure.

These failures usually begin with a small crack
which may develop at the points of discontinuity, an existing subsurface crack or surface faults. Once a crack is developed it propagates with the increase in stress cycle finally leading to failure of the component by fracture. There are mainly two characteristics of this kind of failures:

(a) Progressive development of crack.

(b) Sudden fracture without any warning since yielding is practically absent.

Fatigue failures are influenced by

(1) Nature and magnitude of the stress cycle.

(2) Endurance limit.

(3) Stress concentration.

(4) Surface characteristics.

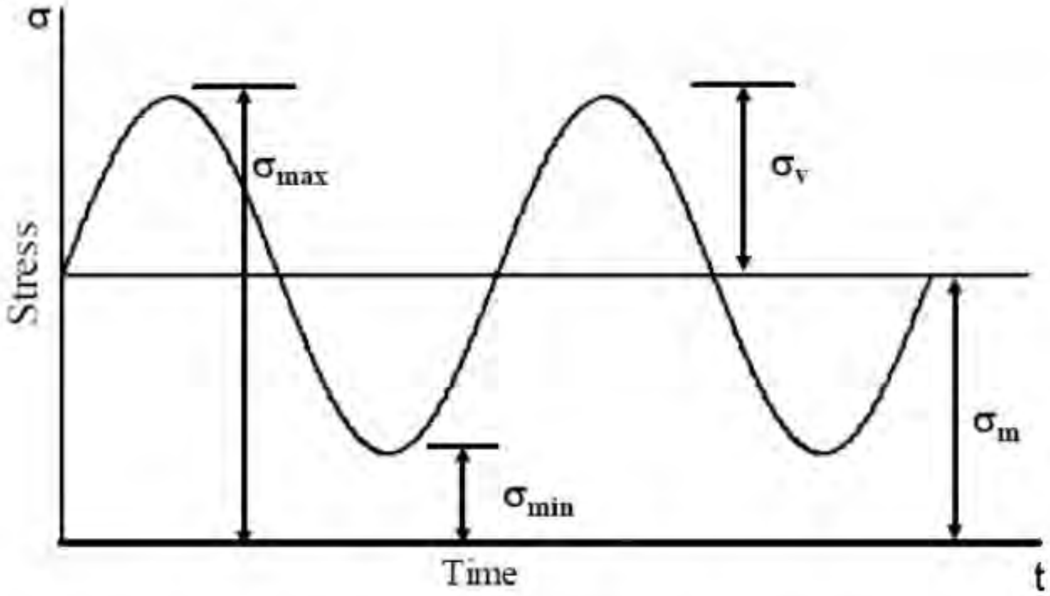
These factors are therefore interdependent. For example, by grinding and polishing, case hardening or coating a surface, the endurance limit may be improved. For machined steel endurance limit is approximately half the ultimate tensile stress. The influence of such parameters on fatigue failures will now be discussed in sequence.

3.3.2 Stress cycle

A typical stress cycle is shown in **figure- 3.3.2.1** where the maximum, minimum, mean and variable stresses are indicated. The mean and variable stresses are given by

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

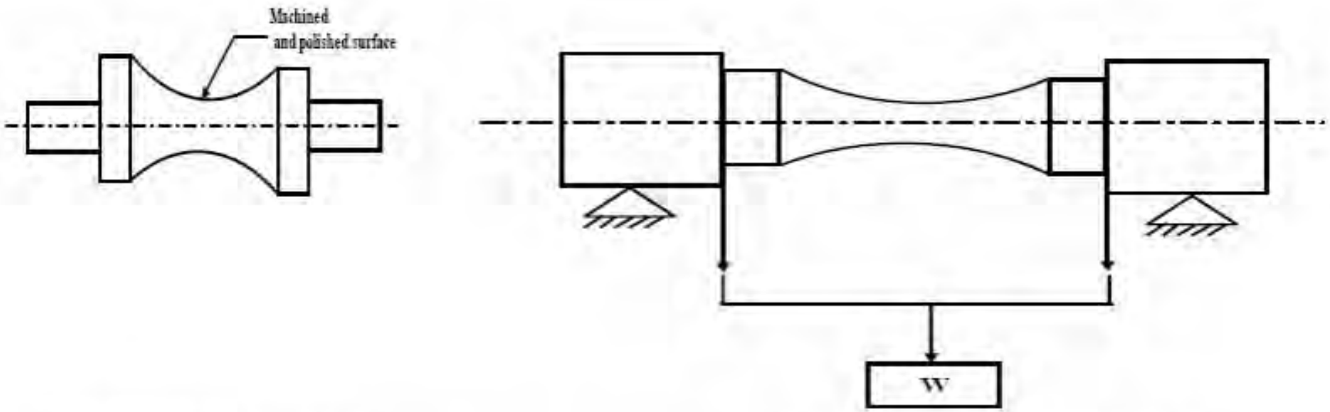
$$\sigma_{variable} = \frac{\sigma_{max} - \sigma_{min}}{2}$$



3.3.2.1 - A typical stress cycle showing maximum, mean and variable stresses.

3.3.3 Endurance limit

Figure- 3.3.3.1 shows the rotating beam arrangement along with the specimen.

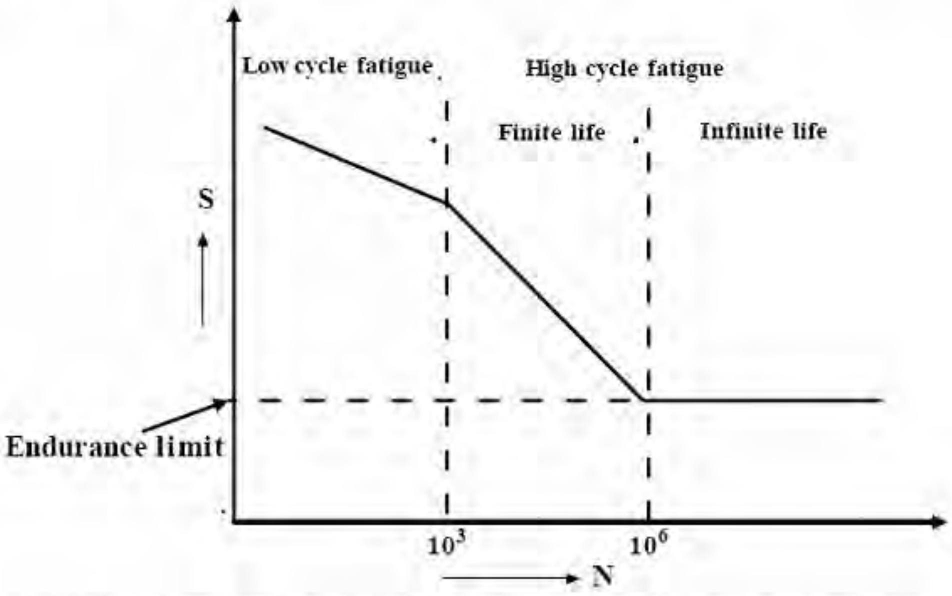


(a) Beam specimen (b) Loading arrangement

***3.3.3.1** - A typical rotating beam arrangement.*

The loading is such that there is a constant bending moment over the specimen ength and the bending stress is greatest at the center where the section is smallest. The arrangement gives pure bending and avoids transverse shear since bending moment is constant over the length. Large number of tests with varying bending loads are carried out to find the number of cycles to fail.

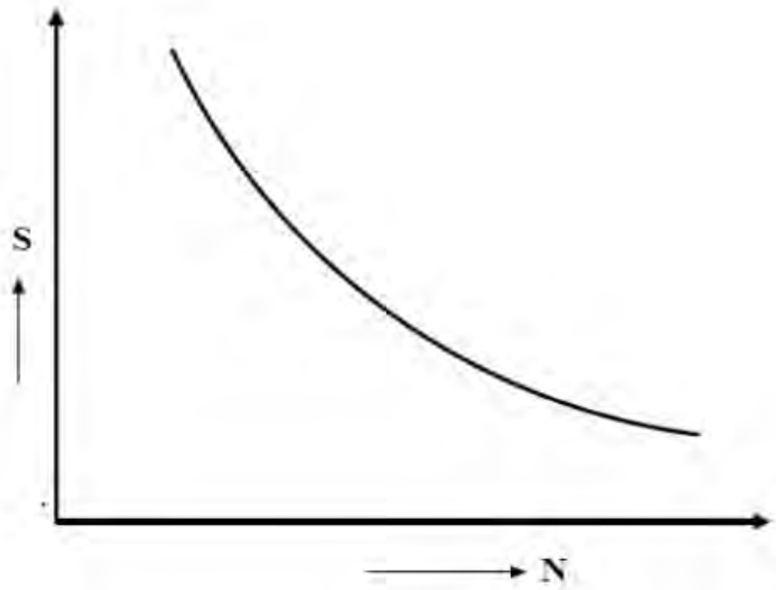
A typical plot of reversed stress (S) against number of cycles to fail (N) is shown in **figure-3.3.3.2**.The zone below 103 cycles is considered as low cycle fatigue, zone between 103 and 106 cycles is high cycle fatigue with finite life and beyond 106 cycles, the zone is considered to be high cycle fatigue with infinite life.



3.3.3.2 - A schematic plot of reversed stress (S) against number of cycles to fail (N) for steel.

The above test is for reversed bending. Tests for reversed axial, torsional or combined stresses are also carried out.For aerospace applications and non-metals axial fatigue testing is preferred.

For non-ferrous metals there is no knee in the curve as shown in **figure- 3.3.3.3** indicating that there is no specified transition from finite to infinite life.



3.3.3.3 *- A schematic plot of reversed stress (S) against number of cycles to fail (N) for non-metals, showing the absence of a knee in the plot.*

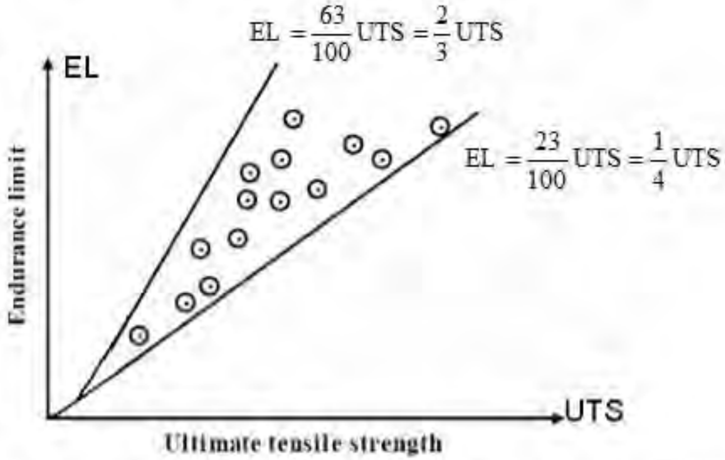
A schematic plot of endurance limit for different materials against the ultimate tensile strengths (UTS) is shown in **figure- 3.3.3.4**. The points lie within a narrow band and the following data is useful:

Steel Endurance limit ~ 35-60 % UTS

Cast Iron Endurance limit ~ 23-63 % UTS

The endurance limits are obtained from standard rotating beam experiments carried out under certain specific conditions. They need be corrected using a number of factors. In general the modified endurance limit σe' is given by

$$\sigma_e' = \sigma_e C_1C_2C_3C_4C_5 / K_f$$



3.3.3.4 - A schematic representation of the limits of variation of endurance limit with ultimate tensile strength.

1. C_1 is the size factor and the values may roughly be taken as

1. $C_1 = 1$, $d \le 7.6mm$
2. $C_1 = 0.85$, $7.6mm \le d \le 50mm$
3. $C_1 = 0.75$, $d \ge 50mm$

For large size $C_1 = 0.6$. Then data applies mainly to cylindrical steel parts.

2. C_2 is the loading factor and the values are given as

$C_2 = 1$, for reversed bending load.

$C_2 = 0.85$, for reversed axial loading for steel parts

$C_2 = 0.78$, for reversed torsional loading for steel parts.

3. C_3 is the surface factor and since the rotating beam specimen is given a mirror polish the factor is used to suit the condition of a machine part.
Since machining process rolling and forging contribute to the surface quality the plots of C_3 versus tensile strength or Brinnel hardness number for different production process, in **figure-3.3.3.5**, is useful in selecting the value of C_3=C_{sur}. Fig 3.3.3.5 we take from handbol

4. C_4 is the temperature factor and the values may be taken as follows:

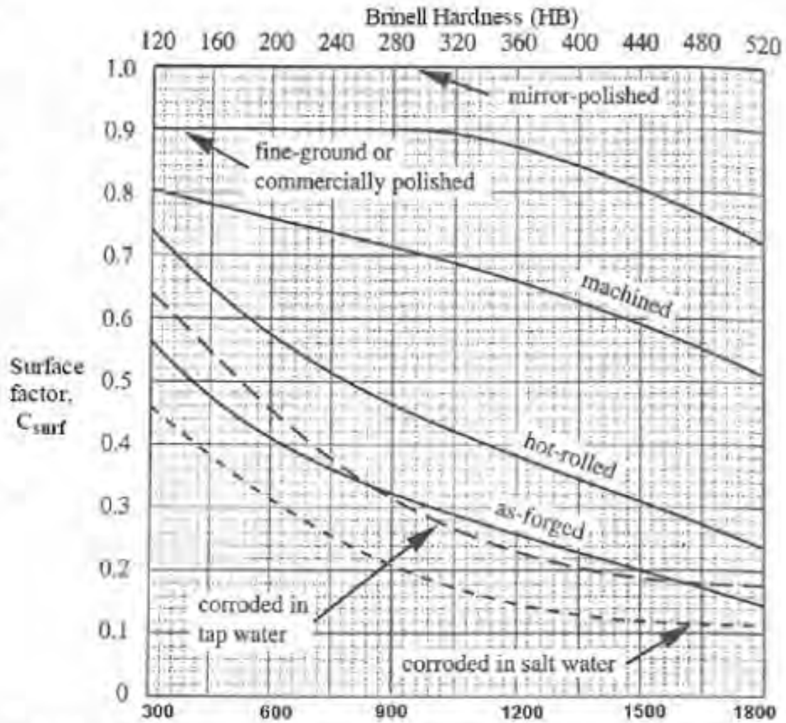
C_4 = 1, for $T \le 450^oC$

C_4= 1-0.0058(T-450) for $450^o < T \le 550^oC$

5. C_5 is the reliability factor and this is related to reliability percentage as follows:

Reliability %	C_5
50	1
90	0.897
99.99	0.702

6. K_f is the fatigue stress concentration factor, discussed in the next section.



3.3.3.5 - Variation of surface factor with tensile strength and Brinnel hardness for steels with different surface conditions (Ref.[2]).

3.3.4 Stress concentration

Stress concentration has been discussed in earlier lessons. However, it is important to realize that stress concentration affects the fatigue strength of machine parts severely and therefore it is extremely important that this effect be considered in designing machine parts subjected to fatigue loading. This is done by using fatigue stress concentration factor defined as

$$k_f = \frac{\text{Endurance limit of a notch free specimen}}{\text{Endurance limit of a notched specimen}}$$

The notch sensitivity 'q' for fatigue loading can now be defined in terms of k_f and the theoretical stress concentration factor k_t and this is given by

$$q = \frac{k_f - 1}{k_t - 1}$$

The value of q is different for different materials and this normally lies between 0 to 0.7. The index is small for ductile materials and it increases as the ductility decreases. Notch sensitivities of some common materials are given in **table- 3.3.4.1** . Which we can take from handbook

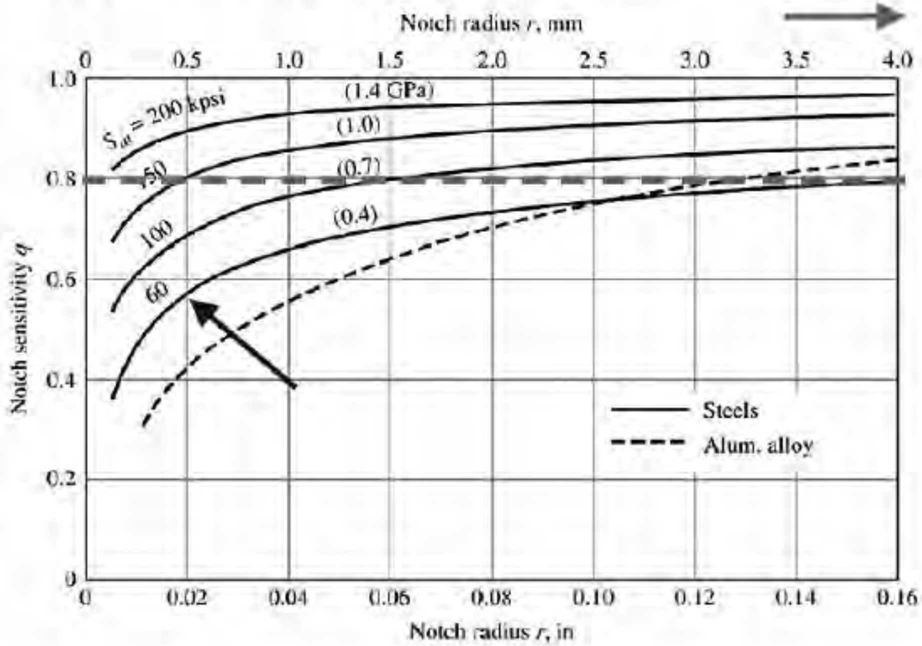
3.3.4.1 - Notch sensitivity of some common engineering materials.

Material	Notch sensitivity index
C-30 steel- annealed	0.18
C-30 steel- heat treated and drawn at 480°C	0.49
C-50 steel- annealed	0.26
C-50 steel- heat treated and drawn at 480°C	0.50
C-85 steel- heat treated and drawn at 480°C	0.57
Stainless steel- annealed	0.16
Cast iron- annealed	0.00-0.05
copper- annealed	0.07
Duraluminium- annealed	0.05-0.13

Notch sensitivity index q can also be defined as

$$q = \frac{1}{1 + \left(\frac{a}{r}\right)^{1/2}} \qquad q = \frac{k_f - 1}{k_t + 1}$$

where, $\sqrt{a}$ is called the Nubert's constant that depends on materials and their heat treatments. A typical variation of q against notch radius r is shown in **figure-3.3.4.2** . This date we can take in handbook.



3.3.4.2 *- Variation of notch sensitivity with notch radius for steel and aluminium alloy with different ultimate tensile strengths*

3.3.5 Surface characteristics

Fatigue cracks can start at all forms of surface discontinuity and this may include surface imperfections due to machining marks also. Surface roughness is therefore an important factor and it is found that fatigue strength for a regular surface is relatively low since the surface undulations would act as stress raisers.

It is, however, impractical to produce very smooth surfaces at a higher machining cost.

Another important surface effect is due to the surface layers which may be extremely thin and stressed either in tension or in compression. For example, grinding process often leaves surface layers highly stressed in tension. Since fatigue cracks are due to tensile stress and they propagate under these conditions and the formation of layers stressed in tension must be avoided. There are several methods of introducing pre-stressed surface layer in compression and they include shot blasting, peening, tumbling or cold working by rolling. Carburized and nitrided parts also have a compressive layer which imparts fatigue strength to such components. Many coating techniques have evolved to remedy the surface effects in fatigue strength reductions.

3.3.6 Problems with Answers

Q.1: A rectangular stepped steel bar is shown in **figure-3.3.6.1**. The bar is loaded in bending. Determine the fatigue stress-concentration factor if ultimate stress of the materials is 689 MPa.

r = 5mm
D = 50 mm
d = 40 mm
b = 1 mm

3.3.6.1

A.1:

From the geometry r/d = 0.125 and D/d = 1.25.

From the stress concentration chart in **figure- 3.2.4.6**

Stress- concentration factor $k_f \approx 1.7$

From **figure- 3.3.4.2**

Notch sensitivity index, $q \approx 0.88$

The fatigue stress concentration factor k_f is now given by $q = \frac{k_f - 1}{k_t - 1}$

$k_f = 1 + q(k_t - 1) = 1.616$

3.3.7 Summary of this Lecture

Design of components subjected to dynamic load requires the concept of variable stresses, endurance limit, low cycle fatigue and high cycle fatigue with finite and infinite life. The relation of endurance limit with ultimate tensile strength is an important guide in such design. The endurance limit needs be corrected for a number of factors such as size, load, surface finish, temperature and reliability. The methods for finding these factors have been discussed and demonstrated in an example.

Lecture

Theme 4

Fasteners

4.1. Types of fasteners: Pins and keys

4.1.1 Introduction: Types of fasteners

A machine or a structure is made of a large number of parts and they need be joined suitably for the machine to operate satisfactorily. Parts are joined by fasteners and they are conveniently classified as permanent or detachable fasteners. They are often sub- divided under the main headings as follows:

Permanent fasteners: Riveted joints
Welded joints

Detachable joints: Threaded fasteners – screws, bolts and nuts, studs.
Cotter joints
Knuckle joints

Keys and Pin joints

Starting with the simple pin and key joints all the main fasteners will be discussed here.

4.1.2 Pin Joints

These are primarily used to prevent sliding of one part on the other, such as, to secure wheels, gears, pulleys, levers etc. on shafts. Pins and keys are primarily used to transmit torque and to prevent axial motion. In engineering practice the following types of pins are generally used.

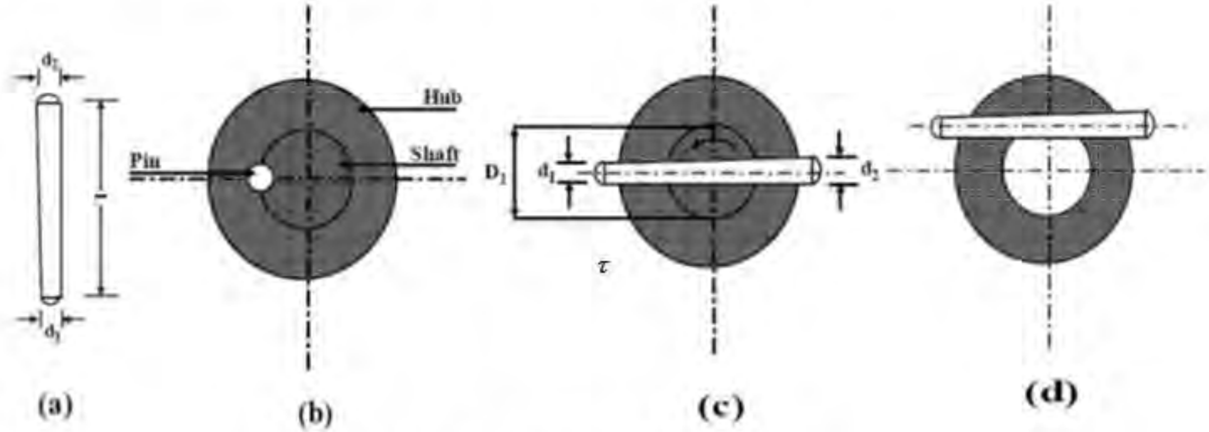
(a) Round pins (b) Taper pins (c) Dowel pins (d) Split pins

Round and taper pins are simple cylindrical pins with or without a taper and they offer effective means of fastening pulleys, gears or levers to a shaft. It may be fitted such that half the pin lies in the hub and the other half in the shaft as shown in **figure-4.1.2.1 (b)**. The pin may be driven through the hub and the shaft as in **figure- 4.1.2.1 (c)** or as in **figure- 4.1.2.1 (d)**. These joints give positive grip and the pins are subjected to a shear load. For example, for the shaft in the assembly shown in **figure- 4.1.2.1 (c)**, the pin is under double shear and we have

$$\tau\left(2\frac{\pi}{4}d^2\right).\frac{D_1}{2} = T$$

where d is the diameter of the pin at hub-shaft interface, τis the yield strength in shear of the pin material and T is the torque transmitted.

where d is the diameter of the pin at hub-shaft interface, τ is the yield strength in shear of the pin material and is the torque transmitted.

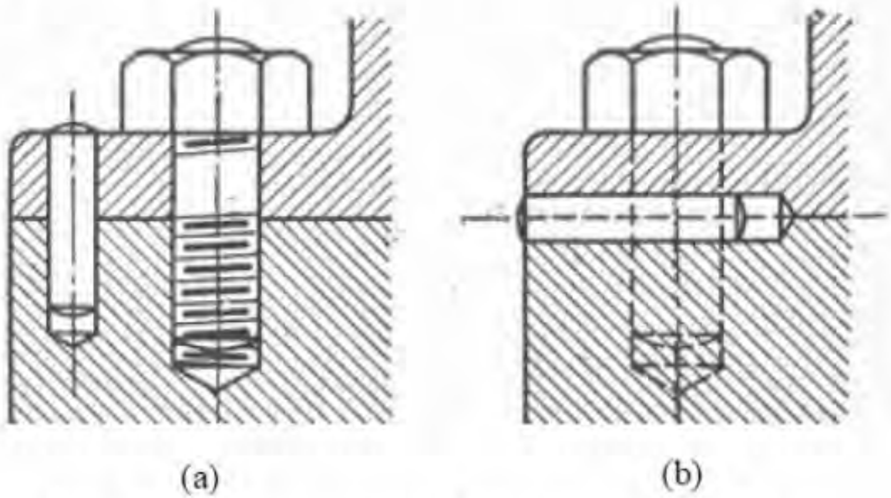


4.1.2.1 *- Different types of pin joints*

A taper pin is preferred over the straight cylindrical pins because they can be driven easily and it is easy to ream a taper hole.

Dowel pins

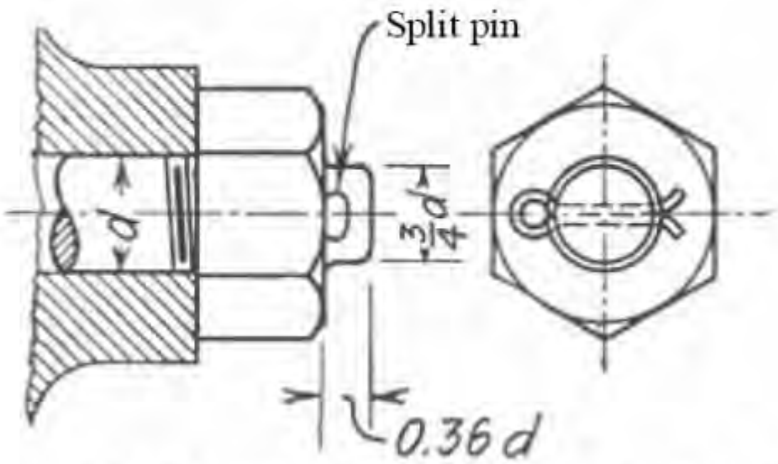
These are used to keep two machine parts in proper alignment. **Figure-4.1.2.2** demonstrates the use of dowel pins. Small cylindrical pins are normally used for this purpose.



4.1.2.2 *- Some uses of Dowel pins (Ref.[6]).*

Split pins

These are sometimes called cotter pins also and they are made of annealed iron or brass wire. They are generally of semi-circular cross section and are used to prevent nuts from loosening as shown in **figure- 4.1.2.3**. These are extensively used in automobile industry.



4.1.2.3 *- Typical use of a split pin (Ref.[6]).*

4.1.3 Keys

Steel keys are widely used in securing machine parts such as gears and pulleys. There is a large variety of machine keys and they may be classified under four broad headings:
Sunk keys, flat keys, saddle keys and pins or round keys

Sunk keys may be further classified into the following categories:

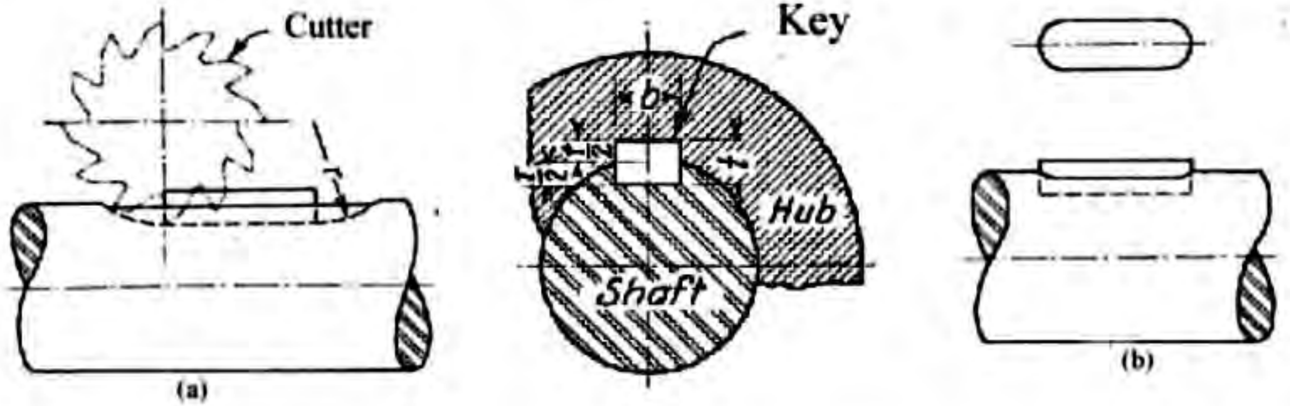
(a) Rectangular sunk keys

(b) Gib head sunk keys

(c) Feather keys

(d) Woodruff keys

Rectangular sunk keys are shown in **figure- 4.1.3.1**. They are the simplest form of machine keys and may be either straight or slightly tapered on one side. The parallel side is usually fitted into the shaft.



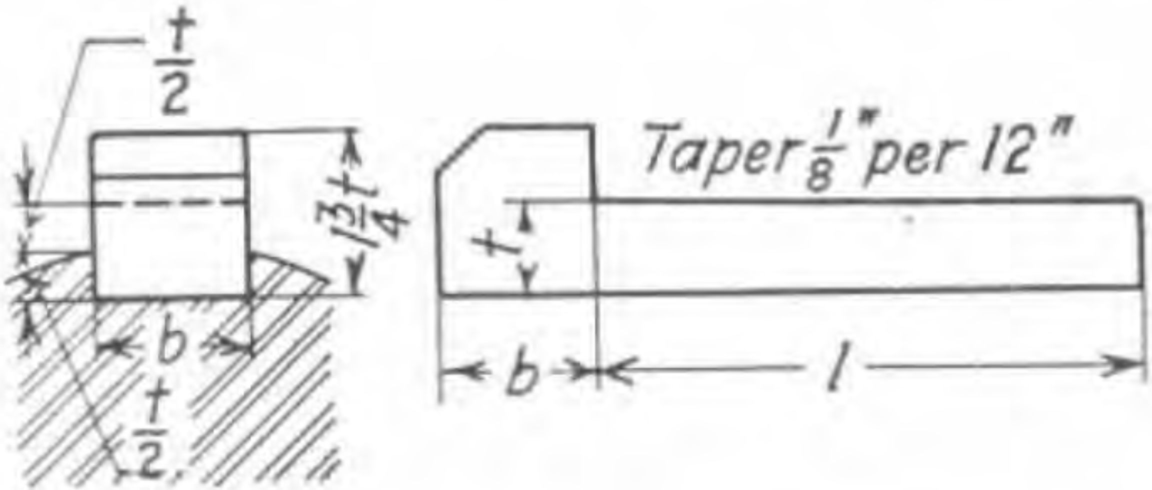
4.1.3.1 *- Rectangular sunk keys (Ref.[6]).*

The slots are milled as shown in **figure- 4.1.3.1(a)**. While transmitting torque a rectangular sunk key is subjected to both shear and crushing or bearing stresses.

Considering shear we may write $\tau.b.l.\frac{D}{2} = T$ where τ is the yield shear stress of the key material, D the shaft diameter and T is torque transmitted. Considering bearing stress we may write $\sigma_{br}.\frac{t.l}{2}.\frac{D}{2} = T$

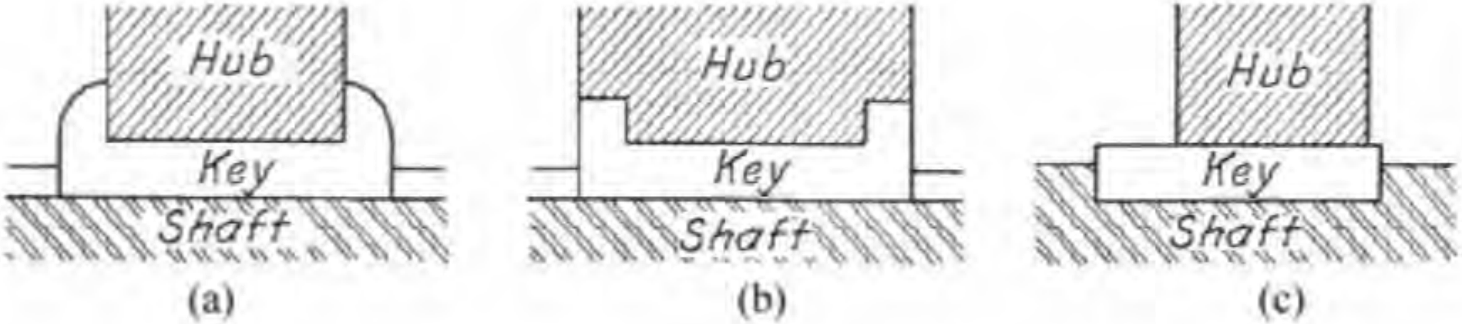
where σ_{br} is the bearing stress developed in the key. Based on these two criteria key dimensions may be optimized and compared with the standard key dimensions available in design hand books.

The **gib head keys** are ordinary sunk keys tapered on top with a raised head on one side so that its removal is easy. This is shown in **figure- 4.1.3.2**



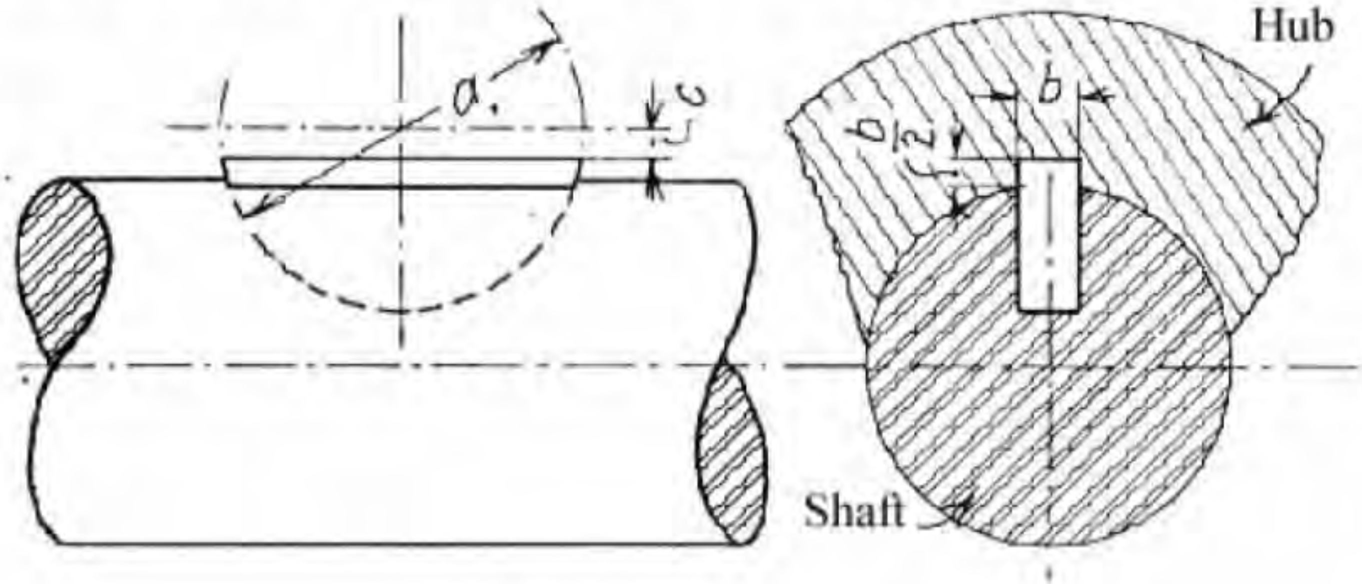
4.1.3.2 *- Gib head key (Ref.[6]).*

Some **feather key** arrangements are shown in **figure- 4.1.3.3**. A feather key is used when one component slides over another. The key may be fastened either to the hub or the shaft and the keyway usually has a sliding fit.



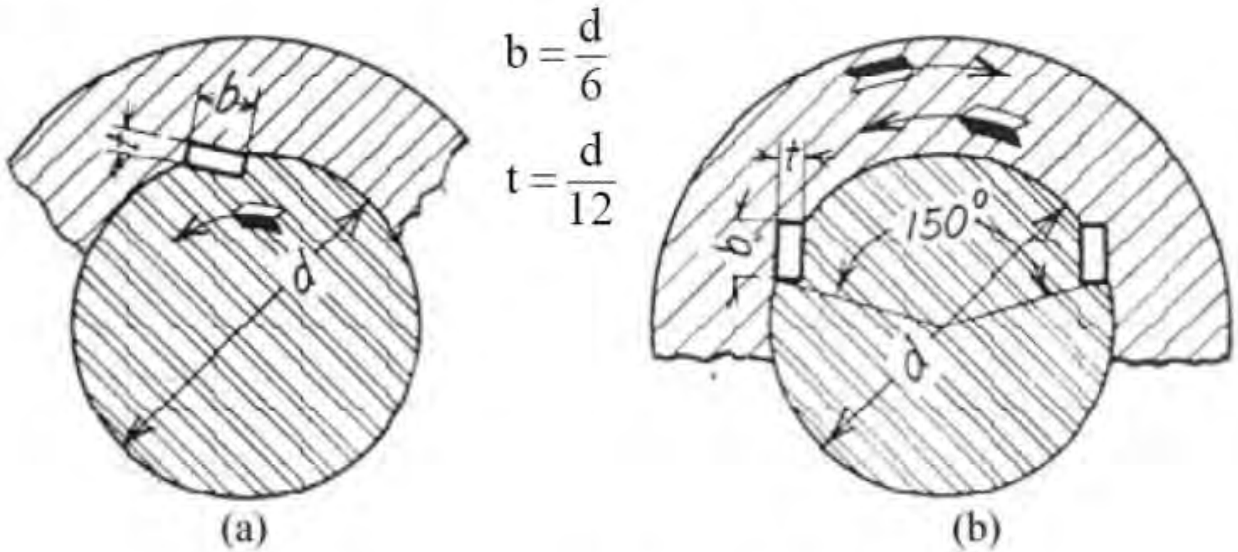
4.1.3.3 *- Some feather key arrangements (Ref.[6]).*

A **woodruff key** is a form of sunk key where the key shape is that of a truncated disc, as shown in **figure- 4.1.3.4**. It is usually used for shafts less than about 60 mm diameter and the keyway is cut in the shaft using a milling cutter, as shown in the **fiaure- 4.1.3.4**. It is widelv used in machine tools and automobiles due to



4.1.3.4 *- Woodruff key (Ref.[6]).*

Lewis keys, shown in **figure- 4.1.3.5**, are expensive but offer excellent service. They may be used as a single or double key. When they are used as a single key the positioning depends on the direction of rotation of the shaft. For heavy load two keys can be used as shown in **figure- 4.1.3.5 (b)**.

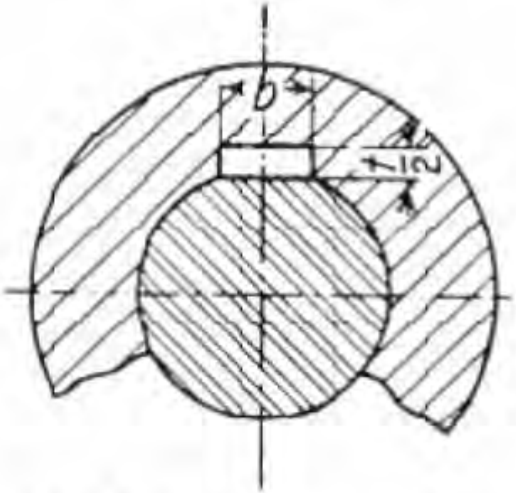


$$b = \frac{d}{6}$$

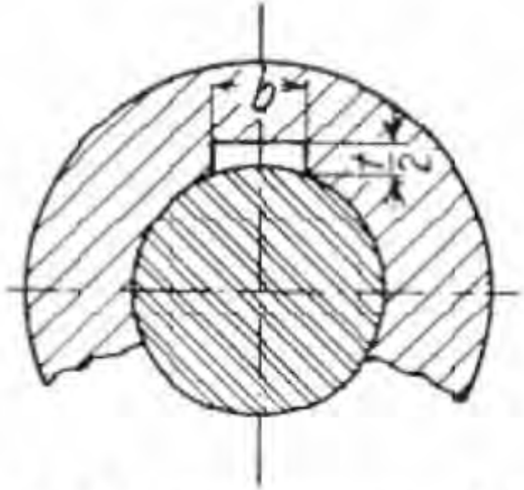
$$t = \frac{d}{12}$$

4.1.3.5 *- Lewis keys (Ref.[6]).*

A **flat key**, as shown in **figure- 4.1.3.6** is used for light load because they depend entirely on friction for the grip. The sides of these keys are parallel but the top is
slightly tapered for a tight fit. Theses keys have about half the thickness of sunk keys.

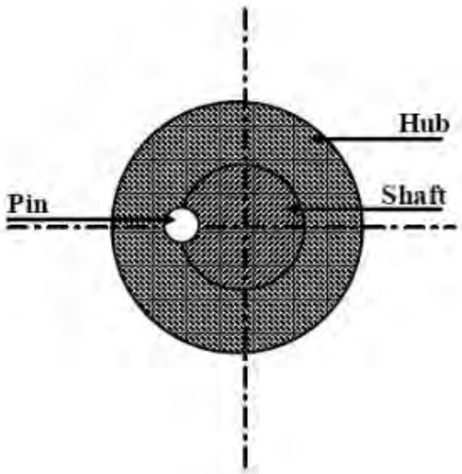


4.1.3.6 *- Flat key (Ref.[6])*.



4.1.3.7 *- Saddle key (Ref.[6]).*

A **saddle key**, shown in **figure- 4.1.3.7**, is very similar to a flat key except that the bottom side is concave to fit the shaft surface. These keys also have friction grip and therefore cannot be used for heavy loads. A simple pin can be used as a key to transmit large torques. Very little stress concentration occurs in the shaft in these cases. This is shown in **figure- 4.1.3.8 (b).**

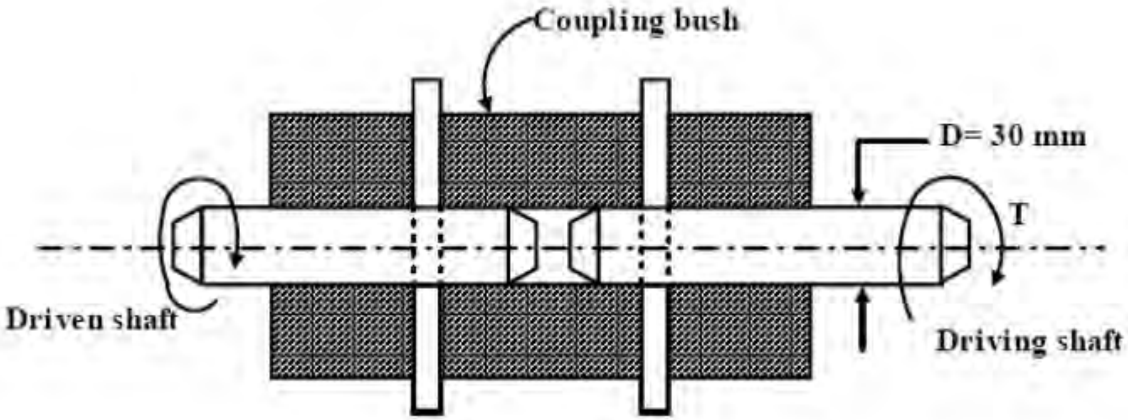


(b)

4.1.3.8.

4.1.4 Problems with Answers

Q.1: Two 30 mm diameter shafts are connected by pins in an arrangement shown in **figure-4.1.4.1**. Find the pin diameter if the allowable shear stress of the pins is 100 MPa and the shaft transmits 5 kW at 150 rpm.



4.1.4.1.

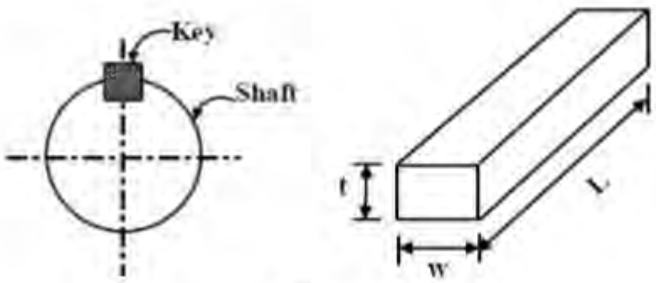
A.1:

The torque transmitted T= Power/$\left(\frac{2\pi N}{60}\right)$. Substituting power = $5x10^3$ Watts and N=150 rpm we have T = 318.3 Nm. The torque is transmitted from the driving shaft to the coupling bush via a pin. The torque path is then reversed and it is transmitted from the coupling bush to the driven shaft via another pin. Therefore both the pins transmit a torque of 318.3 Nm under double shear. We may then write $T = 2.\frac{\pi}{4}.d^2.\tau_y.\frac{D}{2}$. Substituting D=0.03 m, τ_y = 100 MPa and T= 318.3 MPa we have d=11.6 mm ≈ 12 mm.

Q.2: A heat treated steel shaft of tensile yield strength of 350 MPa has a diameter of 50 mm. The shaft rotates at 1000 rpm and transmits 100 kW through a gear. Select an appropriate key for the gear.

A.2:

Consider a rectangular key of width w, thickness t and length L as shown in **figure- 4.1.4.1**. The key may fail (a) in shear or (b) in crushing.



4.1.4.1.

Shear failure: The failure criterion is $T = \tau_y . w . L . \frac{d}{2}$

where torque transmitted is $T = \text{Power} / \left(\frac{2\pi N}{60}\right)$

respectively and τ_y is the yield stress in shear of the key material.

τ_y to be half of the tensile yield stress and substituting the values in equations (1) and (2) we have wL = 2.19 x 10^{-4} m^2.

Crushing failure: $T = \sigma_c . \frac{t.L}{2} . \frac{d}{2}$

Taking σ_c to be the same as σ_y and substituting values in equation (3) we have

tL= 2.19 x 10^{-4} m^2. Some standard key dimensions are reproduced in **table- 4.1.4.1**:

Shaft Diameter (mm)	30-38	38-44	44-50	50-58	58-65	65-75	75-85
Key width, w (mm)	10	12	14	16	18	20	22
Key depth, t (mm)	8	9	9	10	11	12	14
Key length, L (mm)	22-110	28-140	36-160	45-180	50-200	56-220	63-250

4.1.4.1

Based on the standard we may choose w=16 mm. This gives L = 13.6 mm. We may then choose the safe key dimensions as

w = 16 mm   L = 45 mm   t = 10 mm.

4.1.5 Summary of this Lesson

In this lesson firstly the types detachable of fasteners are discussed. Then types and applications of pin and key joints are discussed with suitable illustrations. A brief overview of key design is also included.

Lecture
Theme 4
Fasteners

4.2. Cotter and knuckle joint

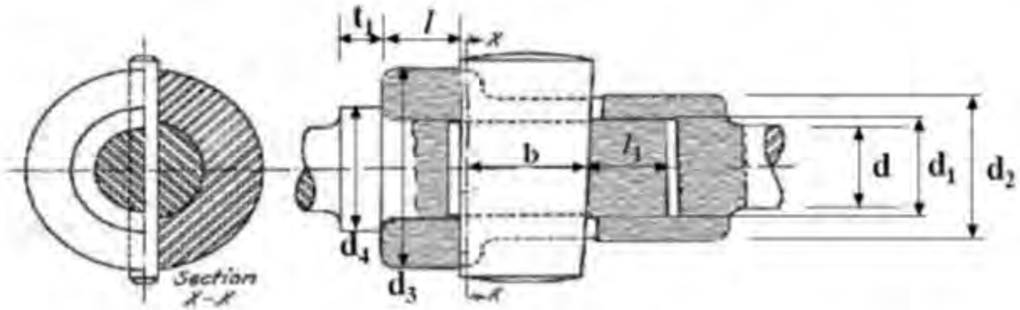
4.2.1 Cotter joint

A cotter is a flat wedge-shaped piece of steel as shown in **figure-4.2.1.1.** This is used to connect rigidly two rods which transmit motion in the axial direction, without rotation. These joints may be subjected to tensile or compressive forces along the axes of the rods.

Examples of cotter joint connections are: connection of piston rod to the crosshead of a steam engine, valve rod and its stem etc.

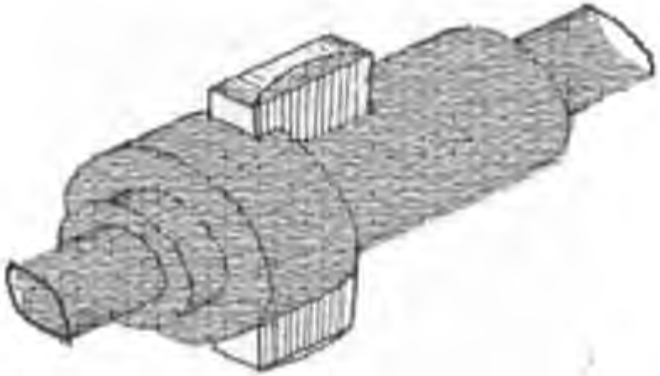
4.2.1.1 *- A typical cotter with a taper on one side only.*

A typical cotter joint is as shown in **figure-4.2.1.2**. One of the rods has a socket end into which the other rod is inserted and the cotter is driven into a slot, made in both the socket and the rod. The cotter tapers in width (usually 1:24) on one side only and when this is driven in, the rod is forced into the socket. However, if the taper is provided on both the edges it must be less than the sum of the friction angles for both the edges to make it self locking i.e $\alpha_1+\alpha_2<\varphi_1+\varphi_2$ where α_1, α_2 are the angles of taper on the rod edge and socket edge of the cotter respectively and φ_1, φ_2 are the corresponding angles of friction.



4.2.1.2 *- Cross-sectional views of a typical cotter joint*

This also means that if taper is given on one side only then $\alpha<\varphi_1+\varphi_2$ for self locking. Clearances between the cotter and slots in the rod end and socket allows the driven cotter to draw together the two parts of the joint until the socket end comes in contact with the cotter on the rod end.



***4.2.1.3** - An isometric view of a typical cotter joint.*

4.2.2 Design of a cotter joint

If the allowable stresses in tension, compression and shear for the socket, rod and cotter be σ_t, σ_c and τ respectively, assuming that they are all made of the same material, we may write the following failure criteria:

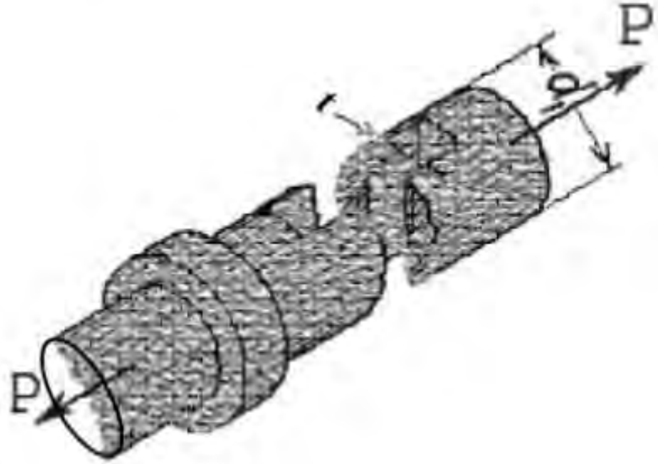
1. Tension failure of rod at diameter d

$$\frac{\pi}{4} d^2 \sigma_t = P$$

4.2.2.1 *- Tension failure of the rod/*

2. Tension failure of rod across slot

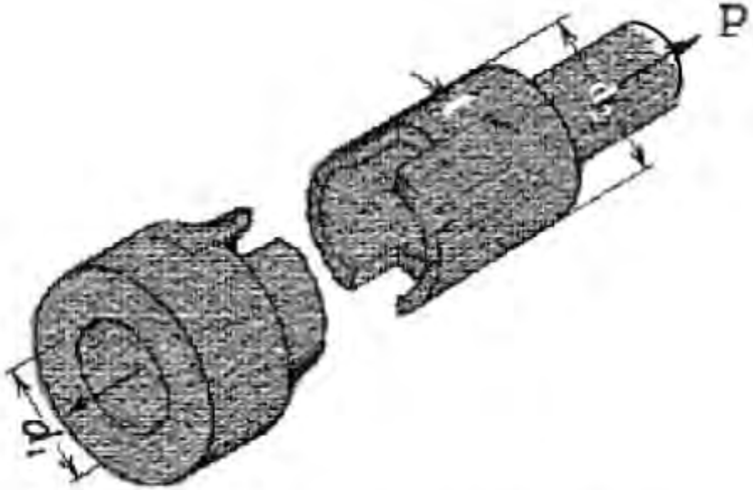
$$\left(\frac{\pi}{4}d_1^2 - d_1 t\right)\sigma_t = P$$



4.2.2.2 *- Tension failure of rod across slot*

3. Tensile failure of socket across slot

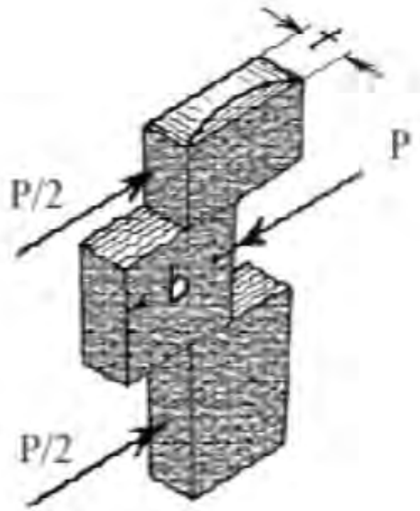
$$\left(\frac{\pi}{4}(d_2^2 - d_1^2) - (d_2 - d_1)t\right)\sigma_t = P$$



4.2.2.3 *- Tensile failure of socket across slot*

4. Shear failure of cotter

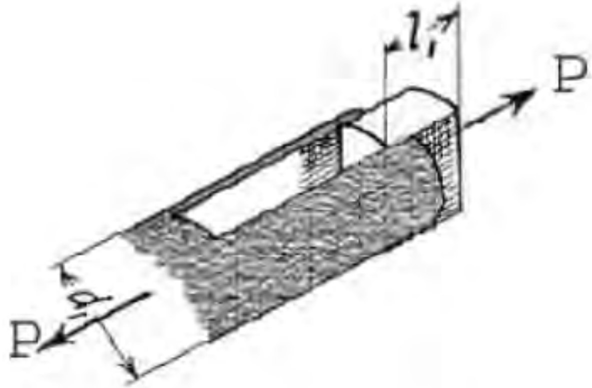
$$2bt\tau = P$$



4.2.2.4 *- Shear failure of cotter*

5. Shear failure of rod end

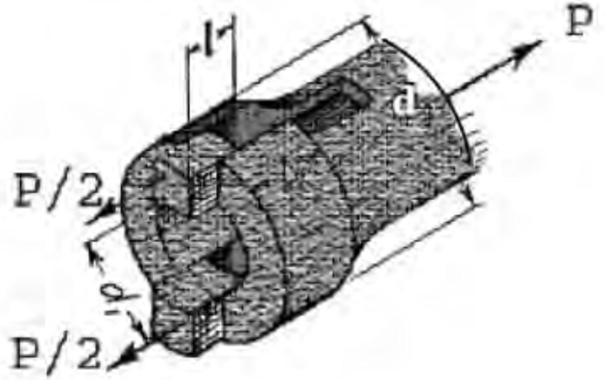
$$2l_1 d_1 \tau = P$$



4.2.2.5 *- Shear failure of rod end*

6. Shear failure of socket end

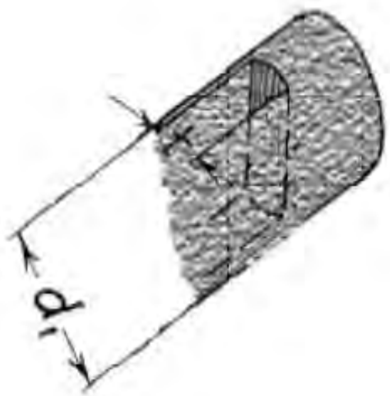
$$2l(d_3 - d_1)\tau = P$$



4.2.2.6 *- Shear failure of socket end*

7. Crushing failure of rod or cotter

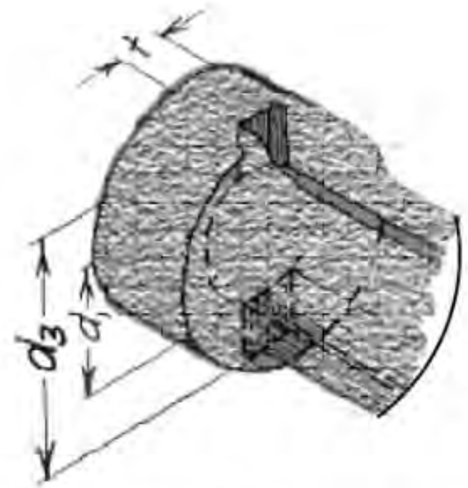
$$d_1 t \sigma_c = P$$



4.2.2.7 *- Crushing failure of rod or cotter*

8. Crushing failure of socket or rod

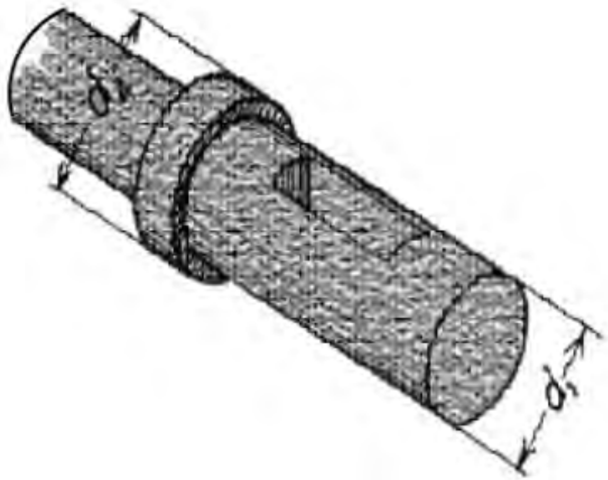
$$\left(d_3 - d_1\right) t\sigma_c = P$$



4.2.2.8 *- Crushing failure of socket or rod*

9. Crushing failure of collar

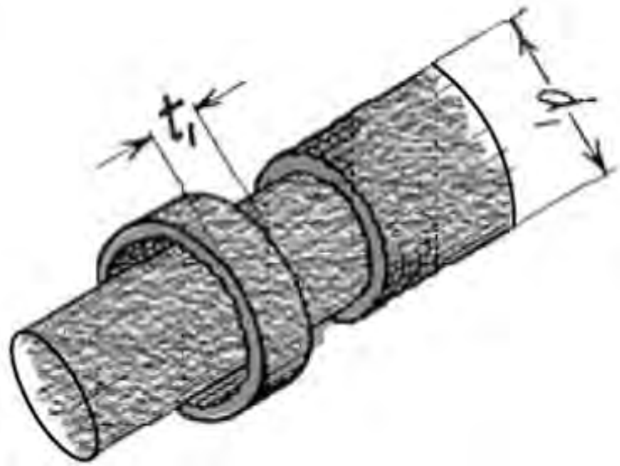
$$\left(\frac{\pi}{4}(d_4{}^2 - d_1{}^2)\right)\sigma_c = P$$



4.2.2.9 *- Crushing failure of collar*

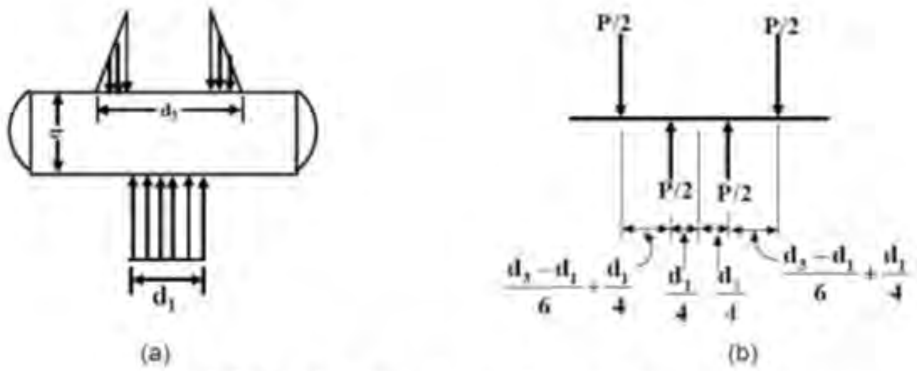
10. Shear failure of collar

$$\pi d_1 t_1 \tau = P$$



4.2.2.10 *- Shear failure of collar.*

Cotters may bend when driven into position. When this occurs, the bending moment cannot be correctly estimated since the pressure distribution is not known. However, if we assume a triangular pressure distribution over the rod, as shown in figure-4.2.2.11 (a), we may approximate the loading as shown in figure-4.2.2.11 (b)



4.2.2.11 *- Bending of the cotter*

This gives maximum bending moment = $\frac{P}{2}\left(\frac{d_3-d_1}{6}+\frac{d_1}{4}\right)$ and

The bending stress, $\sigma_b = \dfrac{\frac{P}{2}\left(\frac{d_3-d_1}{6}+\frac{d_1}{4}\right)\frac{b}{2}}{\frac{tb^3}{12}} = \dfrac{3P\left(\frac{d_3-d_1}{6}+\frac{d_1}{4}\right)}{tb^2}$

Tightening of cotter introduces initial stresses which are again difficult to estimate. Sometimes therefore it is necessary to use empirical proportions to design the joint. Some typical proportions are given below:

$d_1 = 1.21.d$

$d_2 = 1.75.d$

$d_3 = 2.4\ d$

$d_4 = 1.5.d$

$t = 0.31d$

$b = 1.6d$

$l = l_1 = 0.75d$

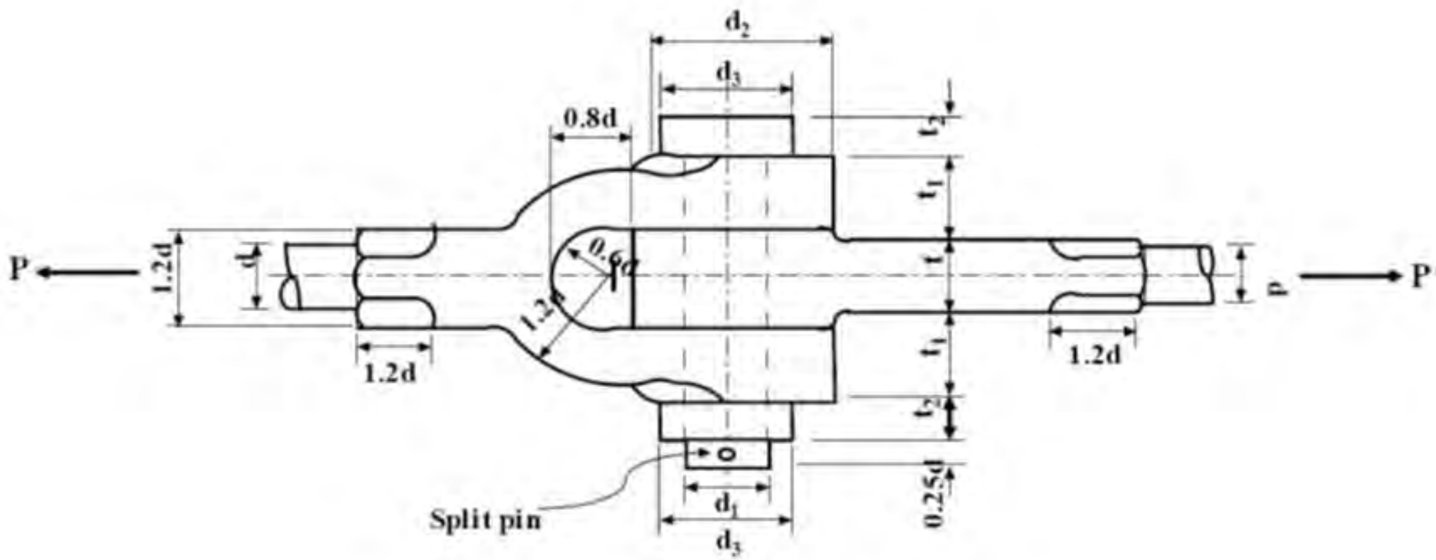
$t_1 = 0.45d$

s= clearance

A design based on empirical relation may be checked using the formulae based on failure mechanisms.

4.2.3 Knuckle Joint

A knuckle joint (as shown in **figure- 4.2.3.1**) is used to connect two rods under tensile load. This joint permits angular misalignment of the rods and may take compressive load if it is guided.



4.2.3.1 *- A typical knuckle joint*

These joints are used for different types of connections e.g. tie rods, tension links in bridge structure. In this, one of the rods has an eye at the rod end and the other one is forked with eyes at both the legs. A pin (knuckle pin) is inserted through the rod-end eye and fork-end eyes and is secured by a collar and a split pin.

d = diameter of rod

d_1= d, t =1.25=d, d_2 =2d, t_1 =0.75d, d_3 =1.5.d, t_2 =0.5d

Mean diameter of the split pin = 0.25 d

However, failures analysis may be carried out for checking. The analyses areshown below assuming the same materials for the rods and pins and the yield stresses in tension, compression and shear are given by σt, σc and τ.

1. Failure of rod in tension: $\frac{\pi}{4}d^2\sigma_t = P$

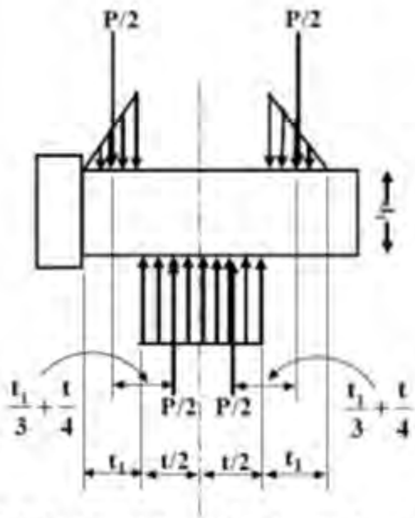
2. Failure of knuckle pin in double shear: $2\frac{\pi}{4}d_1{}^2\tau = P$

3. Failure of knuckle pin in bending (if the pin is loose in the fork)

Assuming a triangular pressure distribution on the pin, the loading on the pin is shown in figure- 4.2.3.2.

Equating the maximum bending stress to tensile or compressive yield stress we have

$$\sigma_t = \frac{16P\left(\frac{t_1}{3} + \frac{t}{4}\right)}{\pi d_1^3}$$



4.2.3.2 *- Bending of a knuckle pin*

4. Failure of rod eye in shear:

$$(d_2 - d_1)t\tau = P$$

5. Failure of rod eye in crushing:

$$d_1 t\sigma_c = P$$

6. Failure of rod eye in tension:

$$(d_2 - d_1)t\sigma_t = P$$

7. Failure of forked end in shear:

$$2(d_2 - d_1)t_1\tau = P$$

8. Failure of forked end in tension:

$$2(d_2 - d_1)t_1\sigma_t = P$$

9. Failure of forked end in crushing:

$$2d_1 t_1\sigma_c = P$$

The design may be carried out using the empirical proportions and then the analytical relations may be used as checks.

For example using the 2nd equation we have $\tau = \frac{2P}{\pi d_1^2}$. We may now put value of d_1 from empirical relation and then find F.S. = $\frac{\tau_y}{\tau}$ which should be more than one.

4.2.4 Problems with Answers

Q.1: Design a typical cotter joint to transmit a load of 50 kN in tension or compression. Consider that the rod, socket and cotter are all made of a material with the following allowable stresses:

Allowable tensile stress σ_y = 150 MPa

Allowable crushing stress σ_c = 110 MPa

Allowable shear stress τ_y = 110 MPa.

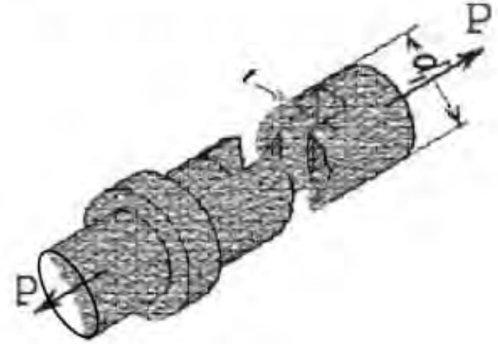
A.1: *Refer to figure- 4.2.1.2 and 4.2.2.1*

Axial load $P=\frac{\pi}{4}d^2\sigma_y$. On substitution this gives d=20 mm. In general standard snaπ size in mm are

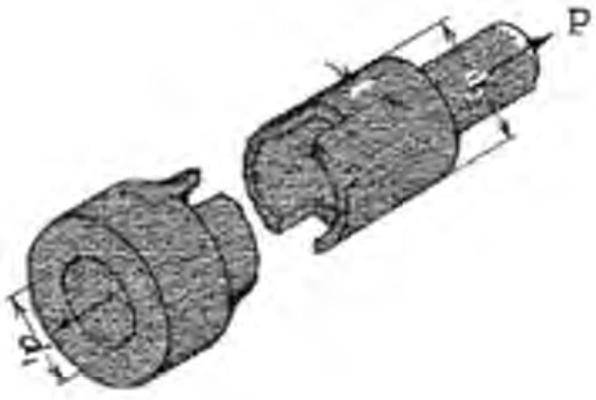
6 mm to 22 mm diameter	2 mm in increment
25 mm to 60 mm diameter	5 mm in increment
60 mm to 110 mm diameter	0 mm in increment
110 mm to 140 mm diameter	15 mm in increment
140 mm to 160 mm diameter	20 mm in increment
500 mm to 600 mm diameter	30 mm in increment

We therefore choose a suitable rod size to be 25 mm.

Refer to figure-4.2.2.2



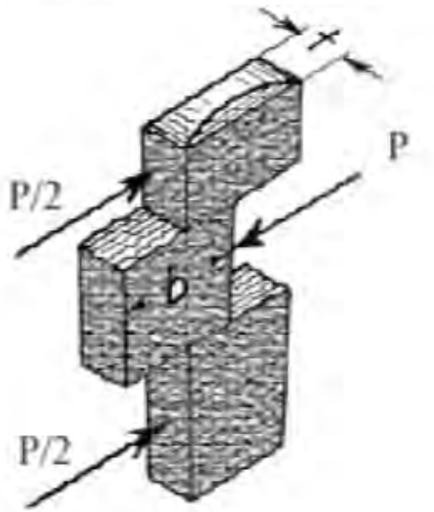
For tension failure across slot $\left(\frac{\pi}{4}d^2 - d_1 t\right)\sigma_y = P$. This gives $d_1 t = 1.58 \times 10^{-4}$ m^2.From empirical relations we may take t=0.4d i.e. 10 mm and this gives d_1= 15.8 mm. Maintaining the proportion let d_1= 1.2 d = 30 mm.



Refer to figure-4.2.2.3

The tensile failure of socket across slot $\left\{\left(\frac{\pi}{4}d_2^2 - d_1^2\right) - (d_2 - d_1)t\right\}\sigma_y = P$

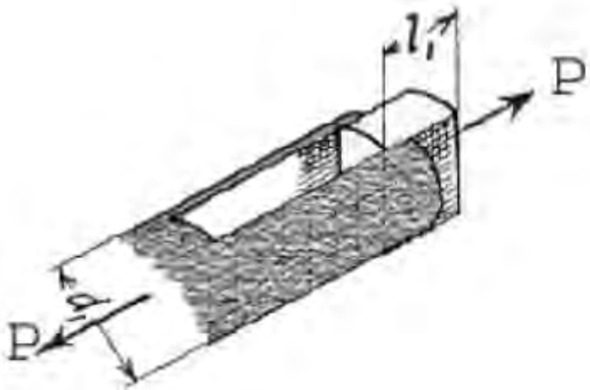
This gives d_2 = 37 mm. Let d_2 = 40 mm



Refer to figure-4.2.2.4

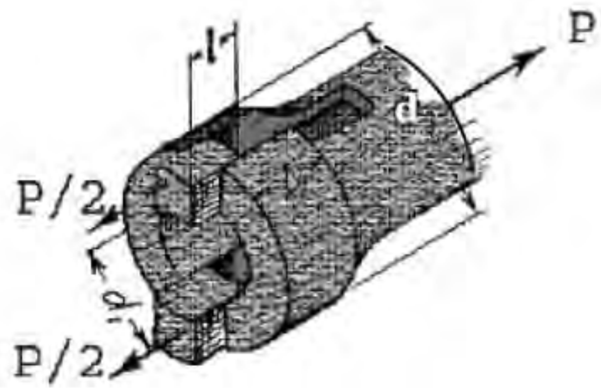
For shear failure of cotter $2bt\tau = P$. On substitution this gives b = 22.72 mm.

Let b = 25 mm.



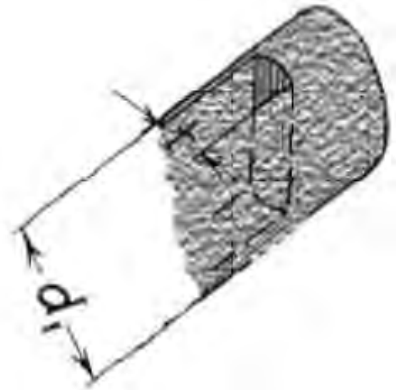
Refer to figure-4.2.2.5

For shear failure of rod end $2l_1d_1\tau = P$ and this gives $l_1 = 7.57$ mm. Let $l_1 =$ 10 mm.



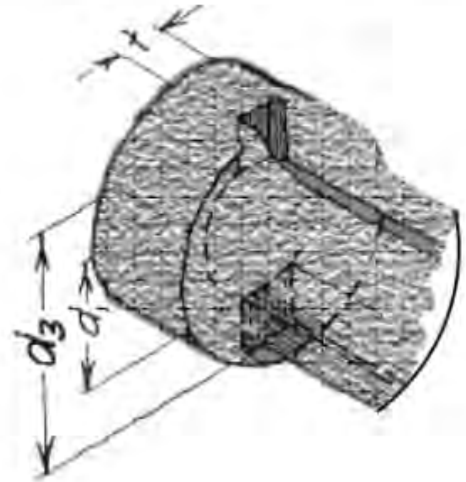
Refer to figure-4.2.2.6

For shear failure of socket end $2l(d_2-d_1)\tau = P$. This gives l= 22.72 mm. Let l=25 mm



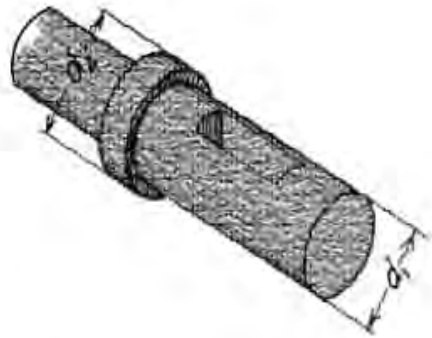
Refer to figure-4.2.2.8

For crushing failure of socket or rod $(d_3-d_1)t\sigma_c = P$. This gives $d_3 = 75.5$ mm. Let $d_3 = 77$ mm.



Refer to figure-4.2.2.9

For crushing failure of collar $\frac{\pi}{4}\left(d_4^2 - d_1^2\right)\sigma_c = P$. On substitution this gives d_4= 38.4 mm. Let d_4= 40 mm.



Refer to figure-4.2.2.10

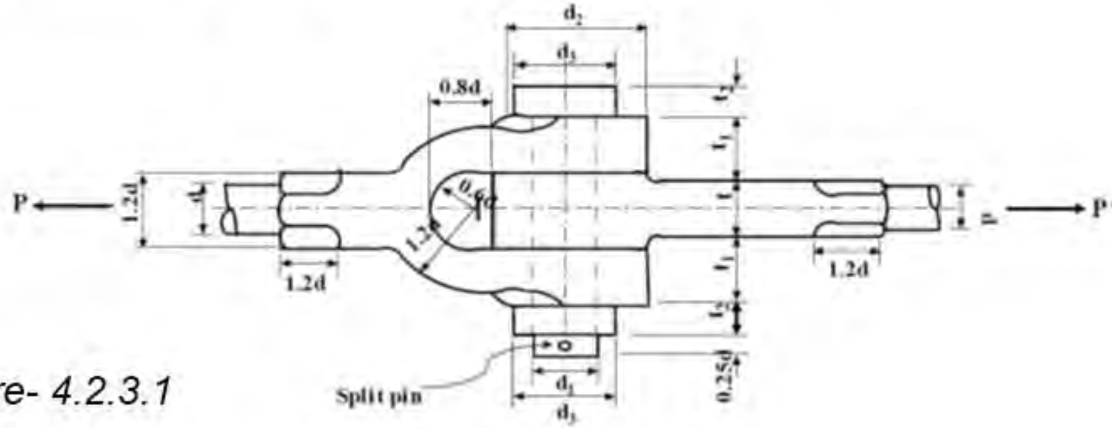
For shear failure of collar $\pi d_1 t_1 \tau$ = P which gives t_1= 4.8 mm. Let t_1 = 5 mm.

Therefore the final chosen values of dimensions are

d= 25 mm; d_1= 30 mm; d_2 = 40 mm; d_3 = 77 mm; d_4 = 40 mm; t= 10 mm; t_1= 5 mm; *l*= 25 mm; l_1= 10 mm; b= 27 mm.

Q.2: Two mild steel rods are connected by a knuckle joint to transmit an axial force of 100 kN. Design the joint completely assuming the working stresses for both the pin and rod materials to be 100 MPa in tension, 65 MPa in shear and 150 MPa in crushing.

Refer to figure- 4.2.3.1



For failure of rod in tension, $P = \frac{\pi}{4} d^2 \sigma_y$. On substituting P=100 kN,

σ_y = 100 MPa we have d= 35.6 mm. Let us choose the rod diameter d = 40 mm which is the next standard size.

We may now use the empirical relations to find the necessary dimensions and then check the failure criteria.

d_1= 40 mm t= 50 mm

d_2 = 80 mm t_1= 30 mm;

d_3 = 60 mm t_2= 20 mm;

split pin diameter = 0.25 d_1 = 10 mm

To check the failure modes:

1. Failure of knuckle pin in shear: $P \Big/ \left(2.\frac{\pi}{4}d_1^2\right) = \tau_y$ which gives τ_y = 39.8 MPa. This is less than the yield shear stress.

2. For failure of knuckle pin in bending: $\sigma_y = \dfrac{16P\left(\frac{t_1}{3}+\frac{t}{4}\right)}{\pi d_1^3}$. On substitution this gives σ_y = 179 MPa which is more than the allowable tensile yield stress of 100 MPa. We therefore increase the knuckle pin diameter to 55 mm which gives σ_y = 69 MPa that is well within the tensile yield stress.

3. For failure of rod eye in shear: $(d_2-d_1)t\tau = P$. On substitution $d_1 = 55$mm $\tau = 80$ MPa which exceeds the yield shear stress of 65 MPa. So d_2 should be at least 85.8 mm. Let d_2 be 90 mm.
4. For failure of rod eye in crushing: $d_1 t\sigma_c = P$ which gives $\sigma_c = 36.36$ MPa that is well within the crushing strength of 150 MPa.
5. Failure of rod eye in tension: $(d_2-d_1)t\sigma_t = P$. Tensile stress developed at the rod eye is then $\sigma_t = 57.14$ MPa which is safe.
6. Failure of forked end in shear: $2(d_2-d_1)t_1\tau = P$. Thus shear stress developed in the forked end is $\tau = 47.61$ MPa which is safe.
7. Failure of forked end in tension: $2(d_2-d_1)t_1\sigma_y = P$. Tensile strength developed in the forked end is then $\sigma_y = 47.61$ MPa which is safe.
8. Failure of forked end in crushing: $2d_1t_1\sigma_c = P$ which gives the crushing stress developed in the forked end as $\sigma_c = 42$ MPa. This is well within the crushing strength of 150 MPa.

Therefore the final chosen values of dimensions are:

$d_1 = 55$ mm	$t = 50$ mm	
$d_2 = 90$ mm	$t_1 = 30$ mm;	and $d = 40$ mm
$d_3 = 60$ mm	$t_2 = 20$ mm;	

4.2.5 Summary of this Lesson

In this lesson two well known joints viz. cotter and knuckle joints used in machinery are discussed. Their constructional detail and working principle have been described. Then the detailed design procedures of both these joints are given with suitable illustrations. Finally two examples, one on cotter joint and the other on cotter joint have been solved.

Lecture

Theme 5

Couplings

5.1. Types and uses of couplings

5.1.1 Introduction

Couplings are used to connect two shafts for torque transmission in varied applications. It may be to connect two units such as a motor and a generator or it may be to form a long line shaft by connecting shafts of standard lengths say 6-8m by couplings. Coupling may be rigid or they may provide flexibility and compensate for misalignment. They may also reduce shock loading and vibration. A wide variety of commercial shaft couplings are available ranging from a simple keyed coupling to one which requires a complex design procedure using gears or fluid drives etc. However there are two main types of couplings:

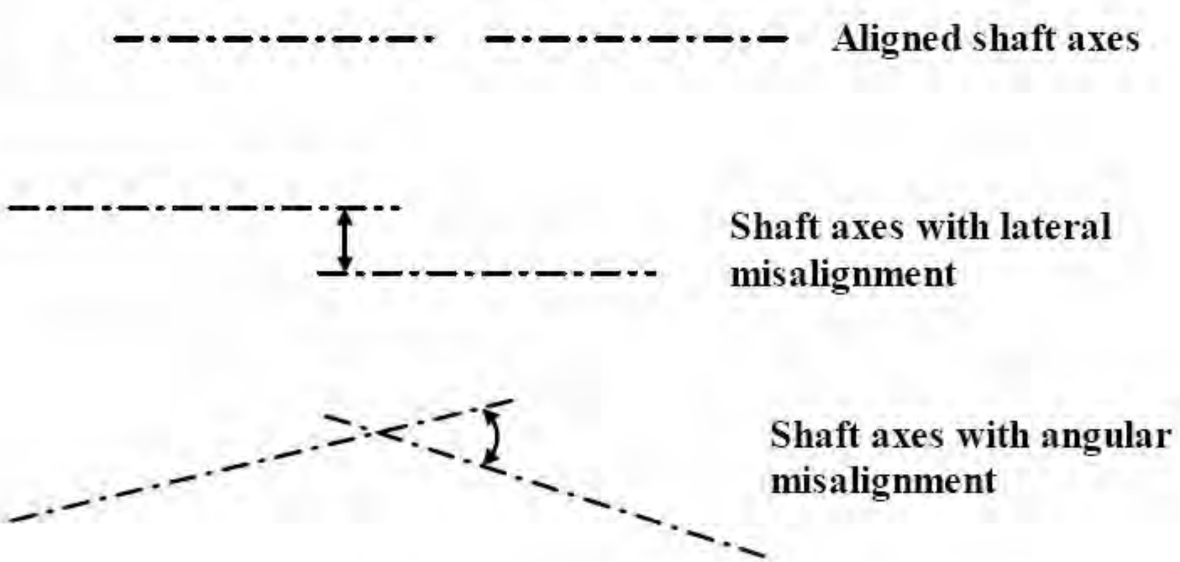
Rigid couplings

Flexible couplings

Rigid couplings are used for shafts having no misalignment while the flexible couplings can absorb some amount of misalignment in the shafts to be connected. In the next section we shall discuss different types of couplings and their uses under these two broad headings.

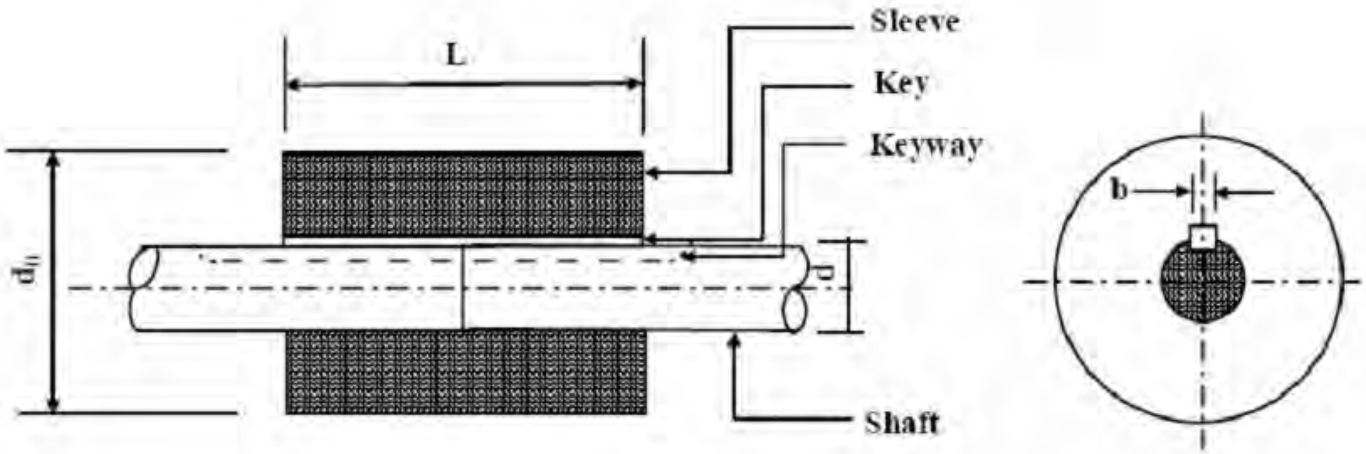
5.1.2 Types and uses of shaft couplings

5.1.2.1 Rigid couplings Since these couplings cannot absorb any misalignment the shafts to be connected by a rigid coupling must have good lateral and angular alignment. The types of misalignments are shown schematically in **figure-5.1.2.1.1**.



5.1.2.1.1. *- Types of misalignments in shafts*

5.1.2.1.1 Sleeve coupling One of the simple type of rigid coupling is a sleeve coupling which consists of a cylindrical sleeve keyed to the shafts to be connected. A typical sleeve coupling is shown in **figure- 5.1.2.1.1.1**.



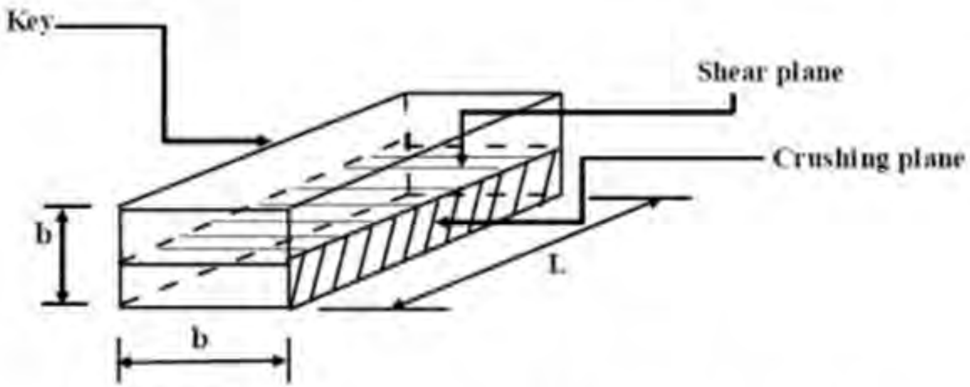
5.1.2.1.1.1 - A typical sleeve coupling

Normally sunk keys are used and in order to transmit the torque safely it is important to design the sleeve and the key properly. The key design is usually based on shear and bearing stresses. If the torque transmitted is T, the shaft radius is r and a rectangular sunk key of dimension b and length L is used then the induced shear stress τ (**figure- 5.1.2.1.1.2**) in the key is given by

$$\tau = T \Big/ \left(b \frac{L}{2} r \right)$$

and for safety

$$(2T/bLr) < \tau_y$$



5.1.2.1.1.2 *- Shear and crushing planes in the key.*

where τ_y is the yield stress in shear of the key material. A suitable factor of safety must be used. The induced crushing stress in the key is given as

$$\sigma_{br} = T \Big/ \left(\frac{b}{2} \frac{L}{2} r \right)$$ and for a safe design $$4T/(bLr) < \sigma_c$$

where σ_c is the crushing strength of the key material.

The sleeve transmits the torque from one shaft to the other. Therefore if d_i is the inside diameter of the sleeve which is also close to the shaft diameter d (say) and d_0 is outside diameter of the sleeve, the shear stress developed in the sleeve is $\tau_{sleeve} = \frac{16Td_0}{\pi\left(d_0^4 - d_i^4\right)}$ and the shear stress in the shaft is given by $\tau_{shaft} = \frac{16T}{\pi d_i^3}$. Substituting yield shear stresses of the sleeve and shaft materials for τ_{sleeve} and τ_{shaft} both d_i and d_0 may be evaluated.

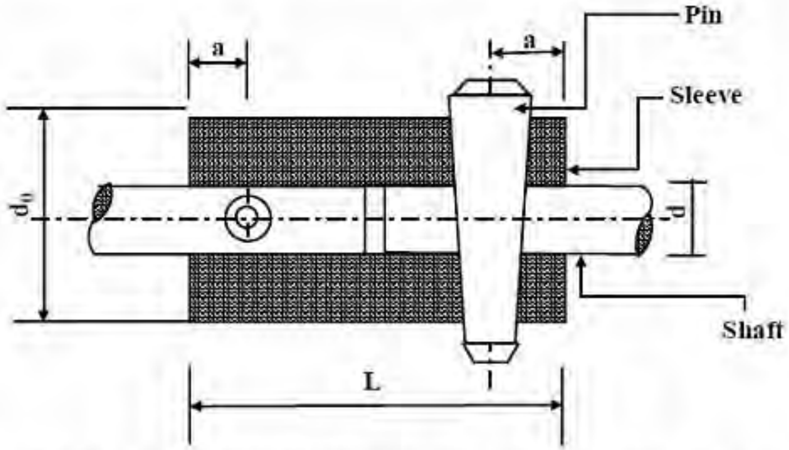
However from the empirical proportions we have:

$d_0 = 2d_i + 12.5$ mm and L=3.5d.

These may be used as checks.

5.1.2.1.2 Sleeve coupling with taper pins

Torque transmission from one shaft to another may also be done using pins as shown in **figure-5.1.2.1.2.1.**



5.1.2.1.2.1 - A representative sleeve coupling with taper pins.

The usual proportions in terms of shaft diameter d for these couplings are:
d_0 = 1.5d, L = 3d and a = 0.75d.

The mean pin diameter d_{mean} = 0.2 to 0.25 d. For small couplings d_{mean} is taken as 0.25d and for large couplings d_{mean} is taken as 0.2d. Once the dimensions are fixed we may check the pin for shear failure using the relation $2\left(\frac{\pi}{4}d_{mean}^2\right)\tau\left(\frac{d}{2}\right) = T$.

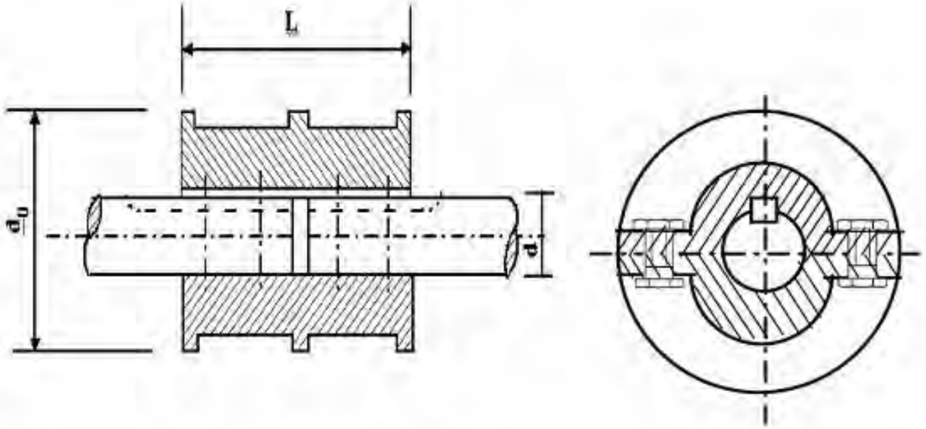
Here T is the torque and the shear stress τ must not exceed the shear yield stress of the pin material. A suitable factor of safety may be used for the shear yield stress.

5.1.2.1.3 Clamp coupling

A typical clamp coupling is shown in **figure-5.1.2.1.3.1**. It essentially consists of two half cylinders which are placed over the ends of the shafts to be coupled and are held together by through bolt.

The length of these couplings 'L' usually vary between 3.5 to 5 times the and the outside diameter 'd0' of the coupling sleeve between 2 to 4 times the shaft diameter d. It is assumed that even with a key the torque is transmitted due to the friction grip. If now the number of bolt on each half is n, its core diameter is dc and the coefficient of friction between the shaft and sleeve material is μ we may find the torque transmitted T as follows:

The clamping pressure between the shaft and the sleeve is given by



5.1.2.1.3.1 - A representative clamp coupling

$$p = \frac{n}{2} x \frac{\pi}{4} d_c^2 x \sigma_t \Big/ (dL/2)$$

where n is the total number of bolts, the number of effective bolts for each shaft is n/2 and σ_t is the allowable tensile stress in the bolt. The tangential force per unit area in the shaft periphery is F = μ p. The torque transmitted can therefore be given by $T = \frac{\pi dL}{2} \mu p. \frac{d}{2}$.

5.1.2.1.4 Ring compression type couplings

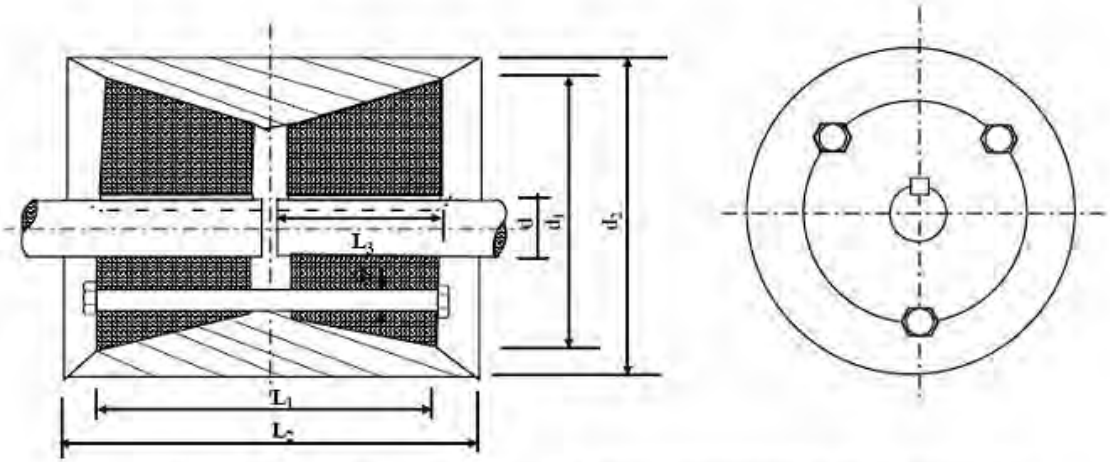
The coupling (**figure-5.1.2.1.4.1)** consists of two cones which are placed on the shafts to be coupled and a sleeve that fits over the cones. Three bolts are used to draw the cones towards each other and thus wedge them firmly between the shafts and the outer sleeve. The usual proportions for these couplings in terms of shaft diameter d are approximately as follows:

$d_1 = 2d + 15.24$ mm $\quad L_1 = 3d$

$d_2 = 2.45d + 27.94$ mm $\quad L_2 = 3.5d + 12.7$ mm

$d_3 = 0.23d + 3.17$ mm $\quad L_3 = 1.5d$

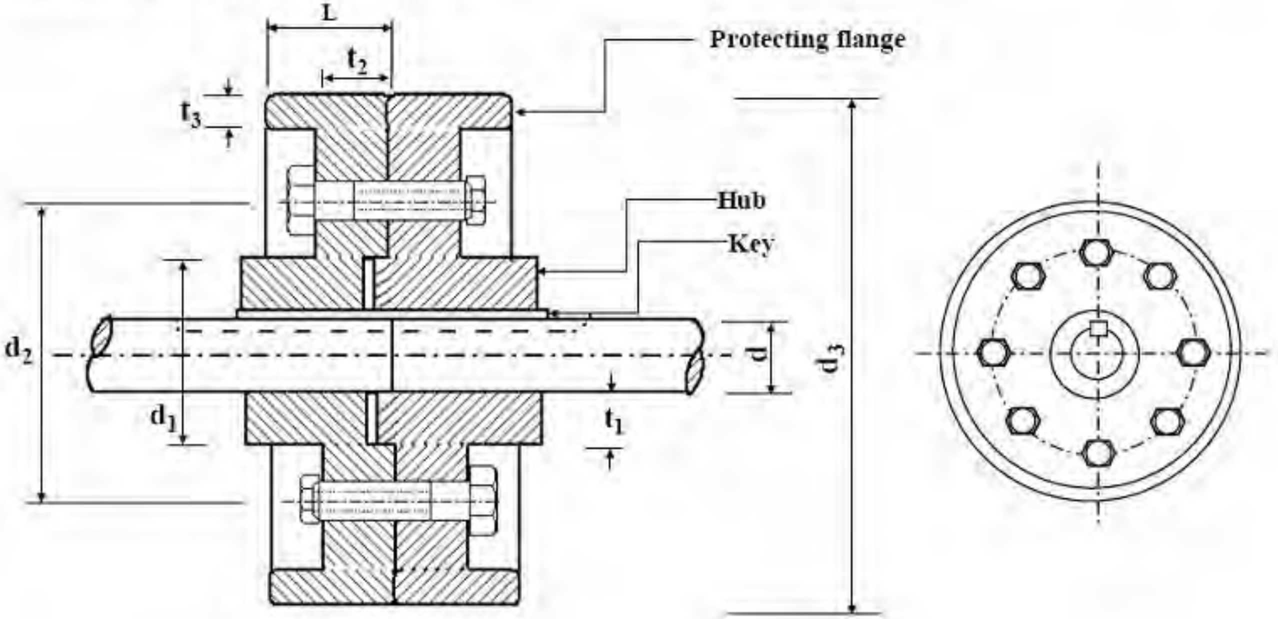
and the taper of the cone is approximately 1 in 4 on diameter.



***5.1.2.1.4.1** - A representative ring compression type coupling.*

5.1.2.1.4 Flange coupling

It is a very widely used rigid coupling and consists of two flanges keyed to the shafts and bolted. This is illustrated in **figure-5.1.2.1.4.2**.



5.1.2.1.4.2 *- A typical flange coupling*

Design details of such couplings will be discussed in the next lesson. The main features of the design are essentially

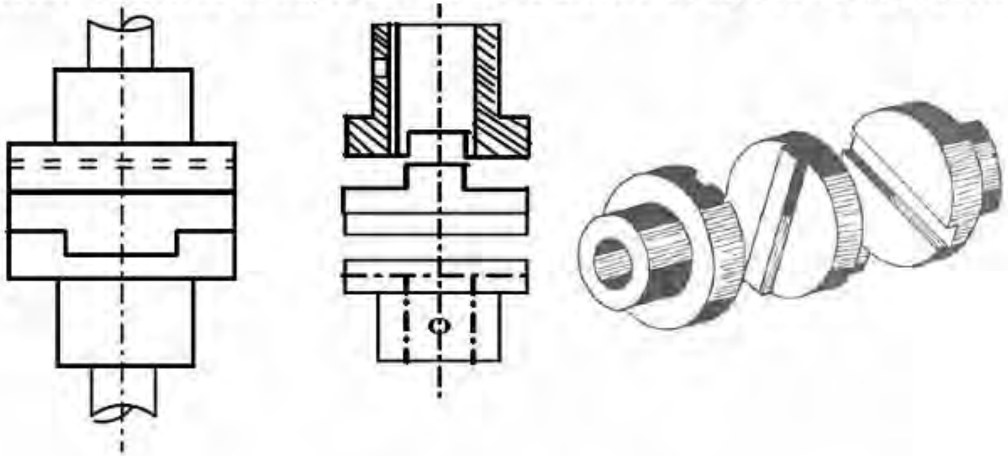
(a) Design of bolts (b) Design of hub (c) Overall design and dimensions.

5.1.2.2 Flexible coupling

As discussed earlier these couplings can accommodate some misalignment and impact. A large variety of flexible couplings are available commercially and principal features of only a few will be discussed here.

5.1.2.2.1 Oldham coupling

These couplings can accommodate both lateral and angular misalignment to some extent. An Oldham coupling consists of two flanges with slots on the faces and the flanges are keyed or screwed to the shafts. A cylindrical
piece, called the disc, has a narrow rectangular raised portion running across each face but at right angle to each other. The disc is placed between the flanges such that the raised portions fit into the slots in the flanges. The disc may be made of flexible materials and this absorbs some misalignment. A schematic representation is shown in **figure- 5.1.2.2.1.1.**



5.1.2.2.1.1 - A schematic diagram of an Oldham coupling

5.1.2.2.2 Universal joints

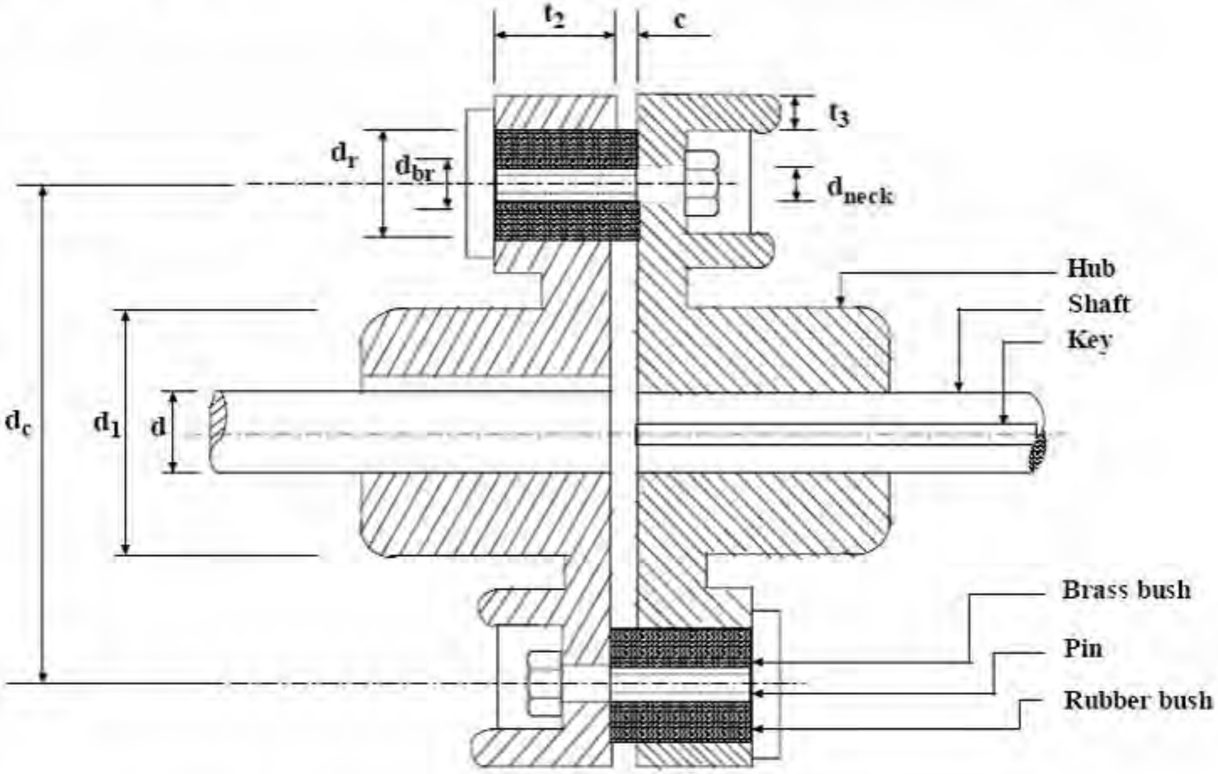
These joints are capable of handling relatively large angular misalignment and they are widely used in agricultural machinery, machine tools and automobiles.A typical universal joint is shown in **figure- 5.1.2.2.2.1.**There are many forms of these couplings, available commercially but they essentially consist of two forks keyed or screwed to the shaft. There is a center piece through which pass two pins with mutually perpendicular axes and they connect the two fork ends such that a large angular misalignment can be accommodated. The coupling, often known as, Hooke's coupling has no torsional rigidity nor can it accommodate any parallel offset.

5.1.2.2.2.1 *- A typical universal joint*

5.1.2.2.2 Pin type flexible coupling

One of the most commonly used flexible coupling is a pin type flexible flange coupling in which torque is transmitted from one flange to the other through a flexible bush put around the bolt. This is shown in the next lesson and is shown in **figure-5.2.2.1.**

These are used when excessive misalignment is not expected such as a coupling between a motor and a generator or a pump mounted on a common base plate. Detail design procedure for these couplings will be discussed in the next lesson.



5.2.2.1 - A typical flexible coupling with rubber bushings.

5.1.3 Summary of this Lecture

Basic function of shaft couplings, their types and uses have been discussed in this lesson. Among the rigid couplings some details of sleeve couplings with key or taper pins, clamp couplings, ring compression type couplings and flange couplings have been described. Among the flexible couplings the Oldham coupling and universal joints are described and the functions of pin type flexible couplings are given briefly.

Lecture

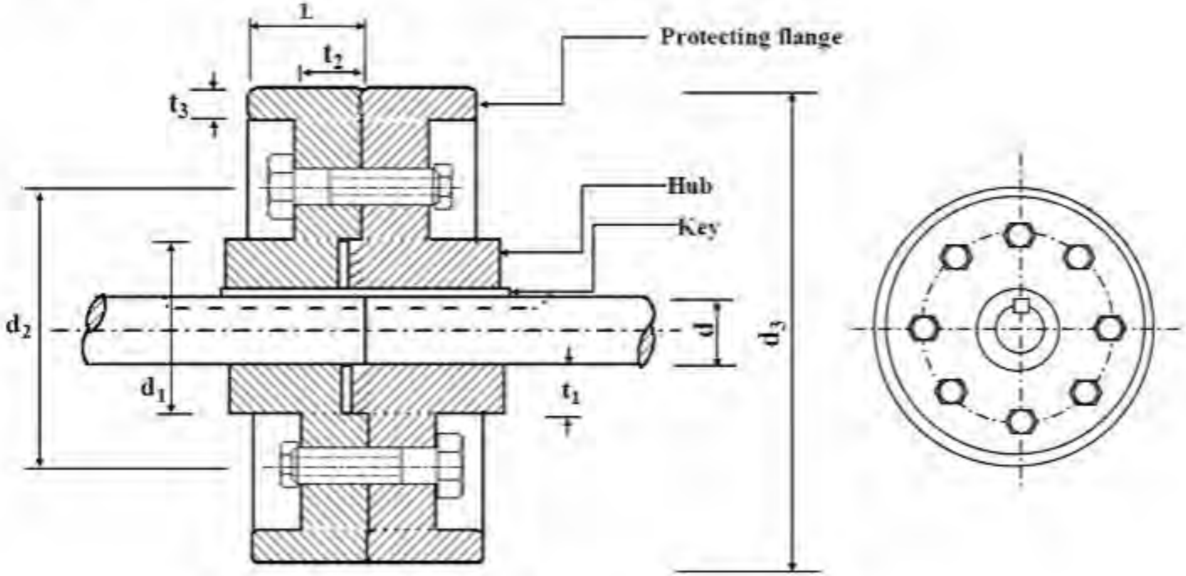
Theme 5

Couplings

5.2. Design procedures for rigid and flexible rubber-bushed couplings

5.2.1 Rigid Flange Coupling

A typical rigid flange coupling is shown in **Figure- 5.2.1.1.**



***5. 2.1.1** - A typical flange coupling*

If essentially consists of two cast iron flanges which are keyed to the shafts to be joined. The flanges are brought together and are bolted in the annular space between the hub and the protecting flange. The rotective flange is provided to guard the projecting bolt heads and nuts. The bolts are placed equi-spaced on a bolt circle diameter and the number of bolt depends on the shaft diameter d. A spigot 'A' on one flange and a recess on the opposing face is provided for ease of assembly.

The design procedure is generally based on determining the shaft diameter d for a given torque transmission and then following empirical relations different dimensions of the coupling are obtained. Check for different failure modes can then be carried out. Design procedure is given in the following steps:

(1) Shaft diameter'd' based on torque transmission is given by $d = \left(\frac{16T}{\pi\tau_s}\right)^{1/3}$

where T is the torque and τ_y is the yield stress in shear.

(2) Hub diameter d1 =1.75d +6.5mm

(3) Hub length L = 1.5d

But the hub length also depends on the length of the key. Therefore this length L must be checked while finding the key dimension based on shear and crushing failure modes.

(4) Key dimensions:

If a square key of sides b is used then b is commonly taken as d/4 . In that case, **for shear failure** we have $\left(\frac{d}{4}.L_k\right).\tau_y.\frac{d}{2} = T$ where τ_y is the yield stress in shear and Lk is the key length.

This gives $L_k = \frac{8T}{d^2\tau_y}$

If Lk determined here is less than hub length L we may assume the key length to be the same as hub length.

For crushing failure we have $\left(\frac{d}{8}.L_k\right)\sigma_c.\frac{d}{2} = T$ where σ_c is crushing stress induced in the key. This gives $\sigma_c = \frac{16T}{L_k d^2}$

and if $\sigma_c < \sigma_{cy}$, the bearing strength of the key material , the key dimensions chosen are in order.

(5) Bolt dimensions :

The bolts are subjected to shear and bearing stresses while transmitting torque.

Considering the shear failure mode we ha $n.\frac{\pi}{4}d_b{}^2\tau_{yb}\frac{d_c}{2}=T$

where n is the number of bolts, d_b the nominal bolt diameter, T is the torque transmitted, τ_{yb} is the shear yield strength of the bolt material and d_c is the bolt circle diameter. The bolt diameter may now be obtained if n is known. The number of bolts n is often given by the

following empirical relation: $n = \frac{4}{150}d+3$

where d is the shaft diameter in mm. The bolt circle diameter must be such that it should provide clearance for socket wrench to be used for the bolts. The empirical relation takes care of this Considering crushing failure we have

$$n.d_b t_2 \sigma_{cyb}\frac{d_c}{2}=T$$

where t_2 is the flange width over which the bolts make contact and σ_{cyb} is the yield crushing strength of the bolt material. This gives t_2. Clearly the bolt length must be more than $2t_2$ and a suitable standard length for the bolt diameter may be chosen from hand book.

(6) A protecting flange is provided as a guard for bolt heads and nuts. The thickness t_3 is less than $t_2/2$. The corners of the flanges should be rounded.

(7) The spigot depth is usually taken between 2-3mm.

(8) Another check for the shear failure of the hub is to be carried out. For this failure mode we may write

$$\pi d_1 t_2 \tau_{yf} \frac{d_1}{2} = T$$

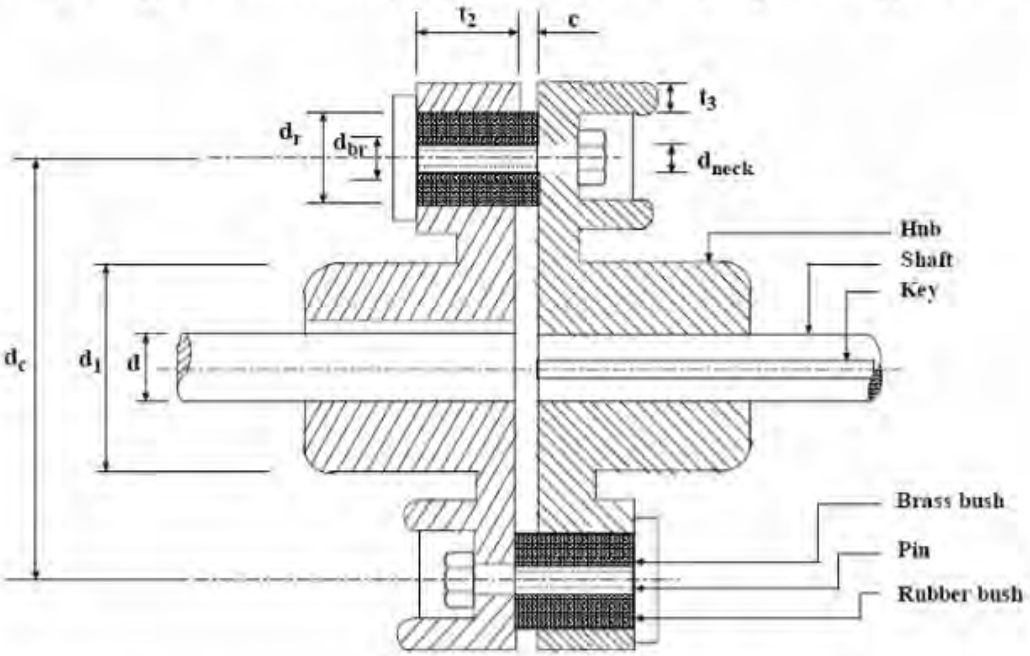
where d_1 is the hub diameter and τ_{yf} is the shear yield strength of the flange material.

Knowing τ_{yf} we may check if the chosen value of t_2 is satisfactory or not.

Finally, knowing hub diameter d_1, bolt diameter and protective thickness t_2 we may decide the overall diameter d_3.

5.2.2 Flexible rubber – bushed couplings

This is simplest type of flexible coupling and a typical coupling of this type is shown in **Figure- 5.2.2.1.**



5.2.2.1 - A typical flexible coupling with rubber bushings.

In a rigid coupling the torque is transmitted from one half of the coupling to the other through the bolts and in this arrangement shafts need be aligned very well.

However in the bushed coupling the rubber bushings over the pins (bolts) (as shown in **Figure-5.2.2.1**) provide flexibility and these coupling can accommodate some misalignment.

Because of the rubber bushing the design for pins should be considered carefully.

(1) Bearing stress Rubber bushings are available for different inside and out side diameters. However rubber bushes are mostly available in thickness between 6 mm to 7.5mm for bores upto 25mm and 9mm thickness for larger bores. Brass sleeves are made to suit the requirements. However, brass sleeve thickness may be taken to be 1.5mm. The outside diameter of rubber bushing d_r is given by

$$d_r = d_b + 2\, t_{br} + 2\, t_r$$

where d_b is the diameter of the bolt or pin , t_{br} is the thickness of the brass sleeve and t_r is the thickness of rubber bushing. Now write

$$n.d_r t_2 p_b \frac{d_c}{2} = T$$

where d_c is the bolt circle diameter and t_2 the flange thickness over the bush contact area. A suitable bearing pressure for rubber is 0.035 N/mm^2 and the number of pin is given by

$n = \frac{d}{25} + 3$. The d_c here is different from what we had for rigid flange bearings. This must be judged considering the hub diameters, out side diameter of the bush and a suitable clearance. From the above torque equation we may obtain bearing pressure developed and compare this with the bearing pressure of rubber for safely.

(2) Shear stress

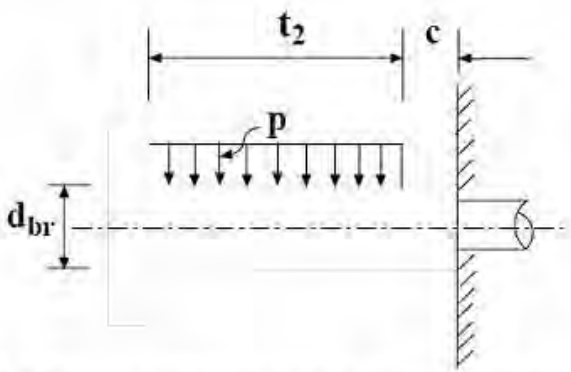
The pins in the coupling are subjected to shear and it is a good practice to ensure that the shear plane avoids the threaded portion of the bolt. Unlike the rigid coupling the shear stress due to torque transmission is given in terms of the tangential force F at the outside diameter of the rubber bush. Shear stress at the neck area is given by

$$\tau_b = \frac{p_b t_2 d_r}{\frac{\pi}{4} d_{neck}^2}$$

where d_{neck} is bolt diameter at the neck i.e at the shear plane.

(3) Bending Stress

The pin loading is shown in **Figure-5.2.2.2.**



5.2.2.2 - Loading on a pin supporting the bushings.

Clearly the bearing pressure that acts as distributed load **F=pt$_2$d** on rubber bush would produce bending of the pin. Considering an equivalent concentrated load the bending stress is

$$\sigma_b = \frac{32F(t_2/2)}{\pi d_{br}^3}$$

Knowing the shear and bending stresses we may check the pin diameter for principal stresses using appropriate theories of failure.

We may also assume the following empirical relations:

Hub diameter = 2d

Hub length = 1.5d

Pin diameter at the neck = $\frac{0.5d}{\sqrt{n}}$

5.2.3 Problems with Answers

Q.1: Design a typical rigid flange coupling for connecting a motor and a centrifugal pump shafts. The coupling needs to transmit 15 KW at 1000 rpm. The allowable shear stresses of the shaft, key and bolt materials are 60 MPa,50 MPa and 25 MPa respectively. The shear modulus of the shaft material may be taken as 84GPa. The angle of twist of the shaft should be limited to 1 degree in 20 times the shaft diameter.

A.1:

1. The shaft diameter based on strength may be given by

$d = \sqrt[3]{\frac{16T}{\pi\tau_y}}$ where T is the torque transmitted and τ_y is the allowable yield stress in shear.

Here $T = \text{Power}/\left(\frac{2\pi N}{60}\right) = \frac{P}{\omega}\ \frac{15x10^3}{\left(\frac{2\pi x1000}{60}\right)} = 143\text{Nm}$, $\omega = \frac{2\pi N}{60}$

Where ω is rate of angular motion in radian, N is rate of angular motion in revolution number

And substituting $\tau_y = 60x10^6$Pa we have

$d = \left(\frac{16x143}{\pi x60x10^6}\right)^{\frac{1}{3}} = 2.29x10^{-2}\text{m} = 23\text{mm}.$

2. **Let us consider a shaft of 25 mm which is a standard size.**

From the rigidity point of view $\frac{T}{J}=\frac{G\theta}{L}$,

substituting T = 143Nm , J = $\frac{\pi}{32}(0.025)^4 = 38.3x10^{-9} m^4, G = 84x10^9 Pa$

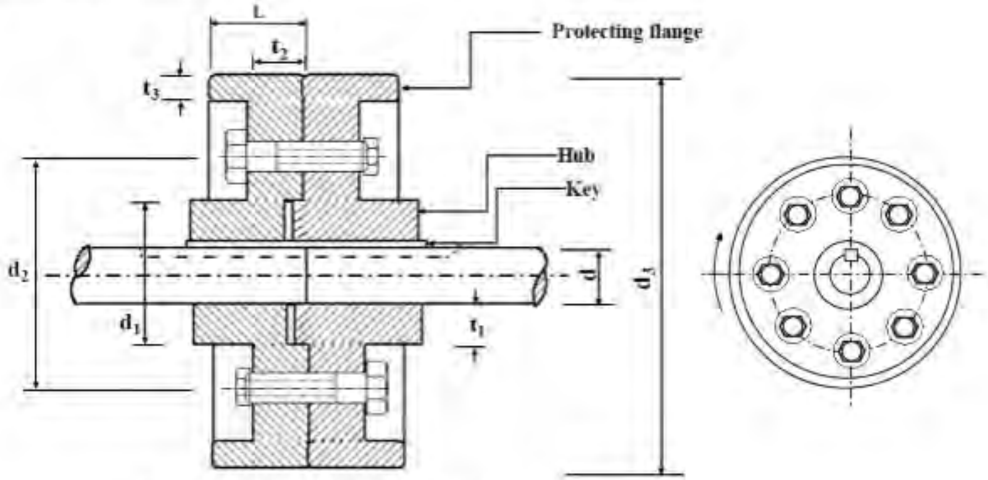
$$\frac{\theta}{L}=\frac{143}{38.3x10^{-9}x84x10^9} = 0.044 \text{ radian per meter.}$$

The limiting twist is 1 degree in 20 times the shaft diameter

which is $\frac{\frac{\pi}{180}}{20x0.025} = 0.035$ radian per meter

Therefore, the shaft diameter of 25mm is safe.

We now consider a typical rigid flange coupling as shown in **Figure 5.2.3.1.**



***5. 2.3.1** - A typical flange coupling*

3. **Hub-**

Using empirical relations

Hub diameter d_1 = 1.75d + 6.5 mm. This gives

d_1 = 1.75x25 + 6.5 = 50.25mm say d_1 = 51 mm

Hub length L=1.5d. This gives L = 1.5x25 = 37.5mm, say L= 38mm.

Hub thickness $t_1 = \frac{d_1 - d}{2} = \frac{51 - 25}{2} = 13\text{mm}$

4. Key –

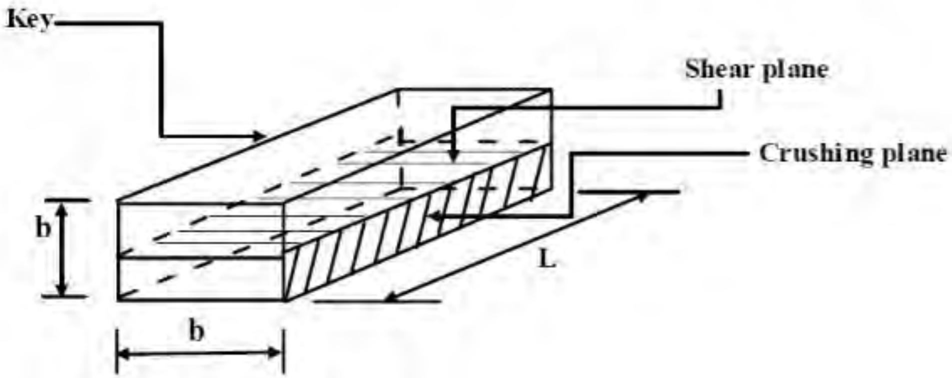
Now to avoid the shear failure of the key (refer to **Figure 5.1.2.1.1.2**)

$$\left(\frac{d}{4}L_k\right).\tau_y.\frac{d}{2}=T$$

where the key width w =d/4 and the key length is L_k

This gives $L_k = \frac{8T}{(\tau_y d^2)}$ i.e.

$$\frac{8x143}{50x10^6x(0.025)^2}=0.0366\ m=36.6\ mm$$



5.2.3.2 - Shear and crushing planes in the key.

The hub length is 37.5 mm. Therefore we take L_k = 37.5mm.

To avoid crushing failure of the key (Ref to **Figure 5.2.3.2**)

$(\frac{d}{8}L_k)\sigma.\frac{d}{2}$ = T where σ is the crushing stress developed in the key. This gives $\sigma=\frac{16T}{L_k d^2}$

Substituting T = 143Nm, L_k = 37.5 x 10^{-3} m and d = 0.025 m $\sigma=\frac{16x143x10^{-6}}{37.5x10^{-3}x(0.025)^2}=97.62MPa$

Assuming an allowable crushing stress for the key material to be 100MPa, the key design is safe. Therefore the key size may be taken as: a square key of 6.25 mm size and 37.5 mm long. However keeping in mind that for a shaft of diameter between 22mm and 30 mm a rectangular key of 8mm width, 7mm depth and length between 18mm and 90mm is recommended. We choose a standard key of 8mm width, 7mm depth and 38mm length which is safe for the present purpose.

5. **Bolts.**

To avoid shear failure of bolts $n\frac{\pi}{4}d_b^{\ 2}\tau_{yb}\frac{d_c}{2} = T$

where number of bolts n is given by the empirical relation $n = \frac{4}{150}d + 3$

where d is the shaft diameter in mm. which gives n=3.66 and we may take n=4 or more.

Here τ_{yb} is the allowable shear stress of the bolt and this is assumed to be 60 MPa.

d_c is the bolt circle diameter and this may be assumed initially based on hub diameter d_1=51 mm and later the dimension must be justified

Let d_c =65mm. Substituting the values we have the bolt diameter d_b as

$$d_b = \left(\frac{8T}{n\pi\tau_{yb}d_c}\right)^{\frac{1}{2}} \text{ i.e. } \left(\frac{8x143}{4\pi x25x10^6 x65x10^{-3}}\right)^{\frac{1}{2}} = 7.48x10^{-3}$$

which gives d_b = 7.48 mm. With higher factor of safety we may take d_b = 10 mm which is a standard size.

We may now check for crushing failure as $nd_b t_2 \sigma_c \frac{d_c}{2} = T$

Substituting n=4, d_b=10mm,σ_c=100MPa,d_c=65mm&T=143Nm and this gives t_2=2.2mm.

However empirically we have $t_2 = \frac{1}{2}t_1 + 6.5 = 13mm$

Therefore we take t_2=13mm which gives higher factor of safety.

6. **Protecting flange thickness.**

Protecting flange thickness t_3 is usually less than $\frac{1}{2}t_2$ we therefore take t_3 = 8mm since there is no direct load on this part.

7. **Spigot depth**

Spigot depth which is mainly provided for location may be taken as 2mm.

Check for the shear failure of the hub

To avoid shear failure of hub we have $\pi d_1 t_2 \tau_f \frac{d_1}{2} = T$

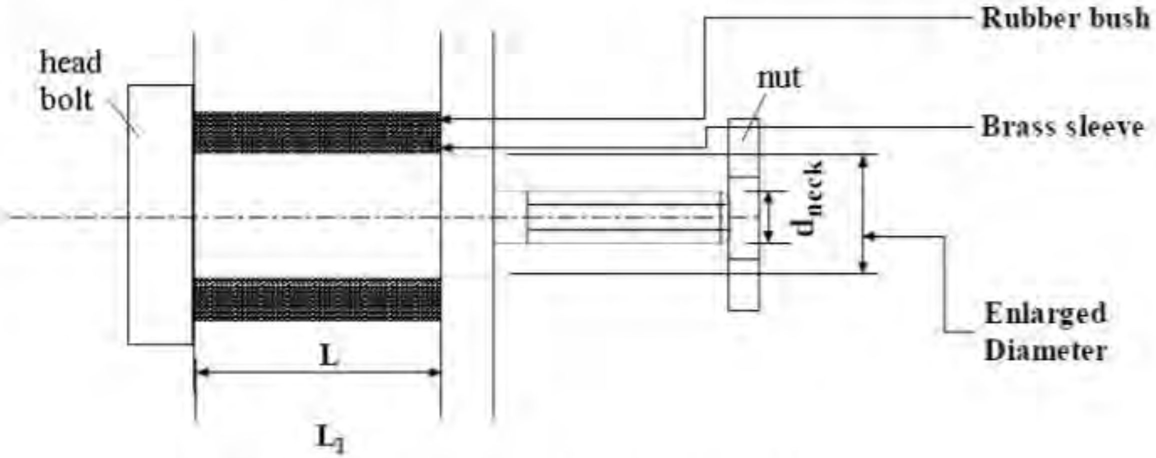
Substituting d_1=51mm, t_2=13mm and T = 143Nm, we have shear stress in flange τ_f as $\tau_f = \frac{2T}{(\pi d^2{}_1 t_2)}$

And this gives τ_f = 2.69 MPa which is much less than the yield shear value of flange material 60MPa.

Q.2: Determine the suitable dimensions of a rubber bush for a flexible coupling to connect of a motor and a pump. The motor is of 50 KW and runs at 300rpm. The shaft diameter is 50mm and the pins are on pitch circle diameter of 140mm. The bearing pressure on the bushes may be taken as 0.5MPa and the allowable shear and bearing stress of the pin materials are 25 MPa and 50 MPa respectively. The allowable shear yield strength of the shaft material may be taken as 60MPa.

A.2:

A typical pin in a bushed flexible coupling is as shown in **Figure-5.2.3.3.**



5.2.3.1 *- A typical pin for the bushings.*

There is an enlarged portion on which a flexible bush is fitted to absorb the misalignment. The threaded portion provided for a nut to tighten on the flange. Considering the whole pin there are three basic stresses developed in the pin in addition to the tightening stresses. There are (a) shear stresses at the unthreaded neck area (b) bending stress over the loaded portion (L) of the enlarged portion of the pin and (c) bearing stress.

However, before we consider the stresses we need to determine the pin diameter and length. Here the torque transmitted $T = \frac{50x10^3}{\left(\frac{2\pi x3000}{60}\right)} = 159\text{Nm}$ Based on torsional shear the shaft diameter $d = \left(\frac{16T}{\pi\tau_y}\right)^{\frac{1}{3}}$

Substituting T=159Nm and τ_y = 60MPa, we have d = 23.8mm. Let the shaft diameter be 25mm. From empirical relations we have. Pin diameter at the neck $d_{neck} = \frac{0.5d}{\sqrt{n}}$ where the number of pins $n = \frac{4d}{150} + 3$.

Substituting d = 25 mm we have n = 3.67 (say) 4 d_{neck} = 6.25 (say) 8mm

On this basis the shear stress at the neck = $\frac{T}{\left[\frac{\pi}{4}d_{neck}^2 n \frac{d_c}{2}\right]}$ which gives 11.29 MPa and this is much less than yield stress of the pin material.

There is no specific recommendation for the enlarged diameter based on d_{neck} but the enlarged diameters should be enough to provide a neck for tightening. We may choose $d_{enlarged}$ = 16mm which is a standard size. Therefore we may determine the inner diameter of the rubber bush as d_{bush} = Enlarged diameter of the pin + 2x brass sleeve thickness. A brass sleeve of 2mm thickness is sufficient and we have d_{bush} = 20mm. Rubber bush of core diameter up to 25mm are available in thickness of 6mm. Therefore we choose a bush of core diameter 20mm and thickness 6mm. In order to determine the bush length we have $T = npLd_{bush}\frac{d_c}{2}$

where p is the bearing pressure, (Ld_{bush}) is the projected area and d_c is the pitch circle diameter. Substituting T= 159Nm, p = 0.5MPa, d_{bush} = 0.02m and d_c = 0.14m we have L = 56.78 mm.

The rubber bush chosen is therefore of 20mm bore size, 6mm wall thickness and 60 mm long.

5.2.4 Summary of this Lecture

Detailed design procedure of a rigid flange coupling has been discussed in which failure modes of different parts such as the shaft, key, bolts and protecting flange are described. Design details of a flexible coupling using
rubber bushings have also been discussed. Here the failure modes of the flexible rubber bushings have been specially considered. Some typical problems have also been solved.

Lecture

Theme 6

Power Screws

6.1. Power Screw drives and their efficiency

6.1.1 Introduction

A power screw is a drive used in machinery to convert a rotary motion into a linear motion for power transmission. It produces uniform motion and the design of the power screw may be such that

(a) Either the screw or the nut is held at rest and the other member rotates as it moves axially. A typical example of this is a screw clamp.

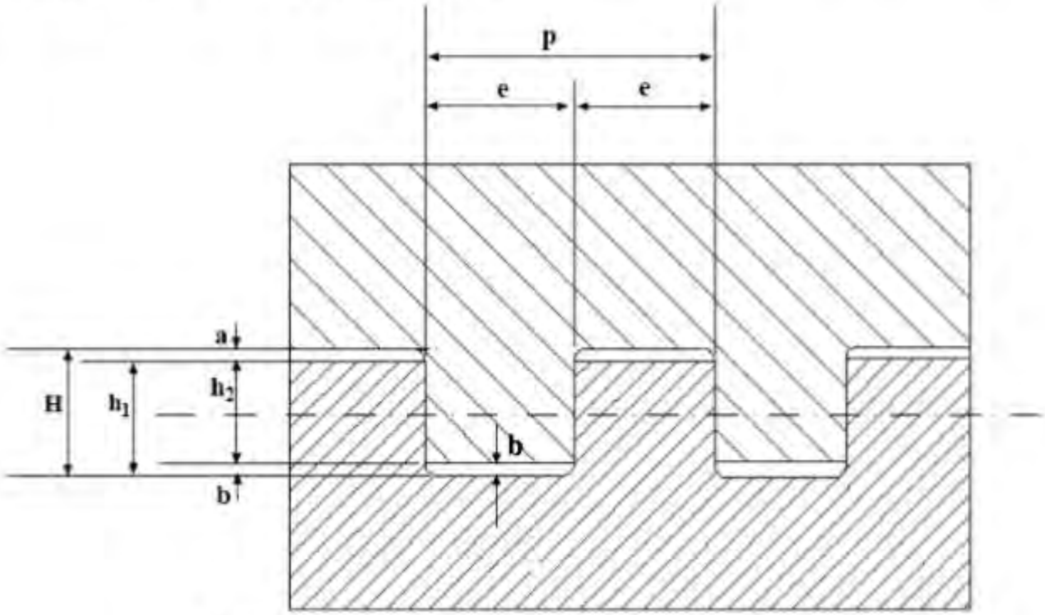
(b) Either the screw or the nut rotates but does not move axially. A typical example for this is a press.

Other applications of power screws are jack screws, lead screws of a lathe, screws for vices, presses etc.

Power screw normally uses square threads but ACME or Buttress threads may also be used. Power screws should be designed for smooth and noiseless transmission of power with an ability to carry heavy loads with high efficiency. We first consider the different thread forms and their proportions:

Square threads-

The thread form is shown in **figure-6.1.1.1**. These threads have high efficiency but they are difficult to manufacture and are expensive. The proportions in terms of pitch are: $h_1 = 0.5\ p$; $h_2 = 0.5\ p - b$; $H = 0.5\ p + a$; $e = 0.5\ p$ a and b are different for different series of threads.



6.1.1.1 *– Some details of square thread form*

There are different series of this thread form and some nominal diameters, corresponding pitch and dimensions a and b are shown in **table-6.1.1.1** as per I.S. 4694-1968.

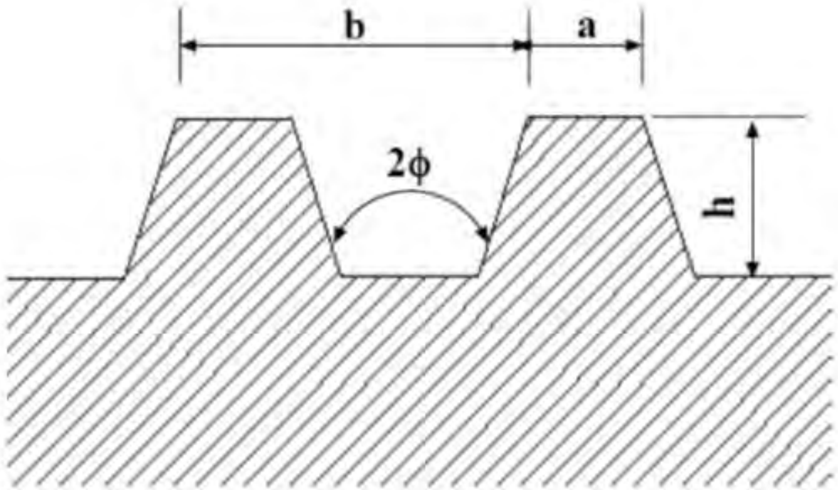
6.1.1.1 – Dimensions of three different series of square thread form.

Fine Series					Normal Series					Coarse Series				
Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)	Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)	Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)
10-22	2	2	0.25	0.25	22-28	2	5	0.25	0.5	22-28	2	8	0.25	0.5
22-62	2	3	0.25	0.25	30-36	2	6	0.25	0.5	30-38	2	10	0.25	0.5
115-175	5	6	0.25	0.5	115-145	5	14	0.5	1	115-130	5	22	0.5	1
250-300	10	12	0.25	0.5	240-260	10	22	0.5	1	250-280	10	40	0.5	1
420-500	20	18	0.5	1	270-290	10	24	0.5	1	290-300	10	44	0.5	1

According to IS-4694-1968, a square thread is designated by its nominal diameter and pitch, as for example, SQ 10 x 2 designates a thread form of nominal diameter 10 mm and pitch 2 mm.

Acme or trapezoidal threads

The acme thread form is shown in **figure- 6.1.1.2**. These threads may be used in applications such as lead screw of a lathe where loss of motion cannot be tolerated. The included angle $2\varphi = 29^o$ and other proportions are $a = \frac{p}{2.7}$ and h = 0.25 p + 0.25 mm where p is pitch



6.1.1.2 *– Some details of acme or trapezoidal thread forms.*

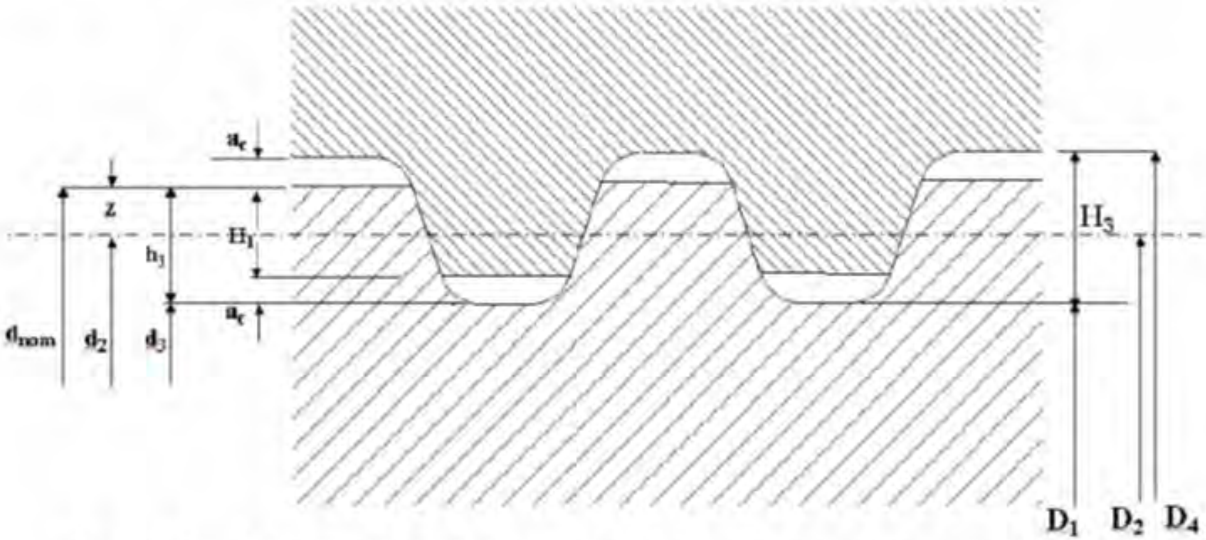
A metric trapezoidal thread form is shown in **figure- 6.1.1.3** and different proportions of the thread form in terms of the pitch are as follows:

Included angle = 30^o ; H_1= 0.5 p ; z= 0.25 p+H_1/2 ; H_3= h_3 = H_1+ a_c = 0.5 p + ac

a_c is different for different pitch, for example

a_c = 0.15 mm for p = 1.5 mm ; a_c = 0.25 mm for p = 2 to 5 mm;

a_c = 0.5 mm for p = 6 to 12 mm ; a_c = 1 mm for p = 14 to 44 mm.



6.1.1.3 *- Some details of a metric Trapezoidal thread form.*

Some standard dimensions for a trapezoidal thread form are given in **table- 6.1.1.2** as per IS 7008 (Part II and III) - 1973:

6.1.1.2 - Dimensions of a trapezoidal thread form.

Nominal Diameter (mm)	8	10	5	25	50	75	100	150	200	250	300
pitch (mm)	1.5	2	4	5	8	10	12	16	18	22	24

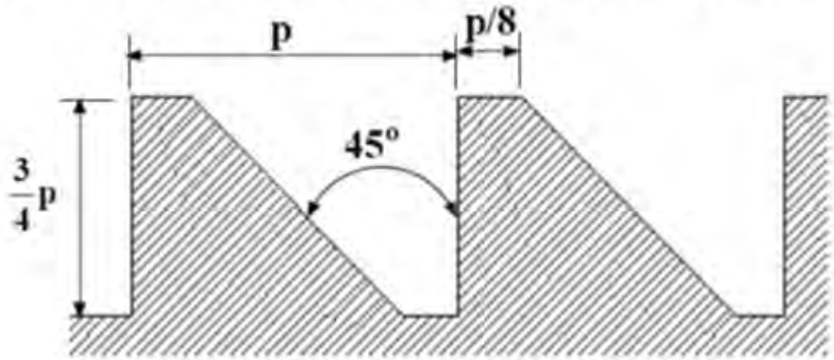
According to IS7008-1973 trapezoidal threads may be designated as, for example, Tr 50 x 8 which indicates a nominal diameter of 50 mm and a pitch of 8 mm.

Buttress thread

This thread form can also be used for power screws but they can transmit power only in one direction. Typical applications are screw jack, vices etc. A Buttress thread form is shown in **figure- 6.1.1.4**. and the proportions are shown in the figure in terms of the pitch.

On the whole the square threads have the highest efficiency as compared to other thread forms but they are less sturdy than the trapezoidal thread forms and the adjustment for wear is difficult for square threads.

When a large linear motion of a power screw is required two or more parallel threads are used. These are called multiple start power drives.



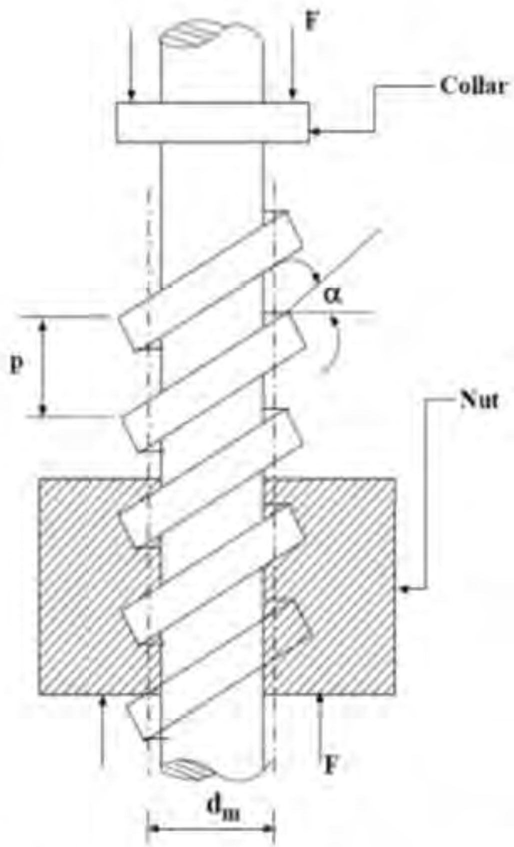
6.1.1.4 *– Some details of a Buttress thread form*

6.1.2 Efficiency of a power screw

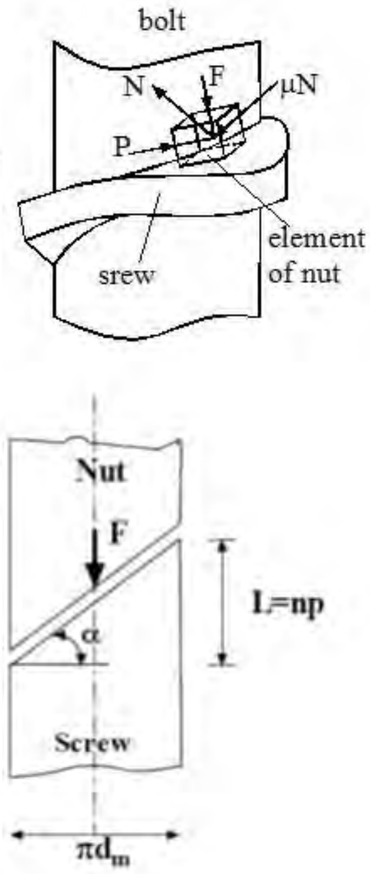
A square thread power screw with a single start is shown in **figure- 6.1.2.1**. Here p is the pitch, α the helix angle, dm the mean diameter of thread and F is the axial load. A developed single thread is shown in **figure- 6.1.2.2** where L = n p for a multi-start drive, n being the number of starts. In order to analyze the mechanics of the power screw we need to consider two cases:

(a) Raising the load

(b) Lowering the load.



***6.1.2.1** – A square thread power screw*



***6.1.2.2** - Development of a single thread*

Raising the load

This requires an axial force P as shown in **figure- 6.1.2.3**. Here N is the normal reaction and μN is the frictional force.

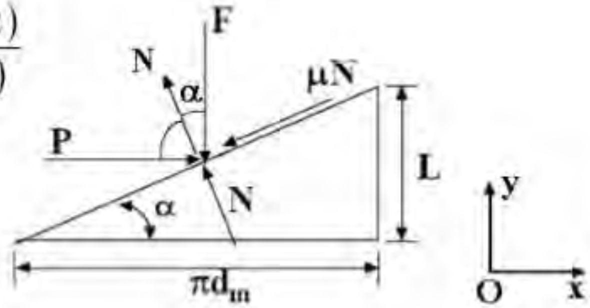
For equilibrium

y: $P - \mu N \cos\alpha - N \sin\alpha = 0$

x: $F + \mu N \sin\alpha - N \cos\alpha = 0$

This gives $N = F/(\cos\alpha - \mu\sin\alpha)$, $P = \dfrac{F(\mu\cos\alpha + \sin\alpha)}{(\cos\alpha - \mu\sin\alpha)}$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$



6.1.2.3 - Forces at the contact surface for raising the load.

Torque transmitted during raising the load is then given by $T_R = P\dfrac{d_m}{2} = F\dfrac{d_m}{2}\dfrac{(\mu\cos\alpha + \sin\alpha)}{(\cos\alpha - \mu\sin\alpha)}$

Since $\tan\alpha = \dfrac{L}{\pi d_m}$ we have

$$T_R = F\frac{d_m}{2}\frac{(\mu\pi d_m + L)}{(\pi d_m - \mu L)}$$

The force system at the thread during lowering the load is shown in **figure- 6.1.2.4**. For equilibrium we have the equations

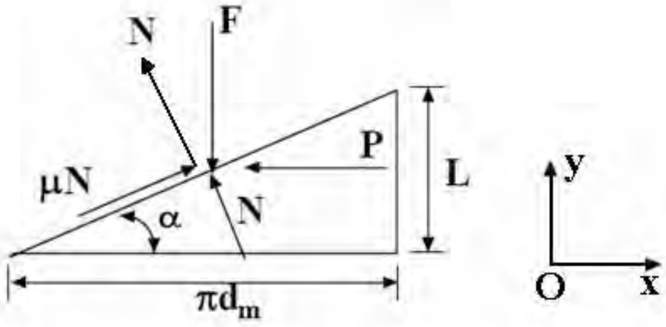
$P - \mu N \cos\alpha + N \sin\alpha = 0$

$F - N \cos\alpha - \mu N \sin\alpha = 0$

This gives

$$N = F/(\cos\alpha + \mu \sin\alpha)$$

$$P = \frac{F(\mu\cos\alpha - \sin\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$



***6.1.2.4** - Forces at the contact surface for lowering the load.*

Torque required to lower the load is given by

$$T_L = P\frac{d_m}{2} = F\frac{d_m}{2}\frac{(\mu\cos\alpha - \sin\alpha)}{(\cos\alpha + \mu\sin\alpha)}$$

And again taking $\tan\alpha = \frac{L}{\pi d_m}$ we have $T_L = F\frac{d_m}{2}\frac{(\mu\pi d_m - L)}{(\pi d_m + \mu L)}$

Condition for self locking

The load would lower itself without any external force if

$$\mu\pi d_m < L$$

and some external force is required to lower the load if

$$\mu\pi d_m \geq L$$

This is therefore the **condition for self locking**.

Efficiency of the power screw is given by

$$\eta = \frac{\text{Work output}}{\text{Work input}}$$

Here work output = F. L

Work input = P. πd_m

This gives $\eta = \frac{F}{P}\tan\alpha$

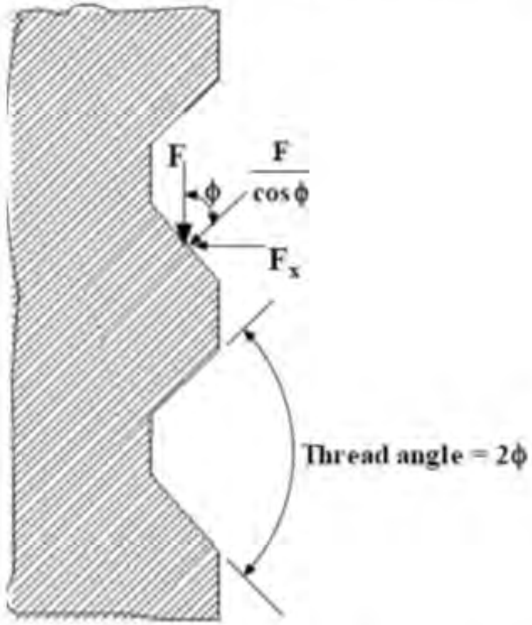
$$w = F \cdot L. \quad \text{Force} \times \frac{\text{vertical}}{\text{dis tan ce}}$$

$$w = P \cdot \pi d_m. \quad \text{Force} \times \text{dis tan ce}$$

The above analysis is for square thread and for trapezoidal thread some modification is required. Because of the thread angle the force normal to the thread surface is increased as shown in **figure- 6.1.2.5.** The torque is therefore given by

$$T = F\frac{d_m}{2}\frac{(\mu\pi d_m \sec\phi + L)}{(\pi d_m - \mu L \sec\phi)}$$

This considers the increased friction due to the wedging action. <u>The trapezoidal threads are not preferred because of high friction but often used due to their ease of machining</u>.



***6.1.2.5** – Normal force on a trapezoidal thread surface*

Bursting effect on the nut

Bursting effect on the nut is caused by the horizontal component of the axial load F on the screw and this is given by (**figure- 6.1.2.5**)

$F_x = F \tan \phi$

For an ISO metric nut $2\phi = 60^\circ$ and $F_x = 0.5777$ F.

Collar friction

If collar friction μc is considered then another term $\mu F d_c/2$ must be added to torque expression. Here dc is the effective friction diameter of the collar. Therefore we may write the torque required to raise the load as

$$T = F\frac{d_m}{2}\frac{(\mu\pi d_m + L)}{(\pi d_m - \mu L)} + \mu_c F\frac{d_c}{2}$$

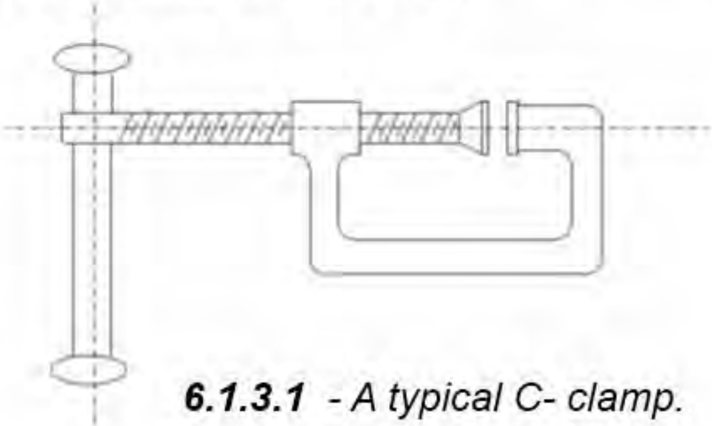
6.1.3 Problems with Answers

Q.1: The C-clamp shown in **figure-6.1.3.1** uses a 10 mm screw with a pitch of 2 mm. The frictional coefficient is 0.15 for both the threads and the collar. The collar has a frictional diameter of 16 mm. The handle is made of steel with allowable bending stress of 165 MPa. The capacity of the clamp is 700 N.

(a) Find the torque required to tighten the clamp to full capacity.

(b) Specify the length and diameter of the handle such that it will not bend unless the rated capacity of the clamp is exceeded. Use 15 N as the handle force. T=F·L

$$d_{nom} = 10mm$$
$$P = 2mm$$
$$\mu = 0.15$$
$$d_c = 16mm$$
$$\sigma_y = 165$$
$$F = 700N$$



***6.1.3.1** - A typical C- clamp.*

A.1.

1. Nominal diameter of the screw, d = 10 mm.

Pitch of the screw, p = 2 mm.

Choosing a square screw thread we have the following dimensions:

Root diameter, $d_3 = d_{nominal} - 2h_3 = 7.5$ mm (since $a_c = 0.25$mm and $h_3 = 0.5p + a_c$)

Pitch diameter, $d_2 = d_{nominal} - 2z = 8$ mm. (since z = 0.5 p)

Mean diameter, dm = (7.5+8)/2 = 7.75 mm.

Torque, $T = F\frac{d_m}{2}\frac{(\mu\pi d_m + L)}{(\pi d_m - \mu L)} + \mu_c F\frac{d_c}{2}$

Here F = 700 N, $\mu = \mu_c = 0.15$, L = p = 2 mm (assuming a single start screw thread) and d_c = 16 mm. Substituting these date. This gives <u>T=1.48Nm</u>. Ans.

Equating the torque required and the torque applied by the handle of length L we have 1.48 = 15 L since the assumed handle force is 15 N. This gives L= 0.0986 m. Let the handle length be <u>100 mm</u>.

The maximum bending stress that may be developed in the handle is

$\sigma = \frac{My}{I} = \frac{32M}{\pi d^3}$ where d is the diameter of the handle.

Taking the allowable bending stress as 165 MPa we have

$$d = \left(\frac{32M}{\pi\sigma_y}\right)^{1/3} = \left(\frac{32x1.48}{\pi x165x10^6}\right)^{1/3} = 4.5x10^{-3}\text{m} = 4.5\text{ mm}$$

With a higher factor of safety let <u>d = 10 mm</u>.

Q.2. A single square thread power screw is to raise a load of 50 KN. A screw thread of major diameter of 34 mm and a pitch of 6 mm is used. The coefficient of friction at the thread and collar are 0.15 and 0.1 respectively. If the collar frictional diameter is 100 mm and the screw turns at a speed of 1 rev s^{-1} find

(a) the power input to the screw.

(b) the combined efficiency of the screw and collar.

A.2. (a) Mean diameter, $d_m = d_{major} - p/2 = 34\text{-}3 = 31$ mm.

Torque $T = F\frac{d_m}{2}\frac{(\mu\pi d_m + L)}{(\pi d_m - \mu L)} + \mu_c F\frac{d_c}{2}$, substituting

Here F=5 x 10^3 N, d_m=31 mm, μ=0.15, μ_c=0.1, L=p= 6 mm and d_c=100 mm

Therefore $T = 50x10^3 x\frac{0.031}{2}\left(\frac{0.15\pi x 0.031 + 0.006}{\pi x 0.031 - 0.15x0.006}\right) + 0.1x50x10^3 x\frac{0.1}{2}$ = 416 Nm. Ans.

(b) The torque to raise the load only (T_0) may be obtained by substituting $\mu = \mu_c = 0$ in the torque equation. This gives

$$T_0 = F\frac{d_m}{2}\left(\frac{L}{\pi d_m}\right) = \frac{FL}{2\pi} = \frac{50x10^3 x0.006}{2\pi} = 47.75$$

Therefore $\eta = \frac{FL/2\pi}{T} = \frac{47.75}{416} = 0.1147$ i.e. 11.47%

6.1.4 Summary of this Lecture

Power screw drive in machinery is firstly discussed and some details of the thread forms used in such drives are given. The force system at the contact surface between the screw and the nut is analyzed and the torque required to raise and lower a load, condition for self locking and the efficiency of a power screw are derived. Typical problems on power screw drives are taken up and discussed.

Lecture

Theme 6

Power Screws

6.2. Design of power screws

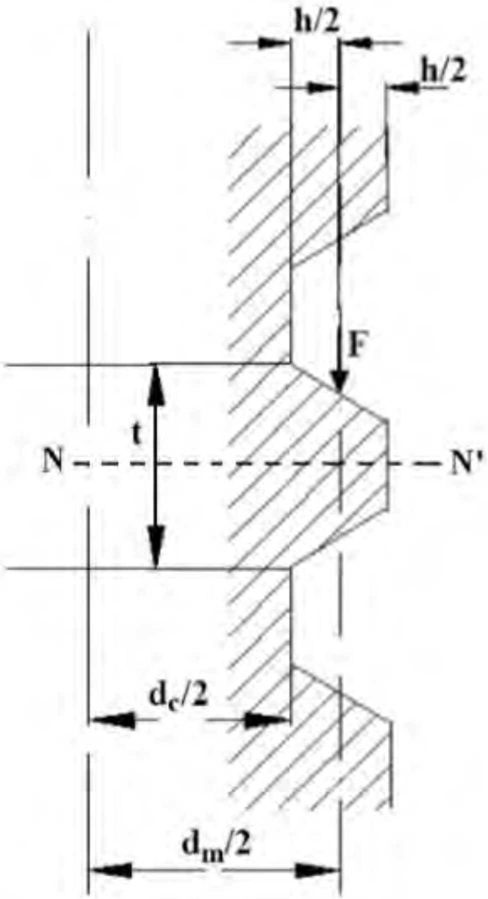
6.2.1 Stresses in power screws

Design of a power screw must be based on the stresses developed in the constituent parts. A power screw is subjected to an axial load and a turning moment. The following stresses would be developed due to the loading:

a) **Compressive stress** is developed in a power screw due to axial load. Depending on the slenderness ratio it may be necessary to analyze for buckling. The compressive stress σ_c is given by σ_c=P/pd_c2 where d_c is the core diameter and if slenderness ratio λ is more than 100 or so buckling criterion must be used. λ is defined as λ = L/K where I=AK2 and L is the length of the screw. Buckling analysis yields a critical load P_c and if both ends are assumed to be hinged critical load is given by $P_C = \pi^2 \frac{EI}{L^2}$. In general the equation may be written as $P_C = n\pi^2 \frac{EI}{L^2}$ where n is a constant that depends on end conditions.

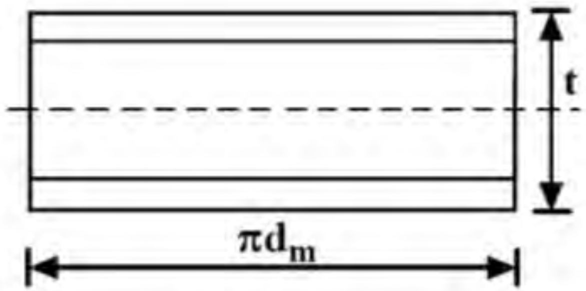
b) Torsional shear stress is developed in the screw due to the turning moment and this is given by $\tau = 16T/\pi\delta_c^3$ where T is the torque applied.

c) Bending stresses are developed in the screw thread and this is illustrated in **figure-6.2.1.1**.

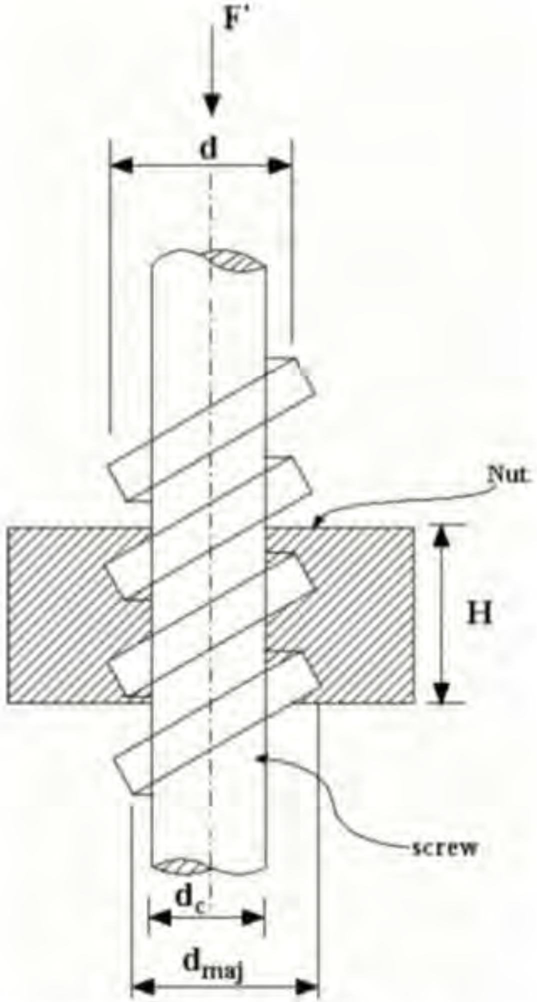


6.2.1.1 *- Loading and bending stresses in screw threads*

The bending moment M=F'h/2 and the bending stress on a single thread is given by σ_b=My/I. Here y=t/2, I=$\pi d_m t^3/12$ and F′ is the load on a single thread. **Figure-6.2.1.2** shows a developed thread and **figure-6.2.1.3** shows a nut and screw assembly. This gives the bending stress at the thread root to be $\sigma_b=3F'h/\pi d_m t^2$. This is clearly the most probable place for failure.



6.2.1.2 *- Dimensions of a developed thread*



6.2.1.3 *- A screw and nut assembly*

Assuming that the load is equally shared by the nut threads

d) Bearing stress $\sigma_b r$ at the threads is given by

$$\sigma_{br} = \frac{F' / n'}{\pi d_m h}$$

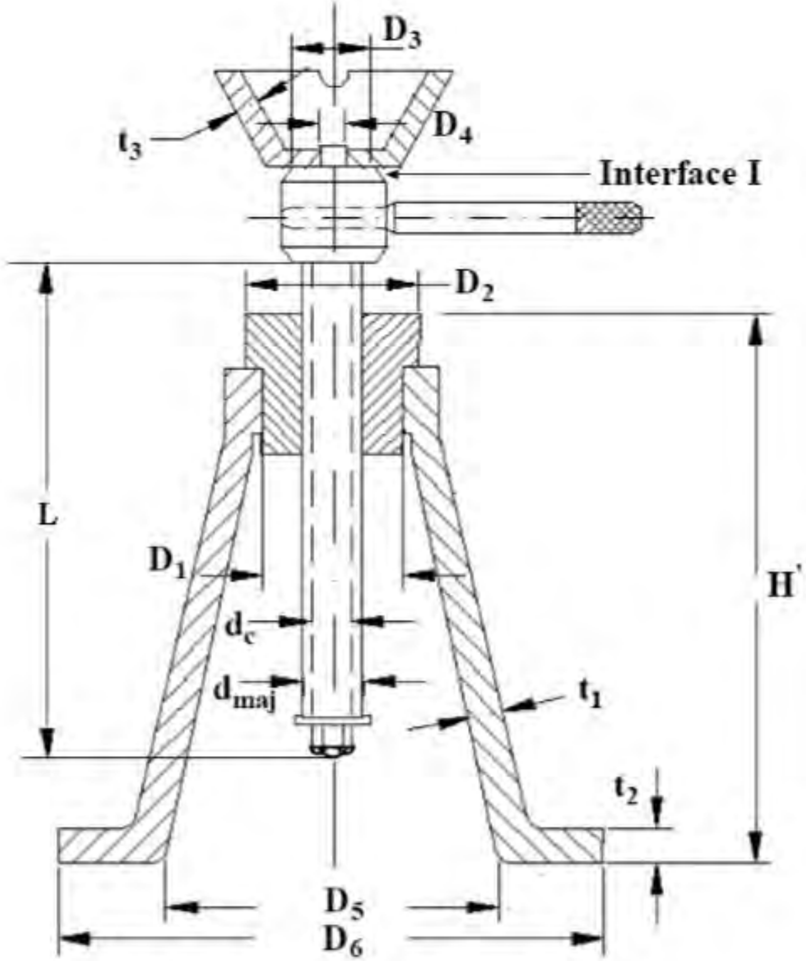
e) Again on similar assumption **shear stress** τ at the root diameter is given by

$$\tau = \frac{F' / n'}{\pi d_c t}$$

Here n is the number of threads in the nut. Since the screw is subjected to torsional shear stress in addition to direct or transverse stress combined effect of bending, torsion and tension or compression should be considered in the design criterion.

6.2.2 Design procedure of a Screw Jack

A typical screw jack is shown in **figure-6.2.2.1** . It is probably more informative to consider the design of a jack for a given load and lift. We consider a reasonable value of the load to be 100KN and lifting height to be 500mm. The design will be considered in the following steps:



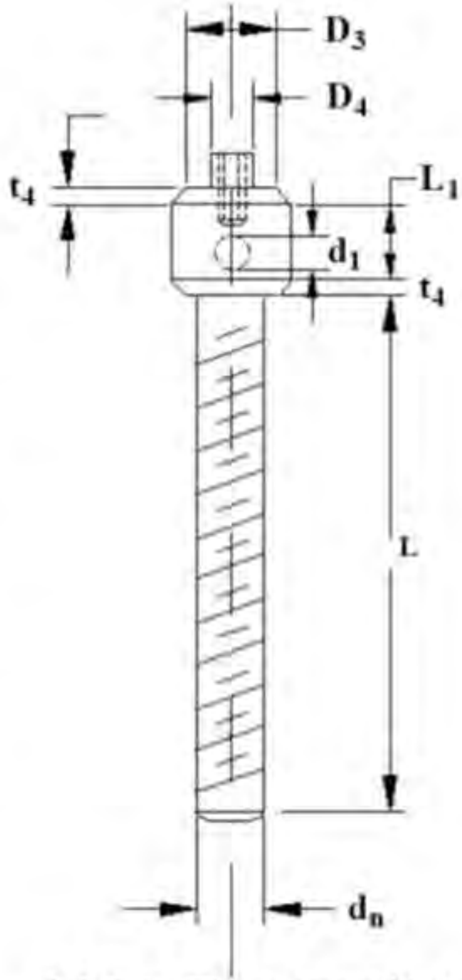
6.2.2.1 *- A typical screw jack*

1. Design of the screw

A typical screw for this purpose is shown in **figure-6.2.2.2.**

Let us consider a mild steel screw for which the tensile and shear strengths may be taken to be approximately 448MPa and 224 MPa respectively. Mild steel being a ductile material we may take the compressive yield strength to be also close to 448MPa. Taking a very high factor of safety of 10 due to the nature of the application and considering the axial compression the core diameter of the screw dc is given by $d_c = \sqrt{\dfrac{100\text{x}10^3}{\dfrac{\pi}{4}\left(\dfrac{448\text{x}10^6}{10}\right)}}$

which gives $d_c \approx 54$ mm.



6.2.2.2 *- The screw with the provision for tommy bar attachment*

From the chart of normal series square threads in **table- 6.1.1.1** the nearest standard nominal diameter of 70 mm is chosen, with pitch p=10 mm.
Therefore, core diameter d_c = 60 mm , Major diameter d_{maj} = 70mm , Mean diameter d_m = 65 mm , Nominal diameter d_n = 70mm.

The torque required to raise the load is given by $T = \frac{Fd_m}{2}\left(\frac{l+\mu\pi d_m}{\pi d_m - \mu l}\right)$

Where l = np, n being the number of starts. Here we have a single start screw and hence l = p =10mm, d_m = 65mm, F = 100X10^{3}N
Taking a safe value of μ for this purpose to be 0.26 and substituting the values we get T = 1027 Nm.

6.1.1.1 – Dimensions of three different series of square thread form.

Fine Series					Normal Series					Coarse Series				
Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)	Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)	Nominal Dia (mm)	Steps (mm)	Pitch (mm)	a (mm)	b (mm)
10-22	2	2	0.25	0.25	22-28	2	5	0.25	0.5	22-28	2	8	0.25	0.5
22-62	2	3	0.25	0.25	30-36	2	6	0.25	0.5	30-38	2	10	0.25	0.5
115-175	5	6	0.25	0.5	115-145	5	14	0.5	1	115-130	5	22	0.5	1
250-300	10	12	0.25	0.5	240-260	10	22	0.5	1	250-280	10	40	0.5	1
420-500	20	18	0.5	1	270-290	10	24	0.5	1	290-300	10	44	0.5	1

Check for combined stress

The screw is subjected to a direct compressive stress σc and a torsional shear stress τ . The stresses are given by

$$\sigma_c = \frac{4F}{\pi d_c^2} = \frac{4x100x10^3}{\pi x(0.06)^2} = 35.3MPa$$

$$\tau = \frac{16T}{\pi d_c^3} = \frac{16x1027}{\pi x(0.060)^3} = 24.22MPa$$

The principal stress can be given by

$$\sigma_{1,2} = \frac{35.3}{2} \pm \sqrt{\left(\frac{35.3}{2}\right)^2 + (24.22)^2} = 47.6MPa \text{ and } -12.31 \text{ MPa}$$

and maximum shear stress τ_{max} = 29.96 MPa.

The factor of safety in compression = 448/12.31=36.4 and in shear = 224/29.96=7.48. Therefore the screw dimensions are safe. Check for buckling and thread stress are also necessary. However this can be done after designing the nut whose height and number of threads in contact is needed to determine the free length of the screw.

2. Design of the nut

A suitable material for the nut, as shown in **figure- 6.2.2.3**, is phosphor bronze which is a Cu-Zn alloy with small percentage of Pb and the yield stresses may be taken as

Yield stress in tension σ_{ty} = 125MPa,

Yield stress in compression σ_{cy} = 150MPa,

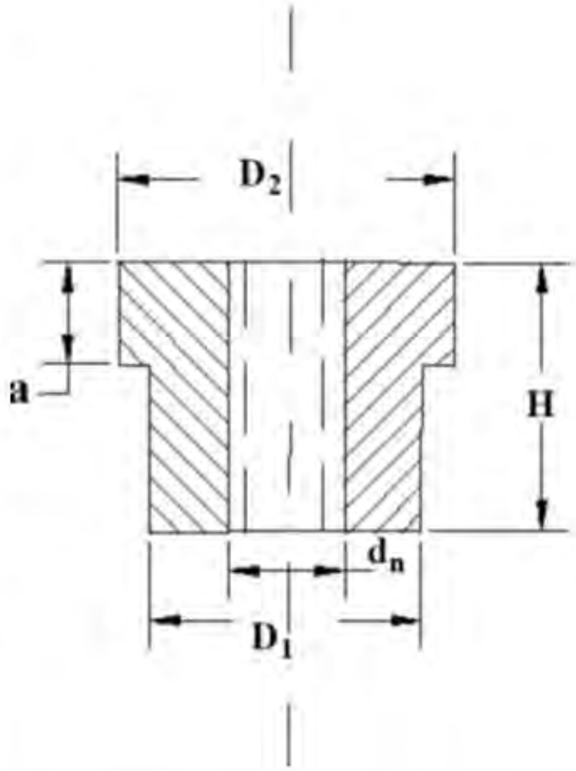
Yield stress in shear τ_y = 105MPa,

Safe bearing pressure P_b = 15MPa.

Considering that the load is shared equally by all threads bearing failure may be avoided if

$$F = \frac{\pi}{4}\left(d_{maj}^{\,2} - d_c^{\,2}\right)P_b n'$$

where n' is the number of threads in contact. Substituting values in the above equation we have n' = 6.52. Let n' =8. Therefore H = $n'p$ = 8X10 = 80mm.



***6.2.2.3** - A phosphor bronze nut for the screw jack*

The nut threads are also subjected to crushing and shear. Considering crushing failure we have

$$F = n' \frac{\pi}{4}\left(d_{maj}^2 - d_c^2\right) \sigma_C$$

This gives σ_c = 12.24 MPa which is adequately safe since σ_{cy} = 150 MPa and therefore crushing is not expected. <u>To avoid shearing of the threads on the nut we may write $F = \pi d_{maj}\, t\, n'\, \tau$</u> where t is the thread thickness which for the square thread is p/2 ie 5. This gives τ=11.37MPa and since τ_y=105MPa <u>shear failure of teeth is not expected</u>. Due to the screw loading the nut needs to be checked for tension also and we may write

$$CF = \frac{\pi}{4}\left(D_1^2 - d_c^2\right) \sigma_{ty}$$

A correlation factor C for the load is used to account for the twisting moment. With C=1.3 and on substitution of values in the equation D_1 works out to be 70mm. But D_1 needs to be larger than d_{maj} and we take D_1 = 100mm.

<u>We may also consider crushing</u> of the collar of the nut and to avoid this we may write

$$F = \frac{\pi}{4}\left(D_2^2 - D_1^2\right)\sigma_{cy} = \pi\left[\left(\frac{D_2}{2}\right)^2 - \left(\frac{D_1}{2}\right)\right]\sigma_{cy}.$$

Substituting values we have D_2 = 110 mm. To allow for the collar margin we take D_2 =120mm. **Considering shearing** of the nut collar $\pi D_1 a \tau_y = F$. Substituting values we have a = 4mm Let a = 15mm.

3. Buckling of the Screw.

Length L of the screw = Lifting height + H.

This gives L= 500+80 = 580 mm

With the nominal screw diameter of 70mm ,

I = $\frac{\pi(0.07)^4}{64} = 1.178X10^{-6}$ and K = $\sqrt{\frac{I}{A}} = \sqrt{\frac{1.178X10^{-6}}{\frac{\pi}{4}(0.07)^2}} = 0.0175\text{mm.}$

The slenderness ration

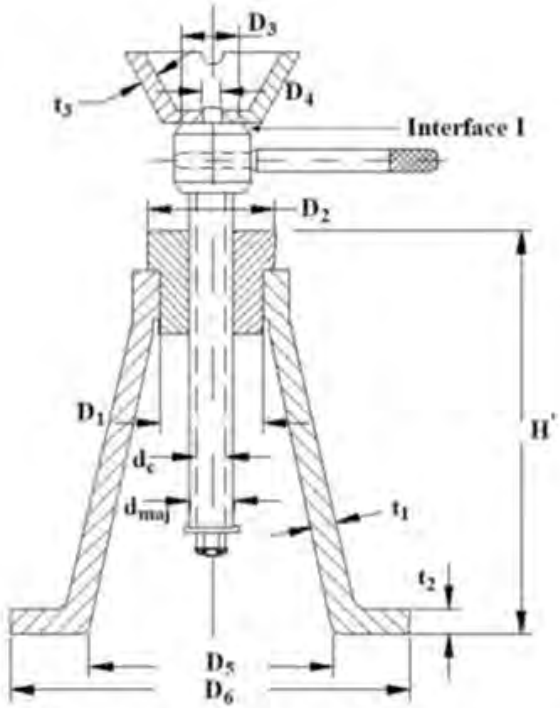
$$\lambda = \frac{L}{K} = \frac{0.58}{0.0175} \simeq 33$$

This value of slenderness ratio is small (< 40) and **the screw may be treated as a short column . No buckling of the screw is therefore expected.**

4. Tommy bar

A typical tommy bar for the purpose is shown in **figure-6.2.2.4.a**.

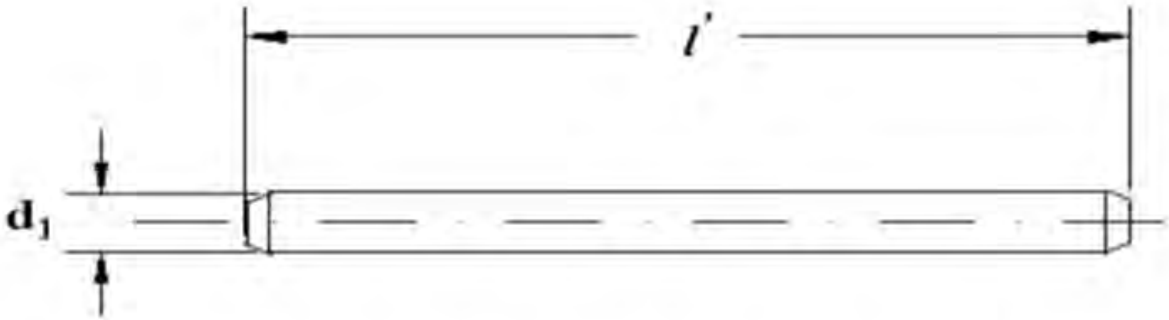
Total torsional moment without the collar friction is calculated in section 6.2.2.1 and T = 1027 Nm. The collar friction in this case (see **figure-6.2.2.1**) occurs at the interface I. However in order to avoid rotation of the load when the screw rotates a loose fitting of the cup is maintained.



6.2.2.1 *- A typical screw jack*

6.2.2.4.a *- A typical tommy bar with a holding end.*

Length $l^{/}$ of the tommy bar = l_1 + and we may write the torque T as 3D $T= F1l^{/}$ Where F_1 is the maximum force applied at the tommy bar end and this may be taken as approximately 400 N . This gives $l^{/}$=1027/400 = 2.56m. This length of the tommy bar is too large and one alternative is to place the tommy bar centrally and apply force at both the ends. This alternative design of the tommy bar is also shown in **figure-6.2.2.4.b**



6.2.2.4.b *- A typical centrally located tommy bar*

The bar is subjected to a bending moment and its maximum value may be taken as1027 Nm. This means to avoid bending we may write $\frac{\pi}{32} d_1^3 \sigma_{ty} = 1027$ where d_1 is the tommy bar diameter as shown in **figure- 6.2.2.4.b** If we choose a M.S bar of σ_{ty} = 448MPa the tommy bar diameter d_1 works out to be $\underline{d_1 = 0.0285m}$.

Let $\underline{d_1 = 30\text{ mm}}$ and we choose $\underline{d_2 = 40\text{mm}}$.

5. Other dimensions

D_3 = (1.5 to 1.7) d. Let D_3 = 112 mm.

$D_4 = D_3/2$= 56 mm.

Let L_1 = 100 mm and t_4 = 10 mm.

Lecture

Theme 7

Design of Springs

7.1.Introduction to Design of Helical Springs

Mechanical springs have varied use in different types of machines. We shall briefly discuss here about some applications, followed by design aspects of springs in general.

7.1.1 Definition of spring: Spring act as a flexible joint in between two parts or bodies

7.1.2 Objectives of Spring

Following are the objectives of a spring when used as a machine member:

7.1.2.1. Cushioning , absorbing , or controlling of energy due to shock and vibration.

Car springs or railway buffers to control energy, springs-supports and vibration dampers.

7.1.2.2. Control of motion

Maintaining contact between two elements (cam and its follower)

In a cam and a follower arrangement, widely used in numerous applications, a spring maintains contact between the two elements. It primarily controls the motion.

Creation of the necessary pressure in a friction device (a brake or a clutch)

A person driving a car uses a brake or a clutch for controlling the car motion. A spring system keep the brake in disengaged position until applied to stop the car. The clutch has also got a spring system (single springs or multiple springs) which engages and disengages the engine with the transmission system.

Restoration of a machine part to its normal position when the applied force is withdrawn (a governor or valve)

A typical example is a governor for turbine speed control. A governor system uses a spring controlled valve to regulate flow of fluid through the turbine, thereby controlling the turbine speed.

7.1.2.3. Measuring forces

Spring balances, gages

7.1.2.4. Storing of energy

In clocks or starters

The clock has spiral type of spring which is wound to coil and then the stored energy helps gradual recoil of the spring when in operation. Nowadays we do not find much use of the winding clocks.

Before considering the design aspects of springs we will have a quick look at the spring materials and manufacturing methods.

7.1.3 Commonly used spring materials

One of the important considerations in spring design is the choice of the spring material. Some of the common spring materials are given below.

Hard-drawn wire:

This is cold drawn, cheapest spring steel. Normally used for low stress and static load. The material is not suitable at subzero temperatures or at temperatures above 1200C.

Oil-tempered wire:

It is a cold drawn, quenched, tempered, and general purpose spring steel. However, it is not suitable for fatigue or sudden loads, at subzero temperatures and at temperatures above 1800C.

When we go for highly stressed conditions then alloy steels are useful.

Chrome Vanadium:

This alloy spring steel is used for high stress conditions and at high temperature up to 220^{0}C. It is good for fatigue resistance and long endurance for shock and impact loads.

Chrome Silicon:

This material can be used for highly stressed springs. It offers excellent service for long life, shock loading and for temperature up to 250^{0}C.

Music wire:

This spring material is most widely used for small springs. It is the toughest and has highest tensile strength and can withstand repeated loading at high stresses. However, it can not be used at subzero temperatures or at temperatures above 120^{0}C.

Normally when we talk about springs we will find that the music wire is a common choice for springs.

Stainless steel:

Widely used alloy spring materials.

Phosphor Bronze / Spring Brass:

It has good corrosion resistance and electrical conductivity. That's the reason it is commonly used for contacts in electrical switches. Spring brass can be used at subzero temperatures.

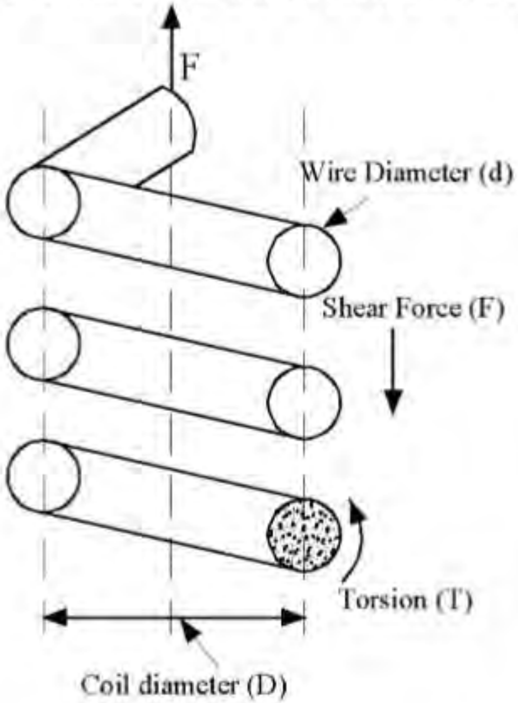
7.1.4 Spring manufacturing processes

If springs are of very small diameter and the wire diameter is also small then the springs are normally manufactured by a cold drawn process through a mangle. However, for very large springs having also large coil diameter and wire diameter one has to go for manufacture by hot processes. First one has to heat the wire and then use a proper mangle to wind the coils.

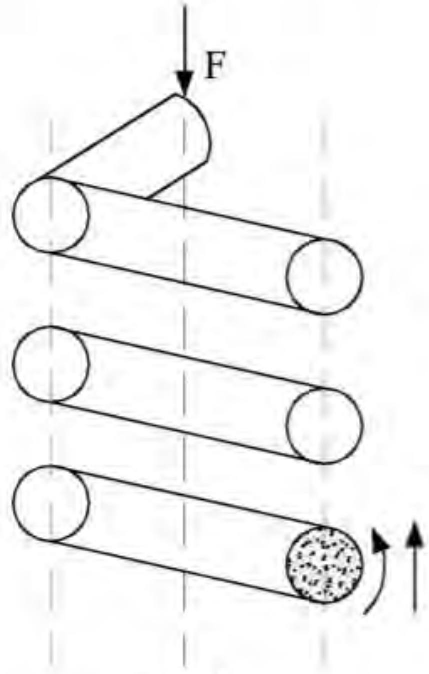
Two types of springs which are mainly used are, **helical springs and leaf springs**. We shall consider in this course the design aspects of two types of springs.

7.1.5. Helical spring

The figures below show the schematic representation of a helical spring acted upon by a tensile load F (Fig.7.1.1) and compressive load F (Fig.7.1.2). The circles denote the cross section of the spring wire. The cut section, i.e. from the entire coil somewhere we make a cut, is indicated as a circle with shade.



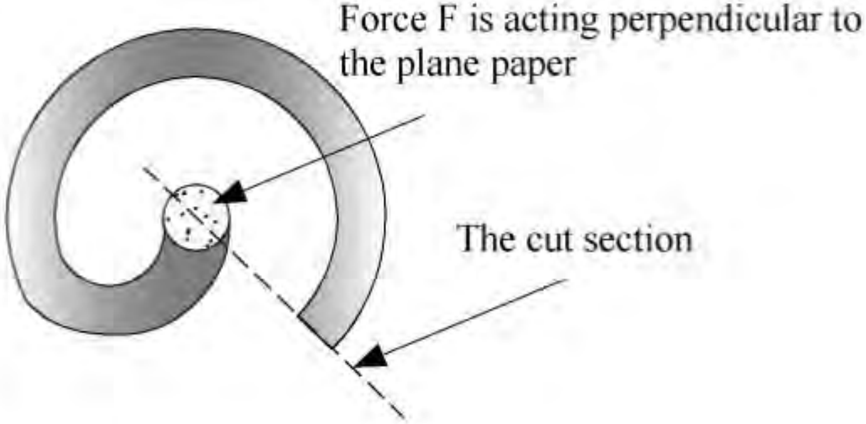
7.1.1



7.1.2

If we look at the free body diagram of the shaded region only (the cut section) then we shall see that at the cut section, vertical equilibrium of forces will give us force, F as indicated in the figure. This F is the shear force. The torque T, at the cut section and it's direction is also marked in the figure. There is no horizontal force coming into the picture because externally there is no horizontal force present. So from the fundamental understanding of the free body diagram one can see that any section of the spring is experiencing a torque and a force. Shear force will always be associated with a bending moment.

However, in an ideal situation, when force is acting at the centre of the circular spring and the coils of spring are almost parallel to each other, no bending moment would result at any section of the spring (no moment arm), except torsion and shear force. The Fig.7.1.3 will explain the fact stated above.

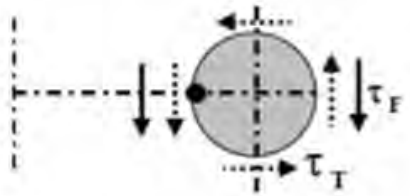


7.1.3

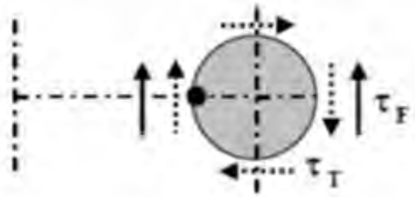
7.1.5.1 Stresses in the helical spring wire:

From the free body diagram, we have found out the direction of the internal torsion T and internal shear force F at the section due to the external load F acting at the centre of the coil.

The cut sections of the spring, subjected to tensile and compressive loads respectively, are shown separately in the Fig.7.1.4 and 7.1.5. The broken arrows show the shear stresses (τ_T) arising due to the torsion T and solid arrows show the shear stresses (τ_F)due to the force F. It is observed that for both tensile load as well as compressive load on the spring, maximum shear stress (τ_T + τ_F) always occurs at the inner side of the spring. Hence, failure of the spring, in the form of crake, is always initiated from the inner radius of the spring.



7.1.4



7.1.5

The radius of the spring is given by D/2. Note that D is the mean diameter of the spring.

The torque T acting on the spring is $T = F \times \frac{D}{2}$ (7.1.1)

If d is the diameter of the coil wire and polar moment of inertia, $I_p = \frac{\pi d^4}{32}$, the shear stress in the spring wire due to torsion is

$$\tau_T = \frac{Tr}{I_p} = \frac{F \times \frac{D}{2} \times \frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{8FD}{\pi d^3} \qquad (7.1.2)$$

Average shear stress in the spring wire due to force F is

$$\tau_F = \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2} \qquad (7.1.3)$$

Therefore, maximum shear stress the spring wire is

$$\tau_T + \tau_F = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2} \quad \text{or} \quad \tau_{max} = \frac{8FD}{\pi d^3}\left(1 + \frac{1}{\frac{2D}{d}}\right)$$

$$\text{or} \quad \tau_{max} = \frac{8FD}{\pi d^3}\left(1 + \frac{1}{2C}\right) \quad \text{where, } C = \frac{D}{d},$$

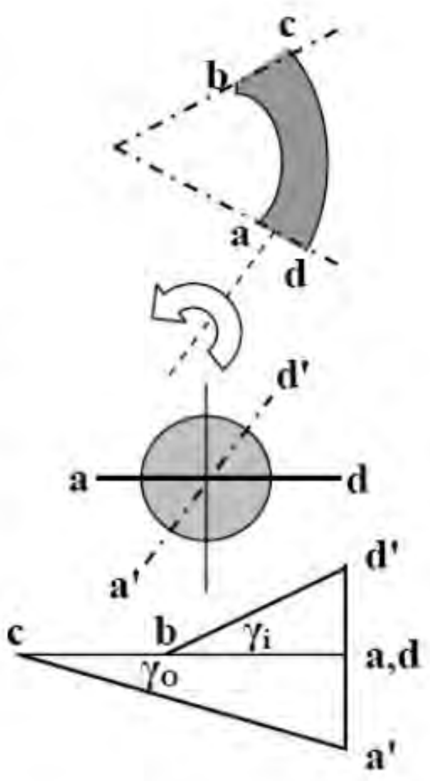
is called the spring index.

$$\text{finally,} \quad \tau_{max} = (K_s)\frac{8FD}{\pi d^3} \quad \text{where, } K_s = 1 + \frac{1}{2C} \qquad (7.1.4)$$

The above equation gives maximum shear stress occurring in a spring. $\mathbf{K_s}$ is the shear stress correction factor.

7.1.5.2 Stresses in helical spring *with curvature effect*

What is curvature effect? Let us look at a small section of a circular spring, as shown in the Fig.7.1.6. Suppose we hold the section b-c fixed and give a rotation to the section a-d in the anti clockwise direction as indicated in the figure, then it is observed that line a-d rotates and it takes up another position, say a'-d'. The inner length a-b being smaller compared to the outer length c-d, the shear strain γ_i at the inside of the spring will be more than the shear strain γ_o at the outside of the spring. Hence, for a given wire diameter, a spring with smaller diameter will experience more difference of shear strain between outside surface and inside surface compared to its larger counter part. The above **phenomenon** is termed as **curvature effect**. **So more is the spring index (C=D/d) the lesser will be the curvature effect.** For example, the suspensions in the railway carriages use helical springs. These springs have large wire diameter compared to the diameter of the spring itself. In this case curvature effect will be predominantly high.



7.1.6

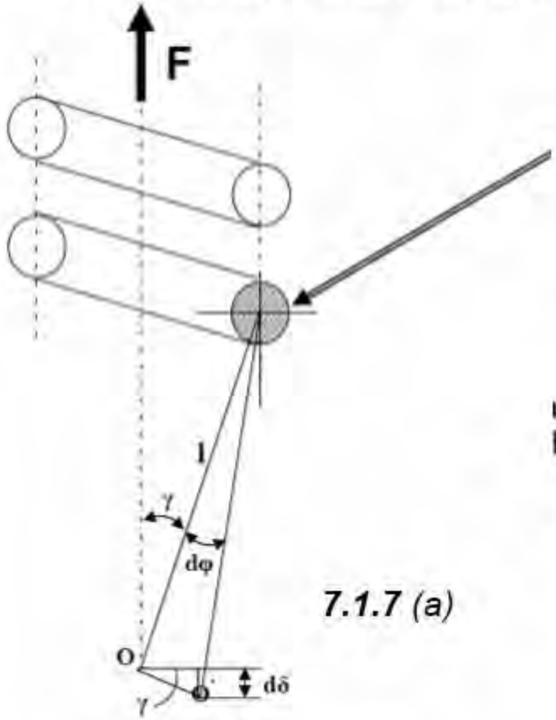
To take care of the curvature effect, the earlier equation for maximum shear stress in the spring wire is modified as,

$$\tau_{max} = (K_w)\frac{8FD}{\pi d^3} \tag{7.1.5}$$

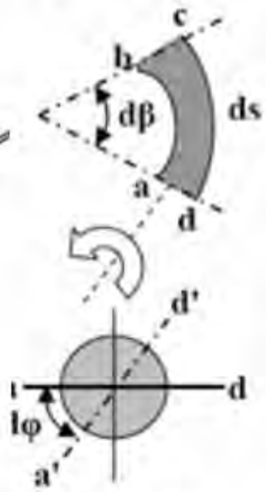
Where, $\mathbf{K_W}$ is Wahl correction factor, which takes care of both curvature effect and shear stress correction factor and is expressed as,

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} \tag{7.1.6}$$

7.1.5.3 Deflection of helical spring



***7.1.7** (a)*



***7.1.7** (b)*

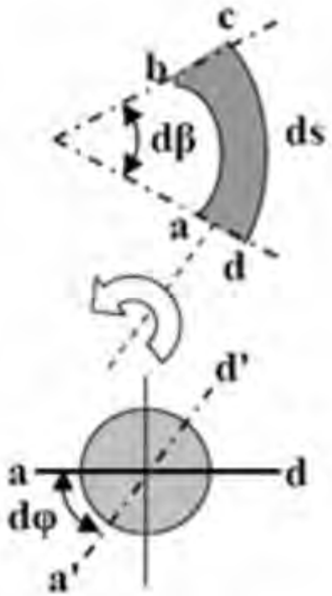
The Fig.7.1.7(a) and Fig.7.1.7 (b) shows a schematic view of a spring, a cross section of the spring wire and a small spring segment of length dl. It is acted upon by a force F. From simple geometry we will see that the deflection, **δ**, in a helical spring is given by the formula,

$$\delta = \frac{8FD^3N}{Gd^4} \tag{7.1.7}$$

Where, N is the number of active turns and G is the shear modulus of elasticity. Now what is an active coil? The force F cannot just hang in space, it has to have some material contact with the spring. Normally the same spring wire e will be given a shape of a hook to support the force F. The hook etc., although is a part of the spring, they do not contribute to the deflection of the spring. Apart from these coils, other coils which take part in imparting deflection to the spring are known as active coils.

7.1.5.4 How to compute the deflection of a helical spring?

Consider a small segment of spring of length ds, subtending an angle of dβ at the center of the spring coil as shown in Fig.7.1.7(b). Let this small spring segment be considered to be an active portion and remaining portion is rigid. Hence, we consider only the deflection of spring arising due to application of force F. The rotation, dφ, of the section a-d with respect to b-c is given as,



7.1.7(b)

$$d\varphi = \frac{Tds}{GI_p} = \frac{F \times \frac{D}{2} \times \frac{D}{2} \times d\beta}{G \times \frac{\pi d^4}{32}} = \frac{8FD^2(d\beta)}{G\pi d^4} \qquad (7.1.8)$$

The rotation, dφ will cause the end of the spring O to rotate to O', shown in Fig.7.1.7(a). From geometry, O-O' is given as,

$$O - O' = l\varphi$$

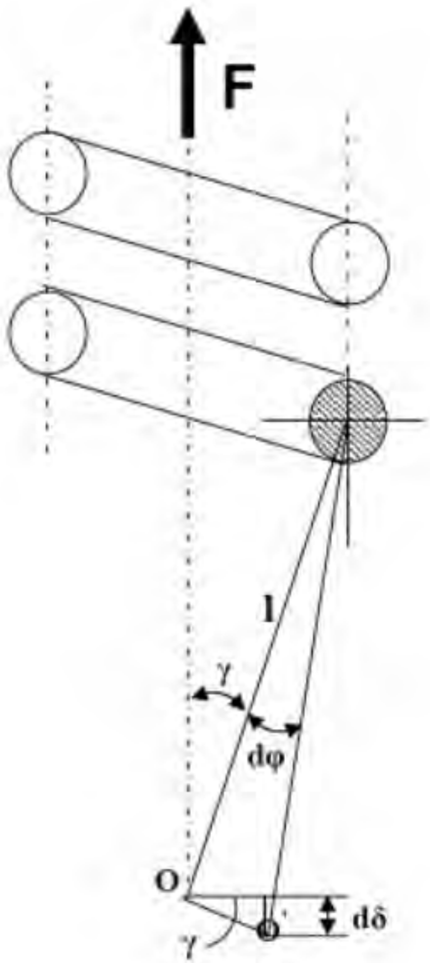
However, the vertical component of O-O' only will contributes towards spring deflection. Due to symmetric condition, there is no lateral deflection of spring, ie, the horizontal component of O-O' gets cancelled.

The vertical component of O-O', dδ, is given as,

$$\mathbf{d\delta = ld\varphi \sin\gamma = ld\varphi \times \frac{D}{2l}}$$

$$\mathbf{= \frac{8FD^2(d\beta)}{G\pi d^4} \times \frac{D}{2}}$$

$$\mathbf{= \frac{4FD^3}{G\pi d^4} d\beta}$$



7.1.7(a)

Total deflection of spring, **δ**, can be obtained by integrating the above expression for entire length of the spring wire.

$$\delta = \int_0^{2\pi N} \frac{4FD^3(d\beta)}{G\pi d^4}$$

Simplifying the above expression we get,

$$\delta = \frac{8FD^3N}{Gd^4} \tag{7.1.9}$$

The above equation is used to compute the deflection of a helical spring. Another important design parameter often used is the spring rate. It is defined as,

$$K = \frac{F}{\delta} = \frac{Gd^4}{8D^3N} \tag{7.1.10}$$

Here we conclude on the discussion for important design features, namely, stress, deflection and spring rate of a helical spring.

Exercise 1.

A helical spring of wire diameter 6mm and spring index 6 is acted by an initial load of 800N.

After compressing it further by 10mm the stress in the wire is 500MPa. Find the number of active coils. **G** = 84000MPa.

Solution 1.

D=spring index(C) x d=36 mm

$$\tau_{max} = (K_w)\frac{8FD}{\pi d^3} \qquad K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.2525$$

$$\text{or, } 500 = 1.2525 \times \frac{8F \times 36}{\pi \times 6^3}$$

(Note that in case of static load one can also use K_S instead of K_W)

$$\therefore F = 940.6N$$

$$K = \frac{F}{\delta} = \frac{940.6 - 800}{10} = 14 N/mm$$

$$K = \frac{Gd^4}{8D^3N}$$

$$\text{or, } N = \frac{Gd^4}{K8D^3N} = \frac{84000 \times 6^4}{14 \times 8 \times 36^3} \approx 21 \text{turns}$$

Problems with Answers

Q1. What are the objectives of a spring?

A1. The objectives of a spring are to cushion, absorb, or controlling of energy arising due to shock and vibration. It is also used for control of motion, storing of energy and for the purpose of measuring forces.

Q2. What is the curvature effect in a helical spring? How does it vary with spring index?

A2. For springs where the wire diameter is comparable with the coil diameter, in a given
segment of the spring, the inside length of the spring segment is relatively shorter than the outside length. Hence, for a given magnitude of torsion, shearing strain is more in the inner segment than the outer segment. This unequal shearing strain is called the curvature effect. Curvature effect decreases with the increase in spring index.

Q3. What are the major stresses in a helical spring?

A3. The major stresses in a helical spring are of two types, shear stress due to torsion and direct shear due to applied load.

Lecture

Theme 7

Design of Springs

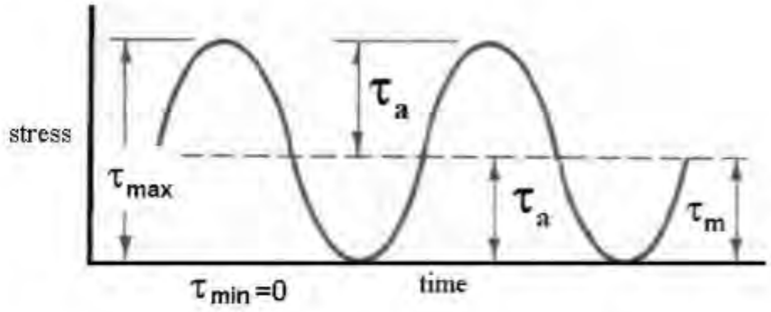
7.2. Design of Helical Springs for Variable Load

7.2.1 Design of helical spring for variable load

In the earlier lecture, we have learned about design of helical springs for static loads. In many applications, as for example in railway carriages or in automobile suspension systems the helical springs used are constantly under variable load. Hence, it is understood that whenever there is a variable load on a spring the design procedure should include the effect of stress variation in the spring wire. The methodology used is the modified Soderberg method. we have learnt about Soderberg method in earlier chapter, here, the necessary modifications applicable to helical spring design will be discussed.

In the case of a spring, whether it is a compression spring or an extension spring, reverse loading is not possible. For example, let us consider a compression spring placed between two plates. The spring under varying load can be compressed to some maximum value and at the most can return to zero compression state (in practice, some amount of initial compression is always present), otherwise, spring will loose contact with the plates and will get displace from its seat. Similar reason holds good for an extension spring, it will experience certain amount of extension and again return to at the most to zero extension state, but it will never go to compression zone.

Due to varying load, the stress pattern which occurs in a spring with respect to time is shown in Fig.7.2.1. The load which causes such stress pattern is called repeated load. The spring materials, instead of testing under reversed bending, are tested under repeated torsion.



7.2.1

From Fig.7.2.1 we see that ,

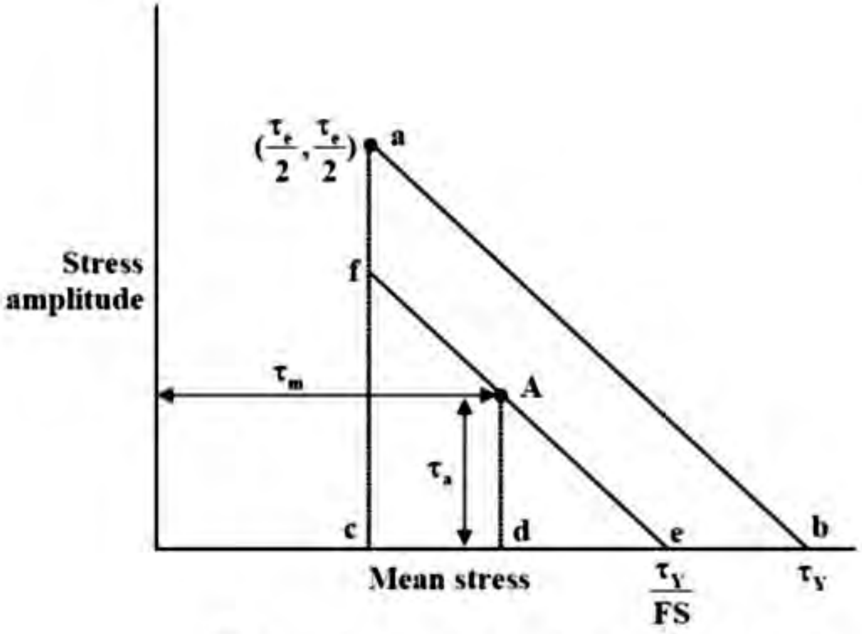
$$\tau_m = \tau_a = \frac{\tau_{max}}{2} \qquad (7.2.1)$$

Where, τ_a is known as the stress amplitude and τ_m is known as the mean stress or the average stress. We know that for varying stress, the material can withstand stress not exceeding endurance limit value. Hence, for repeated torsion experiment, the mean stress and the stress amplitude become,

$$\tau_m = \tau_a = \frac{\tau_{max}}{2} = \frac{\tau_e}{2} \qquad (7.2.2)$$

7.2.1.1 Soderberg failure criterion

The modified Soderberg diagram for repeated stress is shown in the Fig 7.2.2.



Soderberg failure criterion for springs

7.2.2

The stress being repeated in nature, the co-ordinate of the point **a** is $\tau_e/2$, $\tau_e/2$. For safe design, the design data for the mean and average stresses, τ_a and τ_m respectively, should be below the line a-b. If we choose a value of factor of safety (FS), the line a-b shifts to a newer position as shown in the figure. This line e-f in the figure is called a safe stress line and the point A (τ_m, τ_a) is a typical safe design point.

Considering two similar triangles, **abc** and **Aed** respectively, a relationship between the stresses may be developed and is given as,

$$\frac{\tau_a}{\frac{\tau_Y}{FS}-\tau_m}=\frac{\frac{\tau_e}{2}}{\tau_Y-\frac{\tau_e}{2}} \tag{7.2.3}$$

where τY is the shear yield point of the spring material.
In simplified form, the equation for Soderberg failure criterion for springs is

$$\frac{1}{FS}=\frac{\tau_m}{\tau_Y}+\frac{\tau_a}{\tau_Y}(\frac{2\tau_Y}{\tau_e}-1) \tag{7.2.4}$$

The above equation is further modified by considering the shear correction factor, K_s and Wahl correction factor, K_w. It is a normal practice to multiply τm by K_s and to multiply τ_a by K_w.

$$\frac{1}{FS}=\frac{K_s\tau_m}{\tau_Y}+\frac{K_w\tau_a}{\tau_Y}(\frac{2\tau_Y}{\tau_e}-1) \tag{7.2.5}$$

The above equation for Soderberg failure criterion for will be utilized for the designing of springs subjected to variable load.

7.2.1.2 Estimation of material strength

It is a very important aspect in any design to obtain correct material property. The best way is to perform an experiment with the specimen of desired material. Tensile test experiments as we know is relatively simple and less time consuming. This experiment is used to obtain yield strength and ultimate strength of any given material. However, tests to determine endurance limit is extremely time consuming. Hence, the ways to obtain material properties is to consult design data book or to use available relationships, developed through experiments, between various material properties. For the design of springs, we will discuss briefly, the steps normally used to obtain the material properties.

One of the relationships to find out ultimate strength of a spring wire of diameter **d** is,

$$\sigma_{ut} = \frac{A_s}{d^{m_s}} \qquad (7.2.6)$$

For some selected materials, which are commonly used in spring design, the values of As and ms are given in the table below.

Wire Type	A_s	m_s
Hard-drawn wire	1510	0.201
Oil-tempered wire	1610	0.193
Chrome-vanadium wire	1790	0.155
Chrome-silicon wire	1960	0.091
Music wire	2060	0.163

The above formula gives the value of ultimate stress in MPa for wire diameter in mm. Once the value of ultimate strength is estimated, the shear yield strength and shear endurance limit can be obtained from the following table developed through experiments for repeated load.

Wire Type	$\frac{\tau_e}{\sigma_{ult}}$	$\frac{\tau_y}{\sigma_{ult}}$
Hard-drawn wire	0.21	0.42
Oil-tempered wire	0.22	0.45
Chrome-vanadium wire	0.20	0.51
Chrome-silicon wire	0.20	0.51
Music wire	0.23	0.40
302 SS wire	0.20	0.46

Hence, as a rough guideline and on a conservative side, values for shear yield point and shear endurance limit for major types of spring wires can be obtained from ultimate strength as,

$$\frac{\tau_y}{\sigma_{ult}} = 0.40 \quad \text{and} \quad \frac{\tau_e}{\sigma_{ult}} = 0.20 \qquad (7.2.7)$$

With the knowledge of material properties and load requirements, one can easily utilize Soderberg equation to obtain spring design parameters.

7.2.2 Types of springs

There are mainly two types of helical springs, compression springs and extension springs. Here we will have a brief look at the types of springs and their nomenclature.

7.2.2.1 Compression springs

Following are the types of compression springs used in the design.

(a) Plain ends

Plain end spring

7.2.3

Total coils, N_T	: N
Solid length, L_S	: $d(N_T + 1)$
Free length, L	: $L_s + \delta_{max} + \delta_{allowance}$
Pitch, p	: $(L - d)/N$

In the above nomenclature for the spring, **N** is the number of active coils, i.e., only these coils take part in the spring action. However, few other coils may be present due to manufacturing consideration, thus total number of coils, $\mathbf{N_T}$ may vary from total number of active coils.

Solid length, $\mathbf{L_S}$ is that length of the spring, when pressed, all the spring coils will clash with each other and will appear as a solid cylindrical body.

The spring length under no load condition is the free length of a spring. Naturally, the length that we visualise in the above diagram is the free length.

Maximum amount of compression the spring can have is denoted as δ_{max}, which is calculated from the design requirement. The addition of solid length and the δ_{max} should be sufficient to get the free length of a spring. However, designers consider an additional length given as $\delta_{allowance}$. This allowance is provided to avoid clash between to consecutive spring coils. As a guideline, the value of δ allowance is generally 15% of δ_{max}.

The concept of pitch in a spring is the same as that in a screw.

(b) Plain and Ground ends

Total coils, N_T : **N + 1**

Solid length, L_S : **d (N_T)**

Free length, L $L_S + \delta_{max} + \delta_{allowance}$

Pitch, p : **L / (N + 1)**

Plain and Ground end spring

7.2.4

The top and bottom of the spring is grounded as seen in the figure. Here, due to grounding, one total coil is inactive.

(c) **Squared or closed ends**

Total coils, N_T : $N + 2$

Solid length, L_S : $d(N_T + 1)$

Free length, L $L_S + \delta_{max} + \delta_{allowance}$

Pitch, p : $(L - 3d)/N$

Squared or closed end spring

7.2.5

In the Fig 7.2.5 it is observed that both the top as well as the bottom spring is being pressed to make it parallel to the ground instead of having a helix angle. Here, it is seen that two full coils are inactive.

(d) Squared and ground ends

Total coils, N_T : $N + 2$

Solid length, L_S : $d (N_T)$

Free length, L $L_S + \delta_{max} + \delta_{allowance}$

Pitch, p : $(L - 2d) / N$

Squared and ground end spring

7.2.6

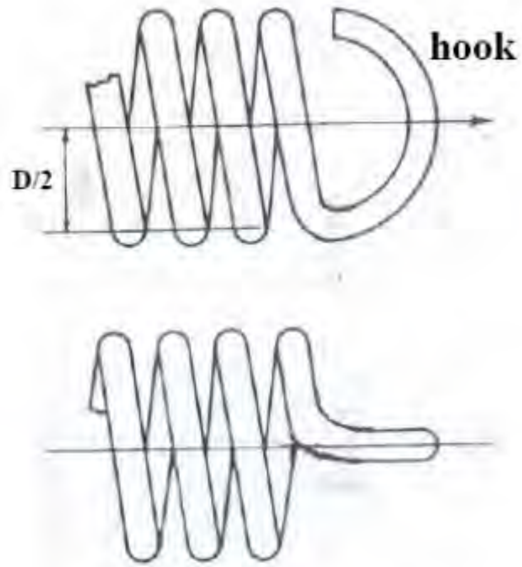
It is observed that both the top as well as the bottom spring, as earlier one, is being pressed to make it parallel to the ground, further the faces are grounded to allow for proper seat. Here also two full coils are inactive.

7.2.2.2 Extension springs

Part of an extension spring with a hook is shown in
Fig.7.2.7. The nomenclature for the extension spring is given below.

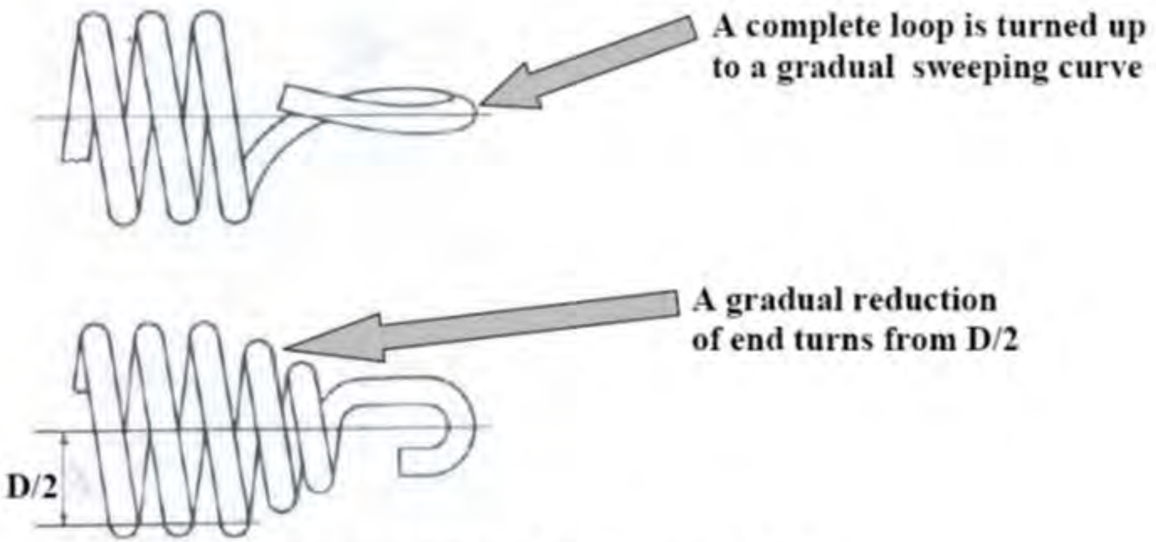
Body length, LB : d (N + 1)
Free length, L : LB + 2 hook diameter.

here, N stands for the number of active coils. By putting the hook certain amount of stress concentration comes in the bent zone of the hook and these are substantially weaker zones than the other part of the spring. One should take up steps so that stress concentration in this region is reduced. For the reduction of stress concentration at the hook some of the modifications of spring are shown in Fig 7.2.8.



Extension spring

7.2.7



Extension springs with improved ends

7.2.8

7.2.3 Buckling of compression spring

Buckling is an instability that is normally shown up when a long bar or a column is applied with compressive type of load. Similar situation arise if a spring is too slender and long then it sways sideways and the failure is known as buckling failure. Buckling takes place for a compressive type of springs. Hence, the steps to be followed in design to avoid buckling is given below.

Free length (L) should be less than 4 times the coil diameter (D) to avoid buckling for most situations. For slender springs central guide rod is necessary.

A guideline for free length (L) of a spring to avoid buckling is as follows,

$$L < \frac{\pi D}{C_e}\sqrt{\frac{2(E-G)}{2G+E}} \qquad (7.2.8)$$

$$L < 2.57\frac{D}{C_e}, \text{ for steel}$$

, Where, C_e is the end condition and its values are given below.

If the spring is placed between two rigid plates, then end condition may be taken as 0.5. If after calculation it is found that the spring is likely to buckle then one has to use a guide rod passing through the center of the spring axis along which the compression action of the spring takes place.

7.2.4 Spring surge (*critical frequency*)

If a load F act on a spring there is a downward movement of the spring and due to this movement a wave travels along the spring in downward direction and a to and fro motion continues. This phenomenon can also be observed in closed water body where a disturbance moves toward the wall and then again returns back to the starting of the disturbance. This particular situation is called surge of spring. If the frequency of surging becomes equal to the natural frequency of the spring the resonant frequency will occur which may cause failure of the spring. Hence, one has to calculate natural frequency, known as the fundamental frequency of the spring and use a judgment to specify the operational frequency of the spring.

The fundamental frequency can be obtained from the relationship given below.

Fundamental frequency:

$$f = \frac{1}{2}\sqrt{\frac{Kg}{W_s}} \qquad\qquad f = \frac{1}{4}\sqrt{\frac{Kg}{W_s}}$$

Both ends within flat plates

(7.2.9) **One end free and other end on flat plate.** (7.2.10)

Where, **K** : **Spring rate**

weight = $2.47\gamma d^2 DN$

W_S : **Spring**

and **d** is the wire diameter, **D** is the coil diameter, **N** is the number of active coils and **γ** is the specific weight of spring material.

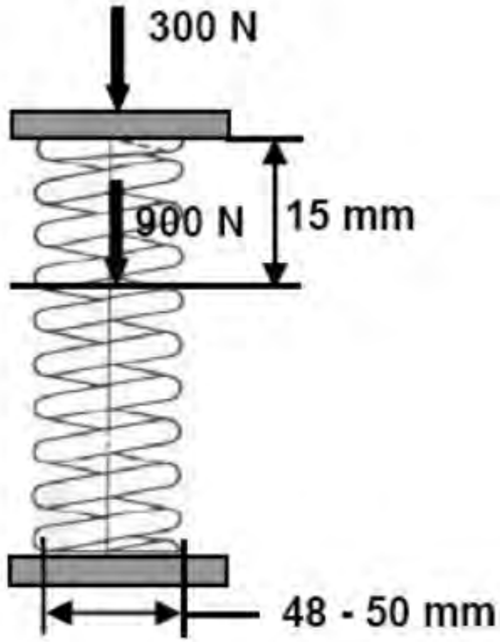
The operational frequency of the spring should be at least 15-20 times less than its fundamental frequency. This will ensure that the spring surge will not occur and even other higher modes of frequency can also be taken care of.

7.2.5 A problem on spring design

A helical spring is acted upon by a varying load of 300 N to 900 N respectively as shown in the figure. The spring deflection will be around 15 mm and outside diameter of the spring should be within 48-50 mm. (Fig. 7.2.5.1)

Solution

To design the spring for the given data, the most important parameter is the spring index. The spring index decides the dimension of the spring with respect to chosen wire diameter. Normally the spring index varies over a wide range from 3-12. For higher value of the spring index the curvature effect will be less, but relatively size of the spring and stress in the spring wire will increase. However, the effects will be some what opposite if the value of spring index is lower. Hence, it is better to start the iteration process with the spring index of 6-7.



7.2.5.1

Let us start the problem with spring index, C=6 and wire diameter, d=7 mm.
The above choice gives us a coil mean diameter, D =42 mm. Thereby, the outside diameter of the coil is 49 mm, which is within the given limit.

Computation of stresses:

The mean load, $F_m = \dfrac{300 + 900}{2} = 600\text{N}$

stress amplitude , $F_a = \dfrac{900 - 300}{2} = 300\text{N}$

Shear stress concentration factor, $k_s = 1 + \dfrac{1}{12} = 1.083.$

Wahl correctionfactor, $k_w = \dfrac{4\text{x}6 - 1}{4\text{x}6 - 4} + \dfrac{0.615}{6} = 1.253.$

So the value of mean shear stress, $\tau_m = 1.083 \times \dfrac{8 \times 600 \times 42}{\pi \times (7)^3} = 202.62\text{MPa}$

and the value of stress amplitude , $\tau_a = 1.253 \times \dfrac{8 \times 300 \times 42}{\pi \times (7)^3} = 117.21\text{MPa}$

Estimation of material properties:

As no specific use of the spring is mentioned in the problem, let us take Chrome Vanadium as the spring material. This alloy spring steel is used for high stress conditions and at high temperatures, it is also good for fatigue resistance and long endurance for shock and impact loads.

Ultimate strength of the material $\sigma_{ult} = \frac{1790}{(7)^{0.155}} = 1324 \text{ MPa}$

From the relationship of σ_{ult} to τ_y (yield point) and endurance limit, τ_e we find that

for $\tau_y = \sigma_{ult} \times 0.51 = 675.2\text{MPa}$

and $\tau_e = \sigma_{ult} \times 0.2 = 264.8\text{MPa}$

From Soderberg equation,

$$\frac{\tau_a}{\frac{\tau_y}{FS}} = \frac{\frac{\tau_e}{2}}{\tau_y - \frac{\tau_e}{2}} \qquad \frac{1}{FS} = \frac{\tau_m}{\tau_y} + \frac{\tau_a}{\tau_y}\left(\frac{2\tau_y}{\tau_e} - 1\right)$$

$$\frac{1}{FS} = \frac{202.62}{675.2} + \frac{117.21}{675.2}\left(\frac{2\times 675.2}{264.8} - 1\right) = 1.01$$

$$\therefore FS \approx 1.00$$

Factor of safety, FS=1.0 implies that the design do not consider any unforeseen effect that may cause extra stresses in the spring. Normally in design of springs it is better to consider a factor of safety which should be in the vicinity of 1.3-1.5.

In order to increase the value of FS, in the next iteration, natural choice for the spring index, C is 5 and d = 8 mm. Because C=7 and d = 6 mm will lead to more stress on the wire and the value of FS will not improve.
With C=5 and d=8 mm and following the similar procedure as in previous iteration we have,

$$k_s = 1.1, \quad k_w = 1.311$$

Therefore,

$$\tau_m = \frac{1.1 \times 8 \times 600 \times 40}{\pi \times 8^3} = 131.3\text{MPa} \qquad \tau_a = \frac{1.311 \times 8 \times 300 \times 40}{\pi \times 8^3} = 78.24\text{MPa}$$

Material properties: $\sigma_{ut} = \dfrac{1790}{(8)^{0.155}} = 1297\text{MPa}$

Finally, $\tau_y = 661.4\text{MPa} \qquad \tau_e = 259.4\text{MPa}$

$$\therefore \frac{1}{FS} = \frac{131.3}{661.4} + \frac{78.24}{661.4}\left(\frac{2 \times 661.4}{259.4} - 1\right) = 0.684 \qquad FS = 1.46$$

The factor of safety obtained is acceptable. Therefore the value of spring index is 5 and corresponding wire diameter is 8mm.

Hence, mean spring diameter, D=40 mm.

Outer diameter of spring, D_o=40+8=48 mm, This value is within the prescribed limit.

Inner diameter of spring, . D_i=32mm.

Spring rate,

$$k = \frac{900 - 300}{15} = 40N/mm = 40 \times 10^3 N/m$$

Once the value of stiffness is known, then the value of number of active turns, N of the spring is,

$$k = \frac{Gd^4}{8D^3N} \qquad \therefore N = \frac{80 \times 10^3 \times 8^4}{8 \times (40)^3 \times k} = 16$$

$$\therefore \delta max = \frac{8 \times 900 \times (40)^3 \times 16}{80 \times 10^3 \times 8^4} = 22.5mm$$

In the above equation, G = 80000 MPa.

Spring Nomenclature:

Let us select the type of spring as squared and ground ends. For this type of spring the value of free length is,

Сейчас не удается отобразить рисунок.

$$L = L_S + \delta_{max} + \delta_{allowance}$$

$$\text{where, } L_S = dN_T = 8.0 \times (16 + 2) = 144\,\text{mm}$$

$$\delta_{allowance} = 15\%\delta_{max}$$

$$L = 18 \times 8 + 22.5 + 15\%\delta_{max} \approx 170\text{mm}$$

$$\text{Pitch, } p = \frac{L - 2d}{N} = \frac{170 - 16}{16} = 9.625\,\text{mm}$$

Check for buckling:

We know that for steel,

$$L < 2.57\frac{D}{C_e} = 206mm$$

Here, for the given spring seat configuration, Ce = 0.5

The free length of the spring, 170 mm is less than the critical length for buckling, 206mm. Therefore the design is safe.

Check for critical frequency:

In order to find the critical frequency of the spring, the weight of the spring is to be first computed,

$$W_s = \frac{\pi d^2}{4}(\pi DN)(\gamma)$$

$$W_s = 2.47\gamma d^2 DN$$

$$W_s = 2.47 \times \left(8X10^{-3}\right)^2 \times \left(40X10^{-3}\right) \times 16 \times 7800 \times 9.81 = 7.74N$$

Therefore,

The fundamental frequency of the spring (for both ends within flat plates),

$$f = \frac{1}{2}\sqrt{\frac{Kg}{W_s}} == \frac{1}{2}\sqrt{\frac{40X10^3 X9.81}{7.74}} \approx 112.6Hz$$

Safe frequency for design should be at least 20 times less than the fundamental frequency to take care of more number of harmonics. Therefore, the spring frequency for should be around 6 Hz.

Problems with Answers

Q1. Do the helical spring experience reverse loading? What is the loading type called when varying load acts on a helical spring?

A1. The helical spring experiences only repeated load. It cannot experience reverse loading, because the spring will lose contact with the end supports.

Q2. What modification in Soderberg diagram is required when it is used for design of helical springs?

A2. In the earlier Soderberg diagram, we have used in the design for varying loads on the machine member, had only stress amplitude in the endurance limit representation, since, endurance limit value was for complete reversed loading. Here, in spring design, we use endurance limit value for repeated loads only. Hence, we have both stress amplitude and mean stress value of equal magnitude, 2eт. Therefore, the endurance limit representation in Soderberg diagram changes to,22eeтт.

Q3. What should be the safe frequency of a helical spring?

A3. Safe frequency for design should be at least 20 times less than the fundamental frequency of the spring to take care of more number of harmonics.

Lecture

Theme 7

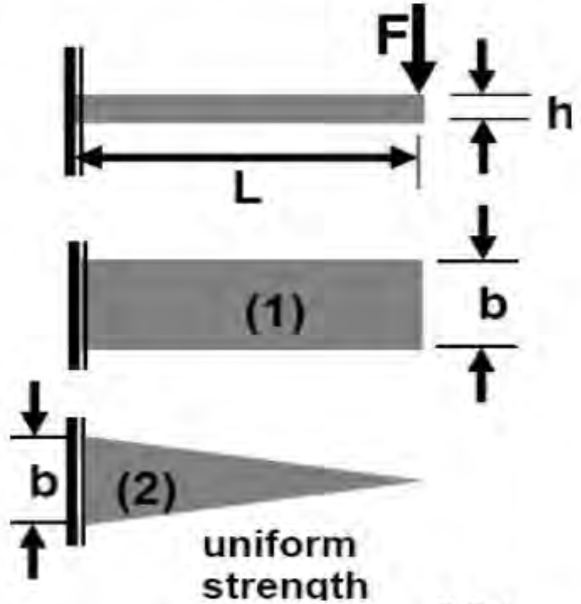
Design of Springs

7.3. Design of Leaf Springs

7.3.1 Leaf Springs

In order to have an idea of working principle of a leaf spring, let us think of the diving board in a swimming pool. The diving board is a cantilever with a load, the diver, at its free end. The diver initiates a to and fro swing of the board at the free end and utilizes the spring action of the board for jumping. The diving board basically is a leaf spring.
The leaf springs are widely used in suspension system of railway carriages and automobiles. But the form in which it is normally seen is laminated leaf spring.

A simple cantilever type leaf spring is shown in the Fig. 7.3.1.
In the cantilever beam type leaf spring, for the same leaf thickness, h, leaf of uniform width, b (case 1) and, leaf of width, which is uniformly reducing from b (case 2) is considered. From the basic equations of bending stress smax and deflection, the maximum stress, and tip deflection, d can be derived.



7.3.1

For case 1(uniform width)

$$\sigma_{max} = \frac{6FL}{bh^2} \qquad\qquad \delta_{max} = \frac{4FL^3}{Ebh^3} \tag{7.3.1}$$

Where, **E** is the Elastic modulus of the spring material.

For case 2(non uniform width)

$$\sigma_{max} = \frac{6FL}{bh^2} \qquad\qquad \delta_{max} = \frac{6FL^3}{Ebh^3} \tag{7.3.2}$$

In the second case it is observed that instead of uniform width leaf, if a leaf of varying width (triangular one as shown in the figure) is used, the bending stress at any cross section is same and equal to σ_{max}. This is called as leaf of a uniform strength. Moreover, the tip deflection being more, comparatively, it has greater resilience than its uniform width counterpart. Resilience, as we know, is the capacity to absorb potential energy during deformation. However, one should keep in mind that in order to withstand the shear force the tip has to have some width. This is shown as a red zone in the figure. In one way non uniform width leaf is a better design than a uniform width leaf.

7.3.2

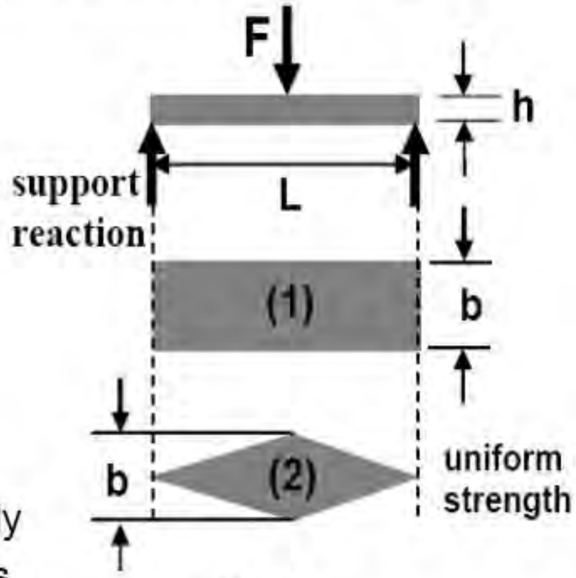
Leaf spring of simply supported beam type is shown in the Fig. 7.3.3, for which the stress and deflection equation are also given as in the case of cantilever.

For case 1(uniform width)

$$\sigma_{max} = \frac{3FL}{bh^2} \qquad \delta_{max} = \frac{2FL^3}{Ebh^3} \qquad (7.3.3)$$

For case 2(non uniform width *Lozenge-shape*)

$$\sigma_{max} = \frac{3FL}{bh^2} \qquad \delta_{max} = \frac{3FL^3}{Ebh^3} \qquad (7.3.4)$$



7.3.3

One of the applications of leaf spring of simply supported beam type is seen in automobiles, where, the central location of the spring is fixed to the wheel axle. Therefore, the wheel exerts the force F (opposite to the direction shown in the figure), on the spring and support reactions at the two ends of the spring come from the carriage. The diamond shaped leaf, shown as case 2, is named as Lozenge shape and it is again a beam of uniform strength.

7.3.2 Design of a leaf spring

Let us consider the simply supported leaf of Lozenge shape for which the maximum stress and maximum deflection are known. From the stress and deflection equations the thickness of the spring plate, h, can be obtained as,

$$\mathbf{h} = \frac{\sigma_{max} L^2}{E\,\delta_{max}} = \frac{\sigma_{des} L^2}{E\,\delta_{des}} \tag{7.3.5}$$

The $\boldsymbol{\sigma_{max}}$ is replaced by design stress $\boldsymbol{\sigma_{des}}$. Similarly, $\boldsymbol{\delta_{max}}$ is replaced by $\boldsymbol{\delta_{des}}$. **E** is the material property and depends on the type of spring material chosen. **L** is the characteristic length of the spring. Therefore, once the design parameters, given on the left side of the above equation, are fixed the value of plate thickness, **h** can be calculated.

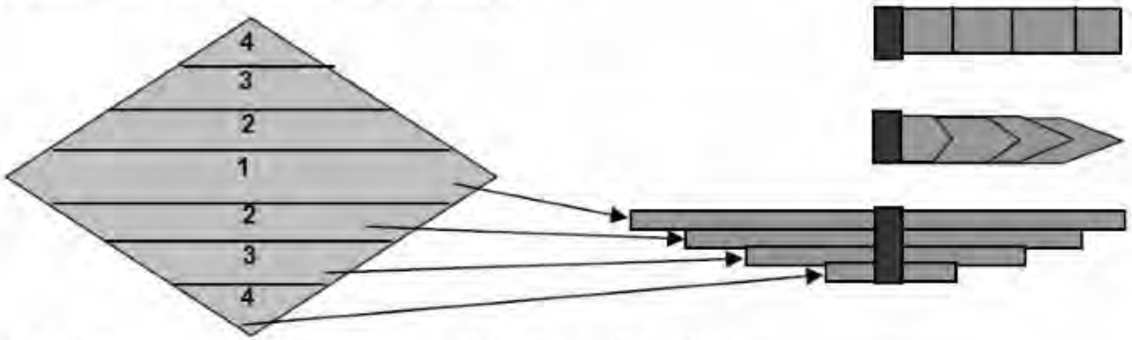
Substitution of **h** in the stress equation above will yield the value of plate width **b**.

$$\mathbf{b} = \frac{3FL}{\sigma_{des} h^2} \tag{7.3.6}$$

In the similar manner **h** and **b** can be calculated for leaf springs of different support conditions and beam types.

7.3.3 Laminated Springs

One of the difficulties of the uniform strength beam, say Lozenge shape, is that the value of width **b** sometimes is too large to accommodate in a machine assembly. One practice is that instead of keeping this large width one can make several slices and put the pieces together as a laminate. This is the concept of laminated spring. The Fig.7.3.4 shows the concept of formation of a laminated spring.



7.3.4 **Laminated Spring**

The Lozenge shaped plate is cut into several longitudinal strips, as indicated in the figure. The central strip, marked 1 is the master leaf which is placed at the top. Then two pieces, marked 2 are put together, side by side to form another leaf and placed below the top leaf. In the similar manner other pairs of strips, marked 3 and 4 respectively are placed in the decreasing order of strip length to form a laminated spring. Here width of each strip, is given as,

$$\mathbf{b_N} = \frac{\mathbf{b}}{\mathbf{N}} \qquad (7.3.7)$$

Where **N** is the number of strips

In practice, strips of width, b_N and lengths, say equal to strip1, strip2 etc., as shown in the example, are cut and put in the laminated form. The stress and deflection equations for a laminated spring is,

$$\sigma_{max} = \frac{pFL}{Nb_N h^2} \text{ and } \delta_{max} = \frac{qFL^3}{ENb_N h^3} \qquad (7.3.8)$$

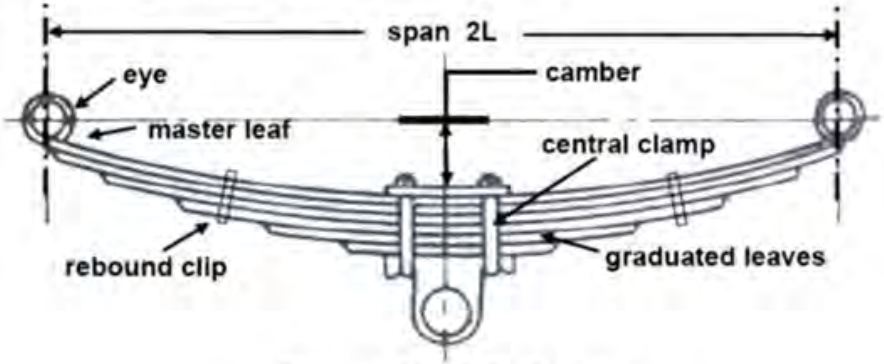
Where, constants **p** and **q** are given as,

		p	q
Simply supported beam	:	**3**	**3**
Cantilever beam	:	**6**	**6**

It is to be noted that the ends of the leaves are not sharp and pointed, as shown in figure. In fact they are made blunt or even made straight to increase the load bearing capacity. This change from ideal situation does not have much effect on the stress equation. However, small effect is there on the deflection equation.

In the following section we will discuss about few more constructional details of a laminated leaf spring.

7.3.4 Laminated semi-elliptic spring



Laminated semi-elliptic spring

7.3.5

The Fig 7.3.5 shows a laminated semi- elliptic spring. The top leaf is known as the master leaf. The eye is provided for attaching the spring with another machine member. The amount of bend that is given to the spring from the central line, passing through the eyes, is known as camber. The camber is provided so that even at the maximum load the deflected spring should not touch the machine member to which it is attached. The camber shown in the figure is known as positive camber. The central clamp is required to hold the leaves of the spring. However, the bolt holes required to engage the bolts to clamp the leaves weaken the spring to some extent. Rebound clips help to share the load from the master leaf to the graduated leaf.

7.3.5 Materials for leaf spring

Materials for leaf spring are not as good as that for the helical spring.
Plain carbon steel, Chromium vanadium steel, Chromium- Nickel- Molybdenum steel, Silicon- manganese steel, are the typical materials that are used in the design of leaf springs.

7.3.6 Standard sizes of leaf spring

Width (mm) : 25-80 mm in steps of 5mm
Thickness (mm) : 2-8 mm in steps of 1mm, 10-16 mm in steps of 2mm

In order to carry heavy load few more additional full length leaves are placed below the master leaf for heavy loads. Such alteration from the standard laminated leaf spring, what we have learnt above, does not change the stress value, but deflection equation requires some correction.

$$\delta_{max} = \frac{\delta_c qFL^3}{ENb_N h^3} \qquad (7.3.9)$$

Where, correction in deflection, δ_c is given as,

$$\delta_c = \frac{1.0 - 4m + 2m^2\{1.5 - \ln(m)\}}{(1.0 - m)^3} \quad \text{where,} \quad m = \frac{N_f}{N}$$

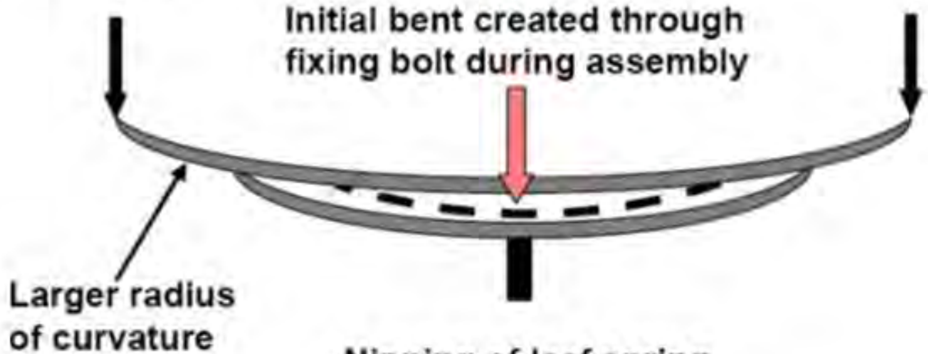
N_f=Number of full length leaves, N=Total number of leaves in the spring

7.3.7 Stresses due to support hinges

The master leaf of a laminated spring is hinged to the supports. The support forces induce, stresses due to longitudinal forces and stresses arising due to possible twist. Hence, the master leaf is more stressed compared to other the graduated leaves. Methods to reduce additional stresses could be,

1. Master leaf is made of stronger material than the other leaves.
2. Master leaf is made thinner than the other leaves. This will reduce the bending stress as evident from stress equation.
3. Another common practice is to increase the radius of curvature of the master leaf than the next leaf.

The last method is explained through Fig 7.3.6.



<u>Nipping of leaf spring</u>

7.3.6

The master leaf has a larger radius of curvature compared to the additional leaf that is placed below so obviously a gap will be created between the two leaves as indicated in the figure. Now, an initial bent is created during assembly by tightening the central bolt. Therefore, some amount of compressive stress will be produced at the inside curvature of the master leaf. Similarly, at the outside curvature of the master leaf tensile stress will be produced. Both these stresses are initial stresses in the master leaf. However, by such operation of tightening the central bolt, the additional leaf that is placed beneath the master leaf has a tendency to flatten out and as a result the stress pattern of the additional leaf will be reverse of that of the master leaf, tensile stress is produced at the inner curvature and compressive stress is produced at the outer curvature. Hence, when the spring is loaded, for both the master leaf and the additional leaf, tensile stress will be produced at the inner curvature and compressive stress will be produced at the outer curvature. Therefore, due to opposite nature of initial stress and loading stress, the master leaf will experience lesser stress on both the surfaces. However, due to same nature of initial stress and loading stress, the additional leaf is stressed more compared to the master leaf. But, it is to be noted that the higher stress on the additional leaf is actually shared between all other leaves than the master leaf. This practice of stress relief in the master leaf is known as *Nipping of leaf spring*. As a matter of fact, all the leaves of a laminated leaf spring do have certain amount of nipping, so that there will be gaps between the leaves, as a result the stresses will be uniformly distributed and accumulated dusts can also be cleaned.

Design problem of a leaf spring

Design a leaf spring to carry a load of 3400N and placed over a span of 800 mm. The spring can deflect by 50mm. Consider, allowable bending stress for the spring material as 350 MPa and E=2 $(10)^5$ MPa.

Let us consider the design to be based on uniform strength beam. Then from section 7.3.2 we find that,

$$\text{Leaf thickness, } h = \frac{\sigma_{des} L^2}{E\delta_{des}} = \frac{350 \times 400^2}{2 \times 10^5 \times 50} = 5.6\text{mm} \approx 6\text{mm}$$

$$\text{Leaf width, } b = \frac{3FL}{\sigma_{des} h^2} = \frac{3 \times 3400 \times 400}{350 \times 50^2} \approx 324\text{mm}$$

It is observed that the width is too large to accommodate as a machine member. Hence, if we consider, say 6 springs, then width of each spring becomes 54mm.

Problems with Answers

Q1. What are the forms of leaf spring ?

A1. Leaf springs are of two forms: cantilever and simply supported type.

Q2. What does the term "uniform strength" in the context of leaf spring mean?

A2. If the leaf spring has a shape of uniformly varying width (say Lozenge shape) then the bending stress at all section remains uniform. The situation is also identical as before in case of varying thickness, the thickness should vary non-uniformly with length to make a beam of uniform strength (L/h^2 = constant). These leaves require lesser material, have more resilience compared to a constant width leaf. These types of springs are called leaf springs of uniform strength.

Q3. What is "nipping" in a laminated spring?

A3. In general the differential curvature between the master leaf and the next leaves is provided in a laminated spring, where, radius of curvature being more for the master leaf. This construction reduces the stress in the master leaf as compared to the other leaves of the spring in a laminated spring. This type of constructional feature is termed as *nipping*.

Lecture

Theme 8

Design of Shaft

8.1. Shaft and its design based on strength

8.1.1 Shaft

Shaft is a common and important machine element. It is a rotating member, in general, has a circular cross-section and is used to transmit power. The shaft may be hollow or solid. The shaft is supported on bearings and it rotates a set of gears or pulleys for the purpose of power transmission. The shaft is generally acted upon by bending moment, torsion and axial force. Design of shaft primarily involves in determining stresses at critical point in the shaft that is arising due to aforementioned loading. Other two similar forms of a shaft are axle and spindle.

Axle is a non-rotating member used for supporting rotating wheels etc. and do not transmit any torque. Spindle is simply defined as a short shaft. However, design method remains the same for axle and spindle as that for a shaft.

8.1.2 Standard sizes of Shafts

Typical sizes of solid shaft that are available in the market are,

Up to 25 mm	0.5 mm increments
25 to 50 mm	1.0 mm increments
50 to 100 mm	2.0 mm increments
100 to 200 mm	5.0 mm increments

8.1.3 Material for Shafts

The ferrous, non-ferrous materials and non metals are used as shaft material depending on the application. Some of the common ferrous materials used for shaft are discussed below.

Hot-rolled plain carbon steel

These materials are least expensive. Since it is hot rolled, scaling is always present on the surface and machining is required to make the surface smooth.

Cold-drawn plain carbon/alloy composition

Since it is cold drawn it has got its inherent characteristics of smooth bright finish. Amount of machining therefore is minimal. Better yield strength is also obtained. This is widely used for general purpose transmission shaft.

Alloy steels

Alloy steel as one can understand is a mixture of various elements with the parent steel to improve certain physical properties. To retain the total advantage of alloying materials one requires heat treatment of the machine components after it has been manufactured. Nickel, chromium and vanadium are some of the common alloying materials. However, alloy steel is expensive.

These materials are used for relatively severe service conditions. When the situation demands great strength then alloy steels are used. They have fewer tendencies to crack, warp or distort in heat treatment. Residual stresses are also less compared to CS(Carbon Steel).

In certain cases the shaft needs to be wear resistant, and then more attention has to be paid to make the surface of the shaft to be wear resistant. The common types of surface hardening methods are,

Hardening of surface

Case hardening and carburizing

Cyaniding and nitriding

8.1.4 Design considerations for shaft

For the design of shaft following two methods are adopted,

Design based on Strength

In this method, design is carried out so that stress at any location of the shaft should not exceed the material yield stress. However, no consideration for shaft deflection and shaft twist is included.

Design based on Stiffness

Basic idea of design in such case depends on the allowable deflection and twist of the shaft.

8.1.5 Design based on Strength

The stress at any point on the shaft depends on the nature of load acting on it. The stresses which may be present are as follows.

Basic stress equations :

Bending stress

$$\sigma_b = \frac{32M}{\pi d_0^3 (1 - k^4)} \tag{8.1.1}$$

Where,

M : Bending moment at the point of interest

d_o : Outer diameter of the shaft

k : Ratio of inner to outer diameters of the shaft (k = 0 for a solid shaft because inner diameter is zero)

Axial Stress

$$\sigma_a = \frac{4\alpha F}{\pi d_0^2 (1 - k^2)} \qquad (8.1.2)$$

Where,

F: Axial force (tensile or compressive)

α: Column-action factor(= 1.0 for tensile load)

The term α has been introduced in the equation. This is known as column action factor. What is a column action factor? This arises due the phenomenon of buckling of long slender members which are acted upon by axial compressive loads. Here, αis defined as,

$$\alpha = \frac{1}{1 - 0.0044(L/K)} \quad \text{for } L/K < 115$$

$$\alpha = \frac{\sigma_{yc}}{\pi^2 nE}\left(\frac{L}{K}\right)^2 \quad \text{for } L/K > 115 \qquad (8.1.3)$$

Where, n = 1.0 for hinged end

n = 2.25 for fixed end

n = 1.6 for ends partly restrained, as in bearing

K = least radius of gyration, L = shaft length

σ_{yc} = yield stress in compression

Stress due to torsion

$$\tau_{xy} = \frac{16T}{\pi d_0^3(1-k^4)} \tag{8.1.4}$$

Where,

T : Torque on the shaft

τ_{xy} : Shear stress due to torsion

Combined Bending and Axial stress

Both bending and axial stresses are normal stresses, hence the net normal stress is given by,

$$\sigma_x = [\frac{32M}{\pi d_0^3(1-k^4)} \pm \frac{4\alpha F}{\pi d_0^2(1-k^2)}] \tag{8.1.5}$$

The net normal stress can be either positive or negative. Normally, shear stress due to torsion is only considered in a shaft and shear stress due to load on the shaft is neglected.

Maximum shear stress theory

Design of the shaft mostly uses maximum shear stress theory. It states that a machine member fails when the maximum shear stress at a point exceeds the maximum allowable shear stress for the shaft material. Therefore,

$$\tau_{max} = \tau_{allowable} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \qquad (8.1.6)$$

Substituting the values of σx and τ_{xy} in the above equation, the final form is,

$$\tau_{allowable} = \frac{16}{\pi d_0^3 (1-k^4)} \sqrt{\left\{M + \frac{\alpha F d_0 (1+k^2)}{8}\right\}^2 + T^2} \qquad (8.1.7)$$

Therefore, the shaft diameter can be calculated in terms of external loads and material properties. However, the above equation is further standarised for steel shafting in terms of allowable design stress and load factors in ASME design code for shaft.

8.1.6 ASME design Code

The shafts are normally acted upon by gradual and sudden loads. Hence, the equation (8.1.7) is modified in ASME code by suitable load factors,

$$\tau_{allowable} = \frac{16}{\pi d_0^3(1-k^4)}\sqrt{\left\{C_{bm}M + \frac{\alpha F d_0(1+k^2)}{8}\right\}^2 + (C_t T)^2}$$

(8.1.8)

where, C_{bm} and C_t are the bending and torsion factors. The values of these factors are given below,

	C_{bm}	C_t
For stationary shaft:		
Load gradually applied	1.0	1.0
Load suddenly applied	1.5 - 2.0	1.5 - 2.0
For rotating shaft:		
Load gradually applied	1.5	1.0
Load suddenly applied (minor shock)	1.5 - 2.0	1.0 - 1.5
Load suddenly applied (heavy shock)	2.0 - 3.0	1.5 - 3.0

ASME code also suggests about the allowable design stress, $\tau_{allowable}$ to be considered for steel shafting,

ASME Code for commercial steel shafting

= 55 MPa for shaft without keyway

= 40 MPa for shaft with keyway

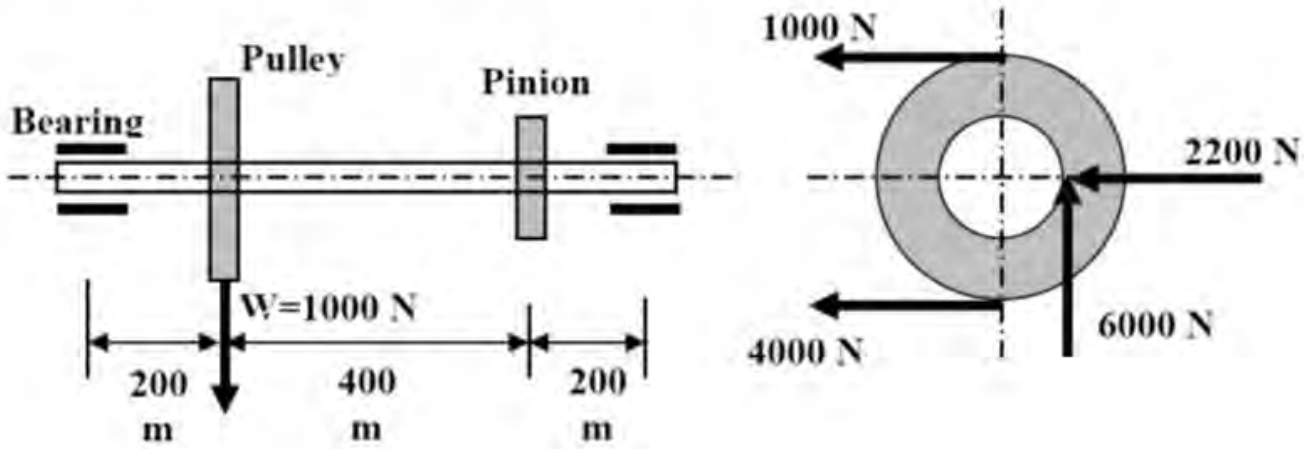
ASME Code for steel purchased under definite specifications

= 30% of the yield strength but not over 18% of the ultimate strength in tension for shafts without keyways. These values are to be reduced by 25% for the presence of keyways.

The equations, (8.1.7) and (8.1.8) are commonly used to determine shaft diameter.

Sample problem

The problem is shown in the given figure. A pulley drive is transmitting power to a pinion, which in turn is transmitting power to some other machine element. Pulley and pinion diameters are 400mm and 200mm respectively. Shaft has to be designed for minor to heavy shock.



Solution

From the given figure, the magnitude of torque,

$T=(4000-1000)\times 200 Nmm = 600\times 10^3$ Nmm

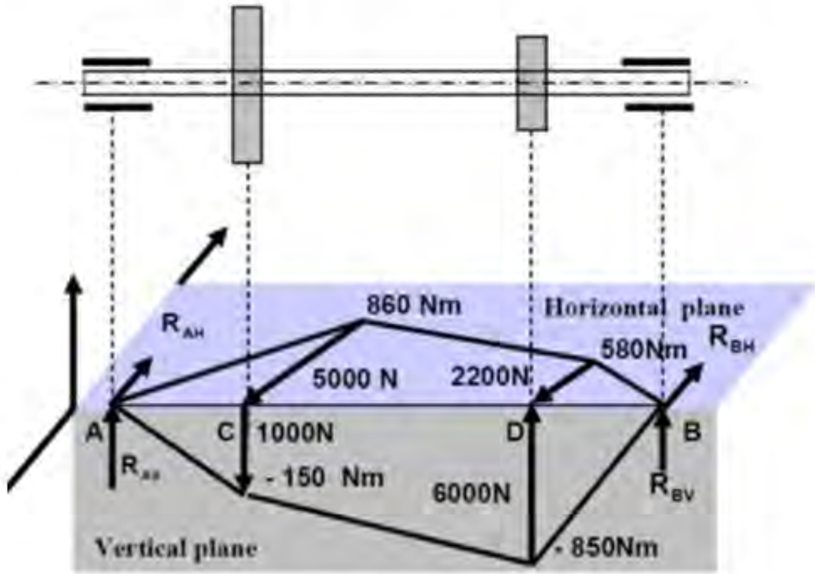
It is observed that the load on the shaft is acting both in horizontal and vertical planes. The loading diagram, corresponding bearing reactions and bending moment diagram is given below.

The bending moment at C:

For vertical plane, M_V: -150 Nm
For horizontal plane, M_H: 860 Nm
Resultant moment: 873 Nm

The bending moment at D:

For vertical plane, M_V: -850 Nm
For horizontal plane, M_H: 580 Nm
Resultant moment: 1029Nm



Loading and Bending Moment Diagram

Therefore, section-D is critical and where bending moment and torsion is 1029 Nm and 600 Nm respectively.

ASME code for shaft design is suitable in this case as no other specifications are provided. In absence of any data for material property, the allowable shear for commercial steel shaft may be taken as 40 MPa, where keyway is present in the shaft.

For the given codition of shock, let us consider C_{bm} = 2.0 and C_t = 1.5.

From the ASME design code, we have,

$$d_o^3 = \frac{16\times10^3}{\tau_d\times\pi}\left(\sqrt{(C_{bm}\times1029)^2+(C_t\times600)^2}\right)$$

$$=\frac{16\times10^3}{40\times\pi}\left(\sqrt{(2.0\times1029)^2+(1.5\times600)^2}\right)$$

$$\therefore d_o = 65.88\text{ mm} \approx 66\text{mm}$$

From standard size available, the value of shaft diameter is also 66mm.

Questions and Answers

Q1. What do you understand by shaft, axle and spindle?

A1. Shaft is a rotating member, in general, has a circular cross-section and is used to transmit power. Axle is a non-rotating member used for supporting rotating wheels etc. and do not transmit any torque. Spindle is simply defined as a short shaft.

Q2. What are the common ferrous materials for a shaft?

A2. Common materials for shaft are, hot-rolled plain carbon steel, cold-drawn plain carbon/alloy composition and alloy steels.

Q3. How do the strength of a steel material for shafting is estimated in ASME design code for shaft?

A3. Material property for steel shaft for ASME code is as follows,

For commercial steel shafting

= 55 MPa for shaft without keyway

= 40 MPa for shaft with keyway

For steel purchased under definite specifications

= 30% of the yield strength but not over 18% of the ultimate strength in tension for shafts without keyways. These values are to be reduced by 25% for the presence of keyways in the shaft.

Lecture

Theme 8

Design of Shaft

8.2. Design of shaft for variable load and based on stiffness

8.2.1 Design of Shaft for variable load

Design of shaft for strength involves certain changes when it is acted upon by variable load. It is required to calculate the mean stress and stress amplitude for all the loads, namely, axial, bending and torsion. Thereafter, any of the design methods for variable load, that is, Soderberg, Goodman or Gerber criteria is utilized. Once again, the familiar design diagram for variable load in terms of the stress amplitude and the mean stress is reproduced below.

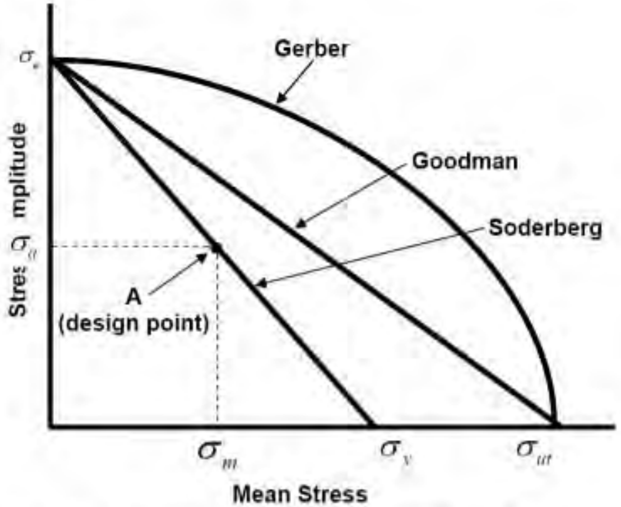


Fig. 8.2.1 Diagram for design under variable load

A is the design point, for which, the stress amplitude is σ_a and mean stress is σ_m. In the Soderberg criterion the mean stress material property is the yield point σ_y , whereas in the Gerber and the Goodman criteria the material property is the ultimate stress σ_{ut} For the fatigue loading, material property is the endurance limit, σ_e in reverse bending. The corresponding equations for all the three above criteria are given as,

Goodman criterion: $$\frac{\sigma_a}{\sigma_e}+\frac{\sigma_m}{\sigma_{ut}}=\frac{1}{FS}$$

Soderberg criterion: $$\frac{\sigma_a}{\sigma_e}+\frac{\sigma_m}{\sigma_y}=\frac{1}{FS} \qquad (8.2.1)$$

Gerber criterion: $$\frac{FS\times\sigma_a}{\sigma_e}+\left(\frac{FS\times\sigma_m}{\sigma_{ut}}\right)^2=1$$

Where,
σ_a = Stress amplitude; σ_e = Endurance limit; σ_m = Mean stress; σ_y = Yield point;
σ_{ut} = Ultimate stress and FS= factor of safety.

Similar equation (8.2.1) also can be written for the shear stress.

For the design of shaft, it is most common to use the Soderberg criterion. Hence, we shall limit our discussion only to Soderberg criterion.

Normal stress equation is given as,

$$\frac{K_f \sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_y} = \frac{1}{FS} \qquad (8.2.2)$$

$$\text{multiplying by } \sigma_y, \ \frac{\sigma_y k_f \ \sigma_a}{\sigma_e} + \sigma_m = \frac{\sigma_y}{FS} = \sigma_{eq}$$

Similarly, *shear stress equation is given as*

$$\frac{K_{fs} \tau_a}{\tau_e} + \frac{\tau_m}{\tau_y} = \frac{1}{FS}$$

$$\text{multiplying by } \tau_y, \ \frac{\tau_y K_{fs} \ \tau_a}{\tau_e} + \tau_m = \frac{\tau_y}{fs} = \tau_{eq} \qquad (8.2.3)$$

In equations (8.2.2) and (8.2.3), to consider the effect of variable load, the normal stress amplitude, σ_a is multiplied by the fatigue stress concentration factor, K_f and the corresponding term, shear stress amplitude is multiplied by a fatigue stress concentration factor in shear, K_{fs}.

The physical significance of equations (8.2.2) and (8.2.3) is that, the effect of variable stress on the machine member (left hand side of the equations), has been effectively defined as an equivalent static stress. Therefore, the problem is treated as a design for static loads. Here, σ_e or τ_e are equivalent to allowable stress, $\frac{\sigma_y}{FS}$ or $\frac{\tau_y}{FS}$. Hereafter, conventional failure theories can be used to complete the design.

Maximum shear stress theory

It states that a machine member fails when the maximum shear stress at a point exceeds the maximum allowable shear stress for the shaft material. Therefore,

$$\tau_{max} = \tau_{allowable} = \sqrt{\left(\frac{\sigma_{eq}}{2}\right)^2 + \tau_{eq}^2} \qquad (8.2.4)$$

substitution of σ_{eq} and τ_{eq} from (8.2.3) will give the required shaft diameter.

8.2.2 Design based on Stiffness

In addition to the strength, design may be based on stiffness. In the context of shaft, design for stiffness means that the lateral deflection of the shaft and/or angle of twist of the shaft should be within some prescribed limit. Therefore, design for stiffness is based on lateral stiffness and torsional rigidity.

8.2.2.1 Lateral stiffness

Let us consider a beam loaded as shown in Fig.8.2.2. The beam deflects by **δ** due to the load P. So the requirement for the design is that where, one has to limit the deflection **δ.** Hence, the design procedure is as follows,

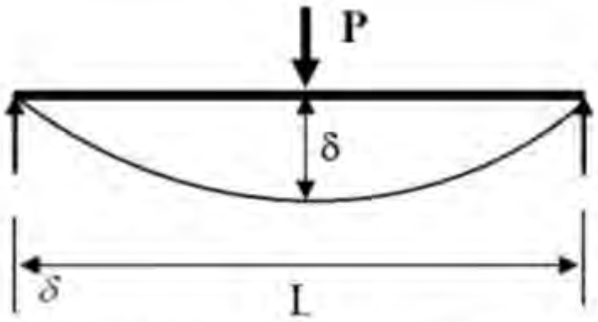


Fig. 8.2.2 Deflection of a beam

Determine the maximum shaft deflection, using any of the following methods,

Integration method
Moment-area method, or
Energy method (Theorem of Castigliano)

Now, the deflection, $\delta = f$ (applied load, material property, moment of inertia and given dimension of the beam).

From the expression of moment of inertia, and known design parameters, including δ, shaft dimension is obtained.

8.2.2.2 Torsional rigidity

To design a shaft based on torsional rigidity, the limit of angle of twist should be known. The angle of twist is given as follows,

$$\theta_{rad} = \frac{TL}{GI_p}$$

$$\text{or,} \quad \theta_{deg} = \frac{584TL}{Gd_0^4(1-k^4)} \qquad (8.2.5)$$

$$\therefore \quad d_O = \sqrt[4]{\frac{584TL}{G(1\text{-}k^4)\theta_{deg}}}$$

Where,

θ = angle of twist
L = length of the shaft

G = shear modulus of elasticity
I_p = Polar moment of inertia

The limiting value of θ varies from 0.3 deg/m to 3 deg/m for machine tool shaft to line shaft respectively. With the knowledge of design parameters, the shaft dimension can be obtained from (8.2.5).

8.2.3 A note on critical speed of rotating shaft

Critical speed of a rotating shaft is the speed where it becomes dynamically unstable. It can be shown that the frequency of free vibration of a non-rotating shaft is same as its critical speed.

The equation of fundamental or lowest critical speed of a shaft on two supports is,

$$f_{critical} = \frac{1}{2\pi}\sqrt{\frac{g(W_1\delta_1 + W_2\delta_2 + \ldots\ldots + W_n\delta_n)}{(W_1\delta_1^2 + W_2\delta_2^2 + \ldots\ldots + W_1\delta_n^2)}} \qquad (8.2.6)$$

Where,

$W_1, W_2 \ldots.$: weights of the rotating bodies

$\delta_1, \delta_2 \ldots.$: deflections of the respective bodies

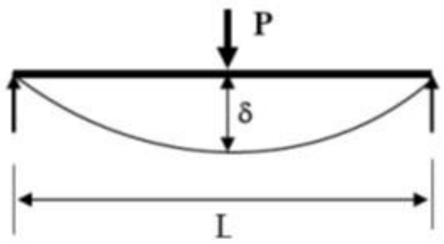
This particular equation (8.2.6) has been derived using the following assumption.

Assumptions: The shaft is weightless The weights are concentrated and Bearings/supports are not flexible

The operating speed of the shaft should be well above or below a critical speed value. There are number of critical speeds depending upon number of rotating bodies.

Sample problem

Design a solid shaft of length 1m, carrying a load of 5 kN at the center and is simply supported as shown in figure. The maximum shaft deflection is 1mm. E=200GPa.



Solution

The maximum deflection of the shaft is given as,

$$\delta_{max} = \frac{PL^3}{48EI}$$

where, for a solid shaft, $I = \frac{\pi d_o{}^4}{64}$

$$\therefore\ d_o = \sqrt[4]{\frac{4PL^3}{3\pi E\delta_{max}}} = \sqrt[4]{\frac{4\times 5000\times 1000^3}{3\times\pi\times 200\times 10^3\times 1}}$$

$$= 57\text{ mm}$$

from standard shaft size, $d_o = 58$ mm

This problem is not a complete one. The magnitude of torque on the shaft is not specified. The design calculations should be first based on strength, where, both bending moment and torsion are required. With the given limits of lateral deflection and angular twist, the design should be checked.

Questions and Answers

Q1. What is an equivalent stress?

A1. When a shaft is subjected to variable load, both the stress amplitude and mean stress can be conveniently represented as equivalent stress. The equivalent stress is conceptually an equivalent static stress.

Q2. What are the limiting values of the angle of twist of a shaft?

A2. The limiting value of angle of twist of a shaft varies from 0.3 deg/m to 3 deg/m for machine tool shaft to line shaft respectively.

Q3. What are the assumptions made to derive the equation for critical frequency? Why critical frequency is important in shaft design?

A3. The assumptions made to derive the equation for critical frequency are,

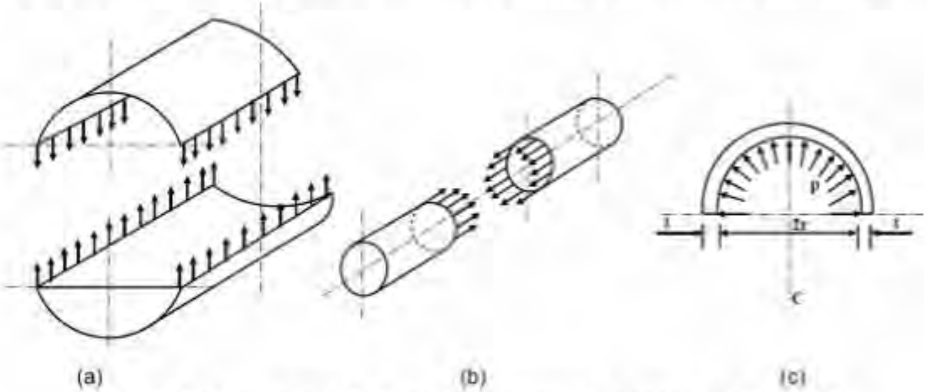
The shaft is weightless, the weights are concentrated and bearings/supports are not flexible. The critical speed value helps a designer to set the limit of shaft speed. To avoid resonance, the shaft speed should be much higher or lower than the critical speed.

Lecture
Theme 9
Thin and thick cylinders

9.1. Thin Cylinders

9.1.1 Stresses in thin cylinders

If the wall thickness is less than about 7% of the inner diameter then the cylinder may be treated as a thin one. Thin walled cylinders are used as boiler shells, pressure tanks, pipes and in other low pressure processing equipments. In general three types of stresses are developed in pressure cylinders viz. circumferential or hoop stress, longitudinal stress in closed end cylinders and radial stresses. These stresses are demonstrated in **figure-9.1.1.1**.



9.1.1.1 - (a) Circumferential stress (b) Longitudinal stress and (c) Radial stress developed in thin cylinders.

In a thin walled cylinder the circumferential stresses may be assumed to be constant over the wall thickness and stress in the radial direction may be neglected for the analysis. Considering the equilibrium of a cut out section the circumferential stress σ_θ and longitudinal stress σz can be found.

Consider a section of thin cylinder of radius r, wall thickness t and length L and subjected to an internal pressure p as shown in **figure-9.1.1.2(a)**. Consider now an element of included angle dθ at an angle of θ from vertical. For equilibrium we may write

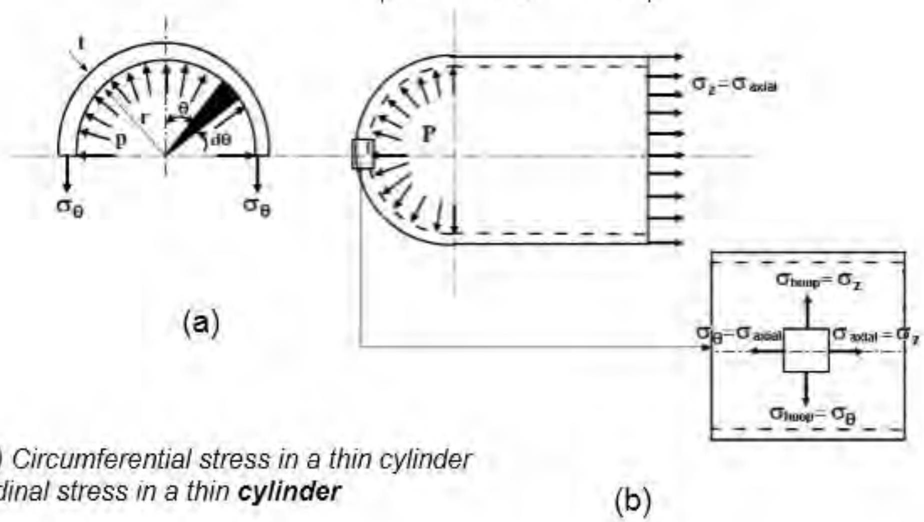
$$2\int_0^{\frac{\pi}{2}} prd\theta L\cos\theta = 2\sigma_\theta tL \qquad \text{This gives } \sigma_\theta = \frac{pr}{t} \qquad (9.1.1)$$

Considering a section along the longitudinal axis as shown in figure-9.1.1.2 (b) we may write

$$p\pi r^2 = \sigma_z \pi (r_o^2 - r_i^2)$$

where r_i and ro are internal and external radii of the vessel and since $r_i \approx r_o = r$ (say) and $r_o - r_i = t$ we have

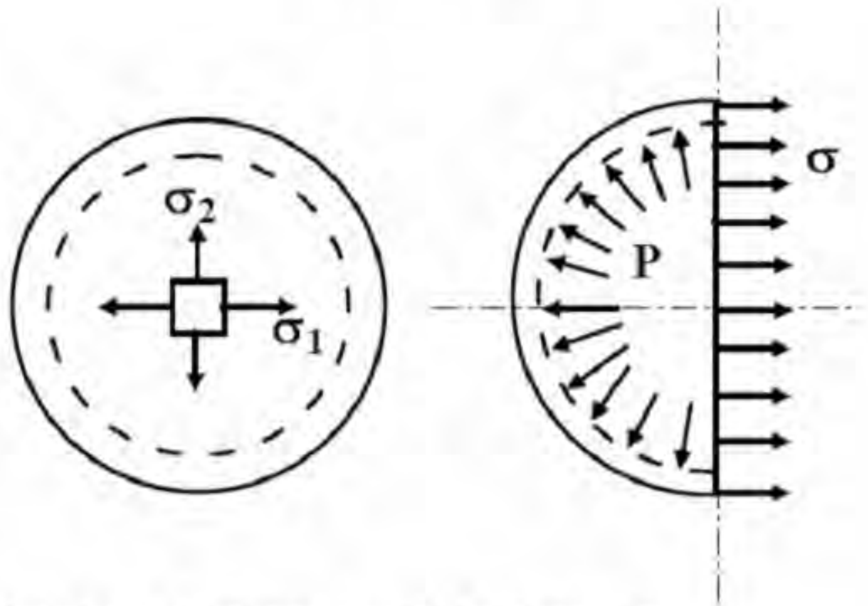
$$\sigma_z = \frac{2pr^2}{2r_o t} = \frac{pr}{2t} \qquad (9.1.2)$$



9.1.1.2 - (a) Circumferential stress in a thin cylinder
(b) Longitudinal stress in a thin ***cylinder***

Thin walled spheres are also sometimes used. Consider a sphere of internal radius r subjected to an internal pressure p as shown in **figure-9.1.1.3**. The circumferential and longitudinal stresses developed on an element of the surface of the sphere are equal in magnitude and in the absence of any shear stress due to symmetry both the stresses are principal stresses. From the equilibrium condition in a cut section we have

$$\sigma_1 = \sigma_2 = p$$



9.1.1.3 - Stresses in a spherical shell

9.1.2 Design Principles

Pressure vessels are generally manufactured from curved sheets joined by welding. Mostly V– butt welded joints are used. The riveted joints may also be used but since the plates are weakened at the joint due to the rivet holes the plate thickness should be enhanced by taking into account the joint efficiency. It is probably more instructive to follow the design procedure of a pressure vessel. We consider a mild steel vessel of 1m diameter comprising a 2.5 m long cylindrical section with hemispherical ends to sustain an internal pressure of (say) 2MPa.

The plate thickness is given by $t \geq \frac{pr}{\sigma_{yt}}$ where σ_{yt} is the tensile yield stress. The minimum plate thickness should conform to the "Boiler code" as given in **table- 9.1.2.1**.

Boiler diameter(m)	≤ 0.90	0.94 to 1.37	1.4 to 1.80	> 1.80
Plate thickness (mm)	6.35	8.00	9.525	12.70

The factor of safety should be at least 5 and the minimum ultimate stresses of the plates should be 385 MPa in the tension, 665 MPa in compression and 308 MPa in shear.

This gives $t_c \geq \frac{2x10^6 x0.5}{(385x10^6/5)}$, i.e., 13 mm. Since this value is more than the value prescribed in the code the plate hickness is acceptable. However for better safety we take t_c =15mm. Thickness ts of the hemispherical end is usually taken as half of this value and we take $t_s \approx$ 8mm.

9.1.3. Welded Joint

The circumferential stress developed in the cylinder $\sigma_\theta = \frac{pr}{t_c}$ With p=2MPa , r=0.5m and t_c = 15 mm,

σ_θ =67 MPa and since this is well below the allowable stress of 100 MPa (assumed) the butt welded joint without cover plate would be adequate.

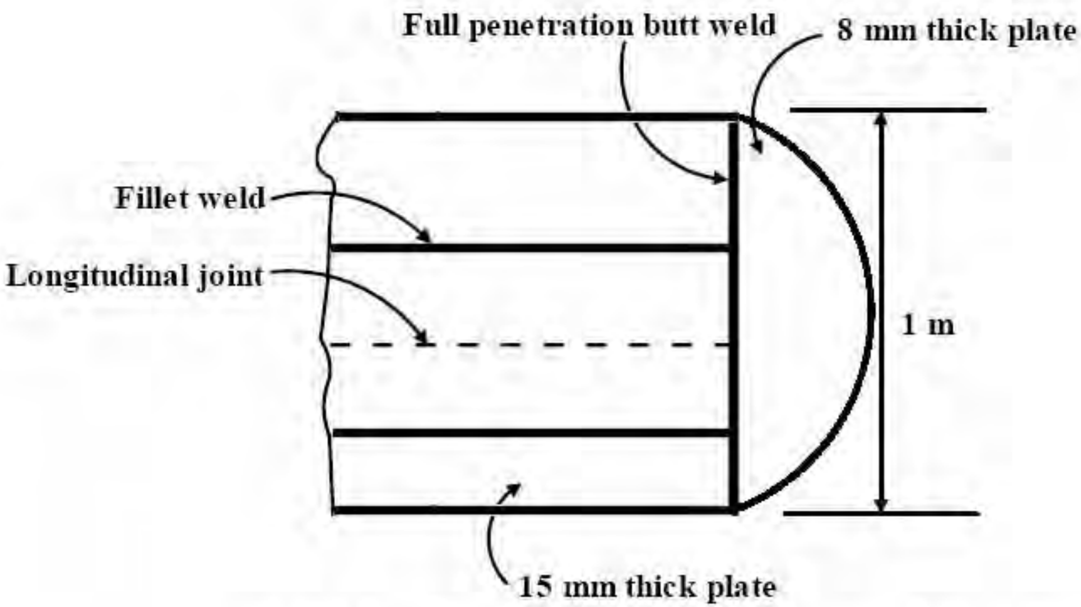
Consider now a butt joint with 10mm cover plates on both sides, as shown in **figure- 9.1.3.1**.

9.1.3.1 - Longitudinal welded joint with cover plates.

The stress induced in the weld σ_w is given by $F_c = 2\sigma_w L t_c \sin 45^0$

where L is the weld length. We may now write $F_c = \sigma_\theta$ t.L and therefore σ_w is given by $\sigma_w = \sigma_\theta \frac{t}{t_c 2\sin 45^0} = 67x\frac{15}{10x2x\sin 45^o}$ which gives σ_w = 71 MPa which again is adequate.

For increased safety we may choose the butt joint with 10mm thick cover plates. The welding arrangement of the vessel is shown in **figure- 9.1.3.2.**



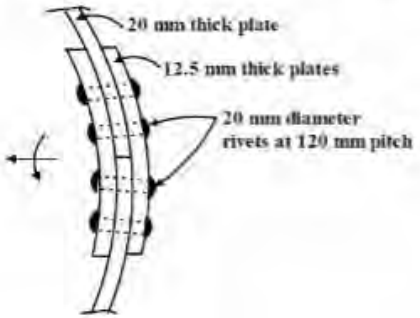
9.1.3.2 - The welding arrangement of the joint.

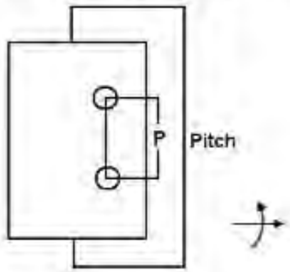
9.1.4. Riveted Joint

The joints may also be riveted in some situations but the design must be checked for safety. The required plate thickness must take account the joint efficiency η.

This gives $t_c = \frac{pr}{\eta \sigma_{ty}}$. Substituting p = 2MPa, r = 0.5 m, η = 70 % and σ_{ty} = (385/5) MPa we have tc = 18.5 mm.

Let us use mild steel plate of 20 mm thickness for the cylinder body and 10mm thick plate for the hemispherical end cover. The cover plate thickness may be taken as 0.625tc i.e. 12.5 mm. The hoop stress is now given by $\sigma_\theta = \frac{pr}{t_c} = 50\text{MPa}$ and therefore the rivets must withstand $\sigma_\theta t_c$ i.e. 1 MN per meter. A longitudinal riveted joint with cover plates is shown in **figure–9.1.4.1** and the whole riveting arrangement is shown in **figure-9.1.4.2.**





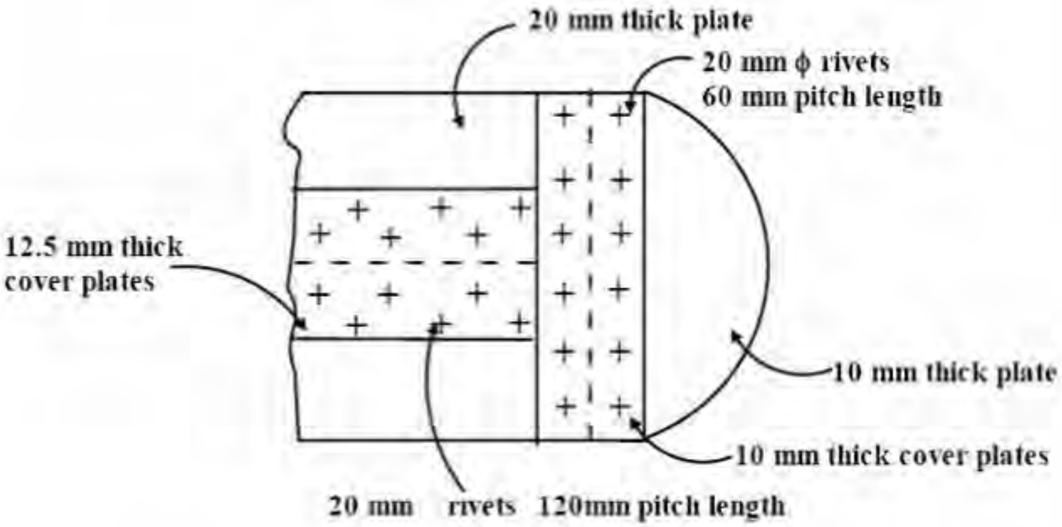
9.1.4.1** – A longitudinal joint with two cover plates* ***9.1.4.2

We may begin with 20mm diameter rivets with the allowable shear and bearing stresses of 100 MPa and 300 MPa respectively. This gives bearing load on a single rivet $F_b = 300\times10^6 \times 0.02 \times 0.02 = 120$ kN

Assuming double shear the shearing load on a single rivet $F_S = 100\times10^6 \times 2 \times \frac{\pi}{4}(0.02)^2 = 62.8\text{kN}$.

The rivet pitch based on bearing load is therefore (120 kN/ 1MN per meter) i.e. 0.12m and based on shearing load is (62.8 kN/ 1MN per meter) i.e. 0.063m. We may therefore consider a minimum allowable pitch of 60mm. This gives approximately 17 rivets of 20 mm diameter per meter. If two rows are used the pitch is doubled to 120mm. For the hemispherical shaped end cover the bearing load is 60 kN and therefore the rivet pitch is again approximately 60 mm.

The maximum tensile stress developed in the plate section is $\sigma t = 1x10^6/[(1-17x0.02)x0.02] = 75.76$ MPa which is a safe value considering the allowable tensile stress of 385 MPa with a factor of safety of 5. The whole riveting arrangement is shown in **figure-9.1.2.4.**



9.1.4.3 - General riveting arrangement of the pressure vessel.

9.1.3 Summary of this Lecture

Stresses developed in thin cylinders are first discussed in general and then the circumferential (σ_θ) and longitudinal stresses (σ_z) are expressed in terms of internal pressure, radius and the shell thickness. Stresses in a spherical shell are also discussed. Basic design principle of thin cylinders are considered. Design of both welded and riveted joints for the shells are discussed.

Lecture

Theme 9

Thin and thick cylinders

9.2. Thick cylinders- Stresses due to internal and external pressures

9.2.1 Stresses in thick cylinders

For thick cylinders such as guns, pipes to hydraulic presses, high pressure hydraulic pipes the wall thickness is relatively large and the stress variation across the thickness is also significant. In this situation the approach made in the previous section is not suitable. The problem may be solved by considering an axisymmetry about z-axis and solving the differential equations of stress equilibrium in polar co-ordinates. In general the stress equations of equilibrium without body forces can be given as

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\frac{\partial \tau_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + 2\frac{\tau_{\theta r}}{r} = 0$$

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{z\theta}}{\partial \theta} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{zr}}{r} = 0$$

For axisymmetry about z-axis $\frac{\partial}{\partial \theta} = 0$ and this gives

$$\begin{aligned}
&\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \\
&\frac{\partial \tau_{\theta r}}{\partial r} + \frac{\partial \tau_{\theta z}}{\partial z} + 2\frac{\tau_{\theta r}}{r} = 0 \qquad (2) \\
&\frac{\partial \tau_{zr}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{zr}}{r} = 0
\end{aligned}$$

In a plane stress situation if the cylinder ends are free to expand $\sigma_z = 0$ and due to uniform radial deformation and symmetry $\tau_{rz} = \tau_{\theta z} = \tau_{r\theta} = 0$. The equation of equilibrium reduces to

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

This can be written in the following form:

$$r\frac{\partial \sigma_r}{\partial r} + \sigma_r = \sigma_\theta \qquad (3)$$

If we consider a general case with body forces such as centrifugal forces in the case of a rotating cylinder or disc then the equations reduce to

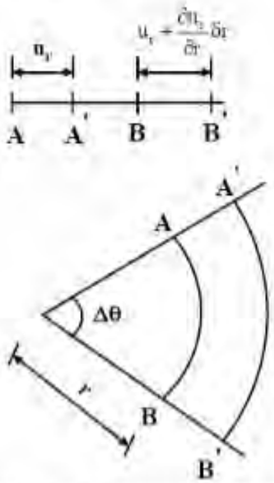
$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + \rho\omega^2 r = 0 \qquad \text{which may be written as}$$

$$r\frac{\partial \sigma_r}{\partial r} - \sigma_\theta + \sigma_r + \rho\omega^2 r^2 = 0 \qquad (4)$$

It is convenient to solve the general equation so that a variety of problems may be solved. Now as shown in **figure- 9.2.1.1**, the strains ε_r and ε_θ may be given by

$$\varepsilon_r = \frac{\partial u_r}{\partial r} = \frac{1}{E}\left[\sigma_r - \nu\sigma_\theta\right] \qquad \text{since } \sigma_z = 0 \qquad (5)$$

$$\varepsilon_\theta = \frac{(r + u_r)\Delta\theta - r\Delta\theta}{r\Delta\theta} = \frac{u_r}{r} = \frac{1}{E}\left[\sigma_\theta - \nu\sigma_r\right] \qquad (6)$$



9.2.1.1 - Representation of radial and circumferential strain.

Combining equation (5) and (6) we have

$$r\frac{\partial\sigma_\theta}{\partial r} - \nu r\frac{\partial\sigma_r}{\partial r} + (1+\nu)(\sigma_\theta - \sigma_r) = 0 \qquad (7)$$

Now from equation (4) we may write

$$\frac{\partial\sigma_\theta}{\partial r} = r\frac{\partial^2\sigma_r}{\partial r^2} + 2\frac{\partial\sigma_r}{\partial r} + 2\rho\omega^2 r$$

and combining this with equation (7) we may arrive at

$$r\frac{\partial^2\sigma_r}{\partial r^2} + 3\frac{\partial\sigma_r}{\partial r} + (3+\nu)\rho\omega^2 r = 0 \qquad (8)$$

For a non-rotating thick cylinder with internal and external pressures pi and po we substitute $\omega = 0$ in equation (8) and this gives

$$r\frac{\partial^2\sigma_r}{\partial r^2} + 3\frac{\partial\sigma_r}{\partial r} = 0 \qquad (9)$$

A typical case is shown in **figure- 9.2.1.2**. A standard solution for equation (9) is $\sigma_r = c\, r^n$ where c and n are constants. Substituting this in equation (9) and also combining with equation (3) we have

$$\sigma_r = c_1 + \frac{c_2}{r^2} \qquad \sigma_\theta = c_1 - \frac{c_2}{r^2}$$ where c_1 and c_2 are constants. (10)

9.2.1.2 – A thick cylinder with both external and internal pressure.

Boundary conditions for a thick cylinder with internal and external pressures p_i and p_o respectively are:

at $\quad r = r_i \quad \sigma_r = -p_i$

and at $\quad r = r_o \quad \sigma_r = -p_o$

The negative signs appear due to the compressive nature of the pressures. This gives

$$c_1 = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} \qquad c_2 = \frac{r_i^2 r_o^2 (p_o - p_i)}{r_o^2 - r_i^2}$$

The radial stress σ_r and circumferential stress σ_θ are now given by

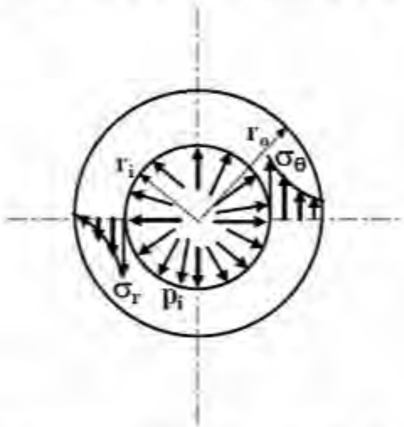
$$\begin{aligned} \sigma_r &= \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{r_i^2 r_o^2 (p_o - p_i)}{r_o^2 - r_i^2} \frac{1}{r^2} \\ \sigma_\theta &= \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{r_i^2 r_o^2 (p_o - p_i)}{r_o^2 - r_i^2} \frac{1}{r^2} \end{aligned} \tag{11}$$

It is important to remember that if σ_θ works out to be positive, it is tensile and if it is negative, it is compressive whereas σ_r is always compressive irrespective of its sign.

Stress distributions for different conditions may be obtained by simply substituting the relevant values in equation (11). For example, if po = 0 i.e. there is no external pressure the radial and circumferential stress reduce to

$$\sigma_r = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(-\frac{r_o^2}{r^2}+1\right) \qquad \sigma_\theta = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(\frac{r_o^2}{r^2}+1\right) \qquad (12)$$

The stress distribution within the cylinder wall is shown in **figure- 9.2.1.3**.



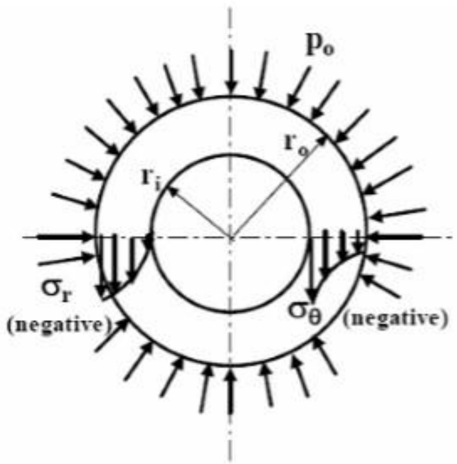
9.2.1.3 - Radial and circumferential stress distribution within the cylinder wall when only internal pressure acts.

It may be noted that $\sigma_r + \sigma_\theta$ = constant and hence the deformation in z-direction is uniform. This means that the cross-section perpendicular to the cylinder axis remains plane. Hence the deformation in an element cut out by two adjacent cross-sections does not interfere with the adjacent element. Therefore it is justified to assume a condition of plane stress for an element in section 9.2.1.

If $p_i = 0$ i.e. there is no internal pressure the stresses σ_r and σ_θ reduce to

$$\begin{aligned} \sigma_r &= \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} - 1\right) \\ \sigma_\theta &= -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} + 1\right) \end{aligned} \tag{13}$$

The stress distributions are shown in **figure-9.2.1.4**.



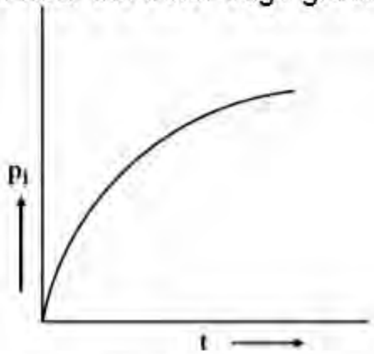
9.2.1.4 - Distribution of radial and circumferential stresses within the cylinder wall when only external pressure acts.

9.2.2 Methods of increasing the elastic strength of a thick cylinder by pre-stressing

In thick walled cylinders subjected to internal pressure only it can be seen from equation (12) that the maximum stresses occur at the inside radius and this can be given by

$$\sigma_{r(max)}\Big|_{r=r_i} = -p_i \qquad\qquad \sigma_{\theta(max)}\Big|_{r=r_i} = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}$$

This means that as pi increases σ_θ may exceed yield stress even when $p_i < \sigma_{yield}$. Furthermore, it can be shown that for large internal pressures in thick walled cylinders the wall thickness is required to be very large. This is shown schematically in **figure- 9.2.2.1**. This means that the material near the outer edge is not effectively used since the stresses near the outer edge gradually reduce (Refer to **figure- 9.2.1.3**).



9.2.2.1 A schematic variation of wall thickness with the internal pressure in a thick walled cylinder.

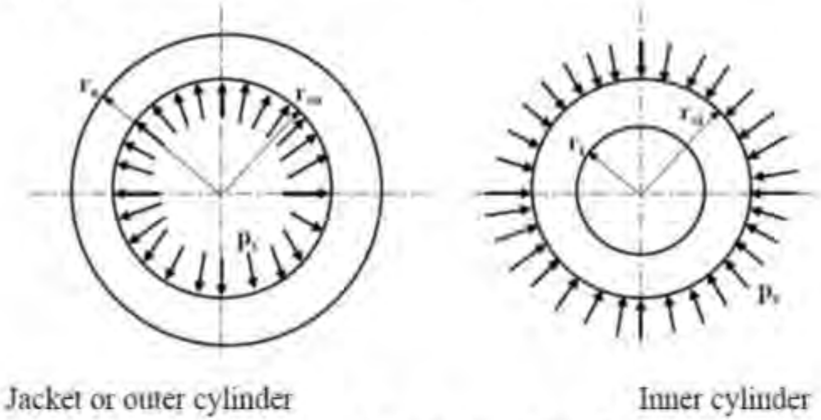
In order to make thick-walled cylinders that resist elastically large internal pressure and make effective use of material at the outer portion of the cylinder the following methods of pre-stressing are used:

1. Shrinking a hollow cylinder over the main cylinder.
2. Multilayered or laminated cylinders.
3. Autofrettage or self hooping.

Composite cylinders

An outer cylinder (jacket) with the internal diameter slightly smaller than the outer diameter of the main cylinder is heated and fitted onto the main cylinder. When the assembly cools down to room temperature a composite cylinder is obtained. In this process the main cylinder is subjected to an external pressure leading to a compressive radial stress at the interface. The outer cylinder or the jacket is subjected to an internal pressure leading to a tensile circumferential stress at the inner wall. Under this condition as the internal pressure increases the compression in the inner cylinder is first released and then only the cylinder begins to act in tension. Gun barrels are normally pre-stressed by hooping since very large internal pressures are generated.

Here the main problem is to determine the contact pressure p_s. At the contact surface the outer radius r_{si} of the inner cylinder is slightly larger than the inside diameter rso of the outer cylinder. However for stress calculations we assume that $r_{so}=r_{si}=r_s$ (say). The inner and outer cylinders are shown in **figure- 9.2.2.2**.



9.2.2.2 - Dimensions and the pressures at the contact surface of the internal and outer cylinders.

For the outer cylinder the radial and circumferential stresses at the contact surface may be given by

$$\sigma_r\big|_{r=r_s} = \frac{p_s r_s^2}{r_o^2 - r_s^2}\left(1 - \frac{r_o^2}{r_s^2}\right) = -p_s$$

$$\sigma_\theta\big|_{r=r_s} = \frac{p_s r_s^2}{r_o^2 - r_s^2}\left(1 + \frac{r_o^2}{r_s^2}\right)$$

In order to find the radial displacements of the cylinder walls at the contact we consider that $\varepsilon_\theta = \frac{u}{r} = \frac{1}{E}(\sigma_\theta - \nu\sigma_r)$. This gives the radial displacement of the inner wall of the outer cylinder as

$$u_{r1} = \frac{p_s r_s}{E}\left[\frac{r_o^2 + r_s^2}{r_o^2 - r_s^2} + \nu\right]$$

Similarly for the <u>inner cylinder</u> the radial and circumferential stresses at the outer wall can be given by

$$\sigma_r\big|_{r=r_s} = -\,p_s \qquad\qquad \sigma_\theta\big|_{r=r_s} = -p_s\,\frac{r_s^2 + r_i^2}{r_s^2 - r_i^2}$$

And following the above procedure the radial displacement of the contact surface of the inner cylinder is given by

$$u_{r2} = -\frac{p_s r_s}{E}\left[\frac{r_s^2 + r_i^2}{r_s^2 - r_i^2} - \nu\right]$$

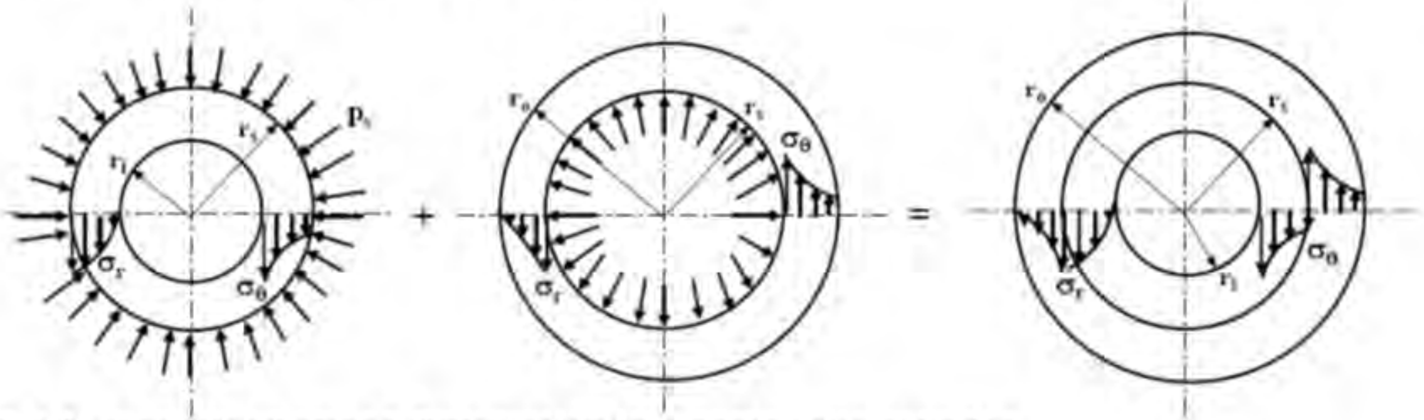
The total interference δ at the contact is therefore given by

$$\delta = \frac{p_s r_s}{E}\left[\frac{r_o^2 + r_s^2}{r_o^2 - r_s^2} + \frac{r_s^2 + r_i^2}{r_s^2 - r_i^2}\right].$$

This gives the contact pressure in terms of the known variables as follows:

$$p_s = \frac{E\delta}{r_s\left[\dfrac{r_o^2 + r_s^2}{r_o^2 - r_s^2} + \dfrac{r_s^2 + r_i^2}{r_s^2 - r_i^2}\right]}$$

The combined stress distribution in a shrink fit composite cylinder is made up of stress distribution in the inner and outer cylinders and this is shown in **figure-9.2.2.3.**



9.2.2.3 - Combined stress distribution in a composite cylinder.

Residual circumferential stress is maximum at r = ri for the inner cylinder and is given by

$$\sigma_{\theta(max)}\Big|_{r=r_i} = -\frac{2p_s r_s^2}{r_s^2 - r_i^2}$$

Residual circumferential stress is maximum at $r = r_s$ for the outer cylinder and is given by

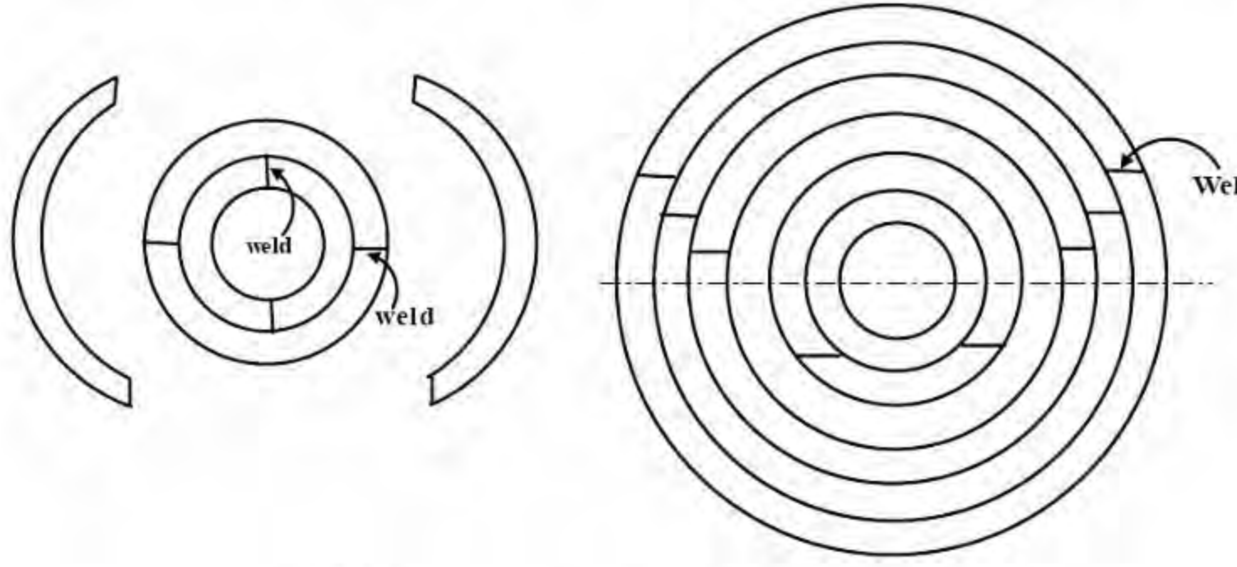
$$\sigma_{\theta(max)}\Big|_{r=r_s} = p_s \frac{r_o^2 + r_s^2}{r_o^2 - r_s^2}$$

Stresses due to fluid pressure must be superimposed on this to find the complete stress distribution.

2. <u>Multilayered or Laminated cylinder</u>

The laminated cylinders are made by stretching the shells in tension and then

welding along a longitudinal seam. This is shown in **figure- 9.2.2.4**.



9.2.2.4 - Method of construction of multilayered cylinder

3. Autofrettage

In some applications of thick cylinders such as gun barrels no inelastic deformation is permitted. But for some pressure vessel design satisfactory function can be maintained until the inelastic deformation that starts at inner bore spreads completely over the wall thickness. With the increase in fluid pressure yielding would start at the inner bore and then with further increase in fluid pressure yielding would spread outward. If now the pressure is released the outer elastic layer would regain its original size and exert a radial compression on the inner shell and tension on the outer region.

This gives the same effect as that obtained by shrinking a hoop over an inner cylinder. This is known as **Self- hooping** or **Autofrettage**. This allows the cylinder to operate at higher fluid pressure. For a given autofrettage fluid pressure a given amount of inelastic deformation is produced and therefore in service the same fluid pressure may be used without causing any additional inelastic deformation.

9.2.3 Summary of this Lecure

Stresses and strains in thick cylinders are first discussed and Lame's equations are derived. Radial and circumferential stress distribution across the wall thickness in thick cylinders have been illustrated. Methods of increasing elastic strength of a thick cylinder by prestressing are then discussed. Interface pressure and displacement during shrinking a hollow cylinder over the main cylinder have been expressed in terms of known variables. Finally multilayered or laminated cylinders and autofrettage are discussed.

Lecture

Theme 9

Thin and thick cylinders

9.3. Design principles for thick cylinders

9.3.1 Application of theories of failure for thick walled pressure vessels.

Having discussed the stresses in thick walled cylinders it is important to consider their failure criterion. The five failure theories will be considered in this regard and the variation of wall thickness to internal radius ratio t/r_i or radius ratio r_o/r_i with p/σ_{yp} for different failure theories would be discussed. A number of cases such as $p_o = 0$, $p_i = 0$ or both non-zero p_o and p_i are possible but here only the cylinders with closed ends and subjected to an internal pressure only will be considered, for an example.

9.3.1.1 Maximum Principal Stress theory

According to this theory failure occurs when maximum principal stress exceeds the stress at the tensile yield point. The failure envelope according to this failure mode is shown in **figure-9.3.1.1.1** and the failure criteria are given by $\sigma_1 = \sigma_2 = \pm\,\sigma_{yp}$. If $p_o = 0$ the maximum values of circumferential and radial stresses are given by

$$\sigma_{\theta(\max)}\Big|_{r=r_i} = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \qquad \sigma_{r(\max)}\Big|_{r=r_i} = -p_i \tag{9.3.1}$$

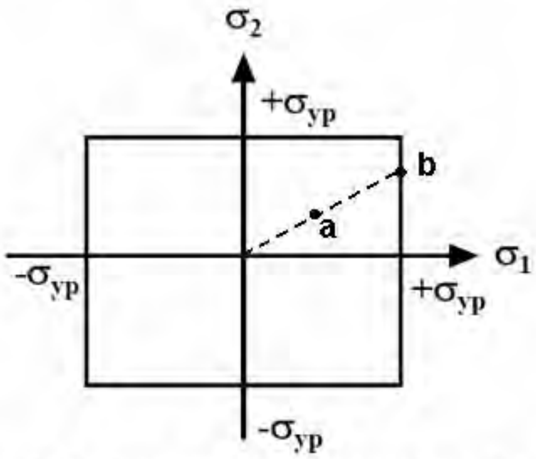
Here both σ_θ and σr are the principal stresses and σ_θ is larger. Thus the condition for failure is based on σ_θ and we have

$$p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} = \sigma_{yp}$$ where σ_{yp} is the yield stress.

$$\because t = r_0 - r_i$$

This gives $$\frac{t}{r_i} = \sqrt{\frac{1 + p_i/\sigma_{yp}}{1 - p_i/\sigma_{yp}}} - 1 \tag{9.3.2}$$

The equality (9.3.2) may by used for safe design.



9.3.1.1.1 - Failure envelope according to Maximum Principal Stress Theory.

9.3.1.2 Maximum Shear Stress theory

According to this theory failure occurs when maximum shear stress exceeds the maximum shear stress at the tensile yield point. The failure envelope according to this criterion is shown in **figure-9.3.1.2.1** and the maximum shear stress is given by

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} \qquad (9.3.3)$$

where the principal stresses σ_1 and σ_2 are given by

$$\sigma_1 = \sigma_\theta = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \qquad \sigma_2 = \sigma_r = -p_i$$

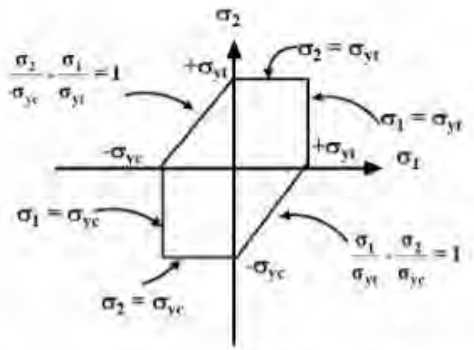
Here σ_1 is tensile and σ_2 is compressive in nature. τmax may therefore be given by

$$\tau_{max} = p_i \frac{r_o^2}{r_o^2 - r_i^2} \qquad (9.3.4)$$

and since the failure criterion is $\tau_{max} = \sigma_{yp} / 2$ we may write

$$\frac{t}{r_i} = \sqrt{\frac{1}{1 - 2\left(p_i / \sigma_{yp} \right)}} - 1 \qquad (9.3.5)$$

The equality (9.3.4) w cam use for safe design.



9.3.1.2.1. - Failure envelope according to Maximum Shear Stress theory.

9.3.1.3 Maximum Principal Strain theory

According to this theory failure occurs when the maximum principal strain exceeds the strain at the tensile yield point.

$$\varepsilon_1 = \frac{1}{E}\{\sigma_1 - \nu(\sigma_2 + \sigma_3)\} = \varepsilon_{yp} \text{ and this gives } \sigma_1 - \nu(\sigma_2 + \sigma_3) = \sigma_{yp} \quad \because \sigma_{xp} = E\varepsilon_{yp}$$

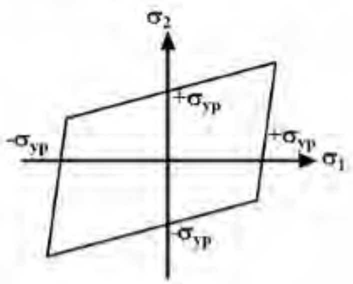
where ε_{yp} and σ_{yp} are the yield strain and stress respectively. Following this the failure envelope is as shown in **figure-9.3.1.3.1.** Here the three principle stresses can be given as follows according to the standard 3D

$$\sigma_1 = \sigma_\theta = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}, \; \sigma_2 = \sigma_r = -p_i \text{ and } \sigma_3 = \sigma_z = \frac{p_i r_i^2}{r_o^2 - r_i^2} \tag{9.3.6}$$

The failure criterion may now be written as

$$p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} + \nu - \frac{\nu r_i^2}{r_o^2 - r_i^2}\right) = \sigma_{yp} \text{ and this gives } \frac{t}{r_i} = \sqrt{\frac{1 + (1-2\nu)p_i/\sigma_{yp}}{1 - (1+\nu)p_i/\sigma_{yp}}} - 1 \tag{9.3.7}$$

Equation (9.3.7) may be used for safe design.



9.3.1.3.1 - Failure envelope according to Maximum Principal Strain theory

9.3.1.4 Maximum Distortion Energy Theory

According to this theory if the maximum distortion energy exceeds the distortion energy at the tensile yield point failure occurs. The failure envelope is shown in **figure-9.3.1.4.1** and the distortion energy Ed is given by

$$E_d = \frac{1+\nu}{6E}\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right\}$$

Since at the uniaxial tensile yield point $\sigma_2 = \sigma_3 = 0$ and $\sigma_1 = \sigma_{yp}$

E_d at the tensile yield point $= \frac{1+\nu}{3E}\sigma_{yp}^2$

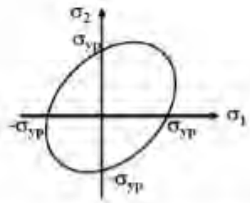
We consider $\sigma_1 = \sigma_\theta$, $\sigma_2 = \sigma_r$ and $\sigma_3 = \sigma_z$ and therefore

$$\sigma_1 = p_i\frac{r_o^2+r_i^2}{r_o^2-r_i^2} \qquad \sigma_r = -p_i \qquad \sigma_z = \frac{p_i r_i^2}{r_o^2-r_i^2} \tag{9.3.8}$$

The failure criterion therefore reduces to

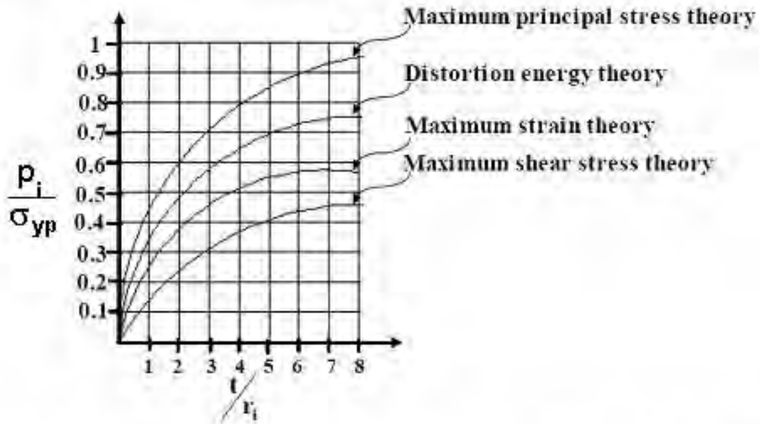
$$\frac{p_i}{\sigma_{yp}} = \frac{1}{\sqrt{3}}\left(\frac{r_o^2-r_i^2}{r_o^2}\right) \quad \text{which gives} \quad \frac{t}{r_i} = \sqrt{\frac{1}{1-\sqrt{3}\,p_i/\sigma_{yp}}}-1 \tag{9.3.9}$$

This equation we can use for safe design.



9.3.1.4.1 - Failure envelope according to Maximum Distortion Energy Theory

Plots of p_i/σ_{yp} and t/r_i for different failure criteria are shown in **figure-9.3.1.4.2**.



9.3.1.4.2 - Comparison of variation of - against t/r$_i$ for different failure criterion.

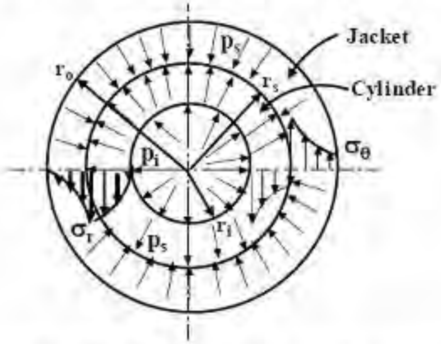
The criteria developed and the plots apply to thick walled cylinders with internal pressure only but similar criteria for cylinders with external pressure only or in case where both internal and external pressures exist may be developed. However, on the basis of these results we note that the rate of increase in p_i/σ_{yp} is small at large values of t/r_i for all the failure modes considered. This means that at higher values of pi small increase in pressure requires large increase in wall thickness. But since the stresses near the outer radius are small, material at the outer radius for very thick wall cylinders are ineffectively used. It is therefore necessary to select materials so that p_i/σ_{yp} is reasonably small. When this is not possible prestressed cylinders may be used.

All the above theories of failure are based on the prediction of the beginning of inelastic deformation and these are strictly applicable for ductile materials under static loading. Maximum principal stress theory is widely used for brittle materials which normally fail by brittle fracture.

In some applications of thick cylinders such as, gun barrels no inelastic deformation can be permitted for proper functioning and there design based on maximum shear stress theory or maximum distortion energy theory are acceptable. For some pressure vessels a satisfactory function is maintained until inelastic deformation that starts from the inner radius and spreads completely through the wall of the cylinder. Under such circumstances none of the failure theories would work satisfactorily and the procedure discussed in section lesson 9.2 is to be used.

9.3.1.5 Failure criteria of pre-stressed thick cylinders

Failure criteria based on the three methods of pre-stressing would now be discussed. The radial and circumferential stresses developed during shrinking a hollow cylinder over the main cylinder are shown in **figure- 9.3.1.5.1**.



9.3.1.5.1 - Distribution of radial and circumferential stresses in a composite thick walled cylinder subjected to an internal pressure.

Following the analysis in section 9.2 the maximum initial (residual) circumferential stress at the inner radius of the cylinder due to the contact pressure p_s is

$$\sigma_\theta\big|_{r=r_i} = -2p_s \frac{r_s^2}{r_o^2 - r_s^2}$$

and the maximum initial (residual) circumferential stress at the inner radius of the jacket due to contact pressure ps is

$$\sigma_\theta\big|_{r=r_s} = p_s \frac{r_o^2 + r_s^2}{r_o^2 - r_s^2}$$

Superposing the circumferential stresses due to pi (considering the composite cylinder as one) the total circumferential stresses at the inner radius of the cylinder and inner radius of the jacket are respectively

$$\sigma_{\theta}\big|_{r=r_i} = -2p_s \frac{r_s^2}{r_s^2 - r_i^2} + p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \qquad \sigma_{\theta}\big|_{r=r_s} = p_s \frac{r_o^2 + r_s^2}{r_o^2 - r_s^2} + p_i \frac{r_i^2}{r_s^2}\left(\frac{r_o^2 + r_s^2}{r_o^2 - r_i^2}\right)$$

These maximum stresses should not exceed the yield stress and therefore we may write

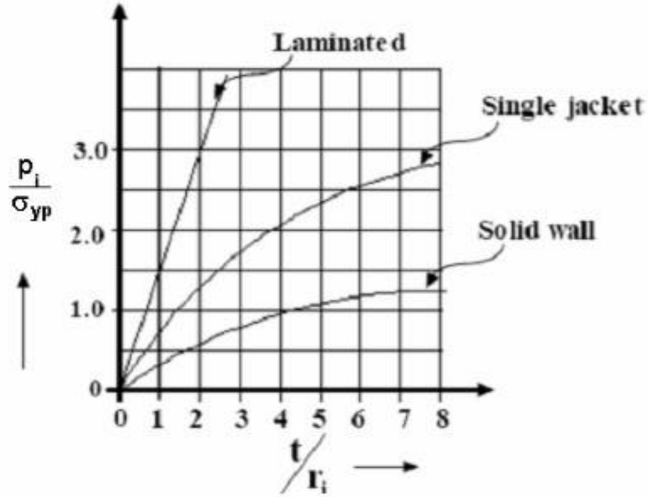
$$\sigma_{\theta}\big|_{r=r_i} = -2p_s \frac{r_s^2}{r_s^2 - r_i^2} + p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} = \sigma_{yp} \qquad (9.3.10)$$

$$\sigma_{\theta}\big|_{r=r_i} = p_s \frac{r_o^2 + r_s^2}{r_o^2 - r_s^2} + p_i \frac{r_i^2}{r_s^2}\left(\frac{r_o^2 + r_s^2}{r_o^2 - r_i^2}\right) = \sigma_{yp} \qquad (9.3.11)$$

It was shown in section-9.2 that the contact pressure p_s is given by

From (9.3.10), (9.3.11) and (9.3.12) it is possible to eliminate p_s and express t/r_i in terms of p_i/σ_{yp} and this is shown graphically in **figure-9.3.1.5.2**.

$$p_s = \frac{E\delta}{r_s\left[\frac{r_o^2 + r_s^2}{r_o^2 - r_s^2} + \frac{r_s^2 + r_i^2}{r_s^2 - r_i^2}\right]} \quad (9.3.12)$$



9.3.1.5.2. - Plot of p_i/σ_{yp} vs t/r_i for laminated multilayered, single jacket and solid wall cylinders.

This shows that even with a single jacket there is a considerable reduction in wall thickness and thus it contributes to an economic design.

As discussed earlier autofrettage causes yielding to start at the inner bore and with the increase in pressure it spreads outwards. If now the pressure is released the outer elastic layer exerts radial compressive pressure on the inner portion and this in turn causes radial compressive stress near the inner portion and tensile stress at the outer portion. For a given fluid pressure during autofrettage a given amount of inelastic deformation is produced and therefore in service the same fluid pressure may be used without causing any additional elastic deformation.

The self hooping effect reaches its maximum value when yielding just begins to spread to the outer wall. Under this condition the cylinder is said to have reached a fully plastic condition and the corresponding internal fluid pressure is known as fully plastic pressure, say, p_f. This pressure may be found by using the reduced equilibrium equation (3) in section- 9.2.1 which is reproduced here for convenience

$$\sigma_\theta = \sigma_r + r\frac{d\sigma_r}{dr} \tag{9.3.13}$$

Another equation may be obtained by considering that when the maximum shear stress at a point on the cylinder wall reaches shear yield value τ_{yp} it remains constant even after further yielding. This is given by

$$\frac{1}{2}(\sigma_\theta - \sigma_r) = \tau_{yp} \tag{9.3.14}$$

However experiments show that fully plastic pressure is reached before inelastic deformation has sp
to every point on the wall. In fact Luder's lines appear first. Luder's lines are spiral bands across the wall such that the material between the bands retains elasticity. If the cylinder is kept under fully pla
pressure for several hours uniform yielding across the cylinder wall would occur.

This gives $\frac{d\sigma_r}{dr} = 2\tau_{yp}/r$ and on integration we have $\sigma_r = 2\tau_{yp}\log r + c$

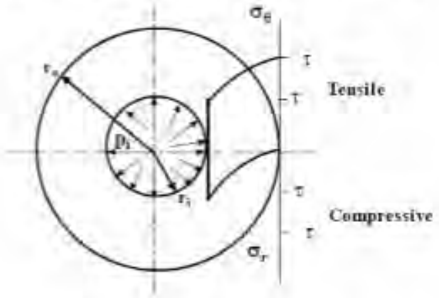
Applying the boundary condition at $r = r_o$ $\sigma_r = 0$ we have

$$\sigma_r = 2\tau_{yp}\log\left(\frac{r}{r_o}\right) \text{ and } \sigma_\theta = 2\tau_{yp}\left\{1+\log\left(\frac{r}{r_o}\right)\right\} \qquad (9.3.15)$$

Also applying the boundary condition at $r = r_i$ $\sigma_r = -p_f$ we have

$$p_f = -2\tau_{yp}\log\left(\frac{r_i}{r_o}\right) \qquad (9.3.16)$$

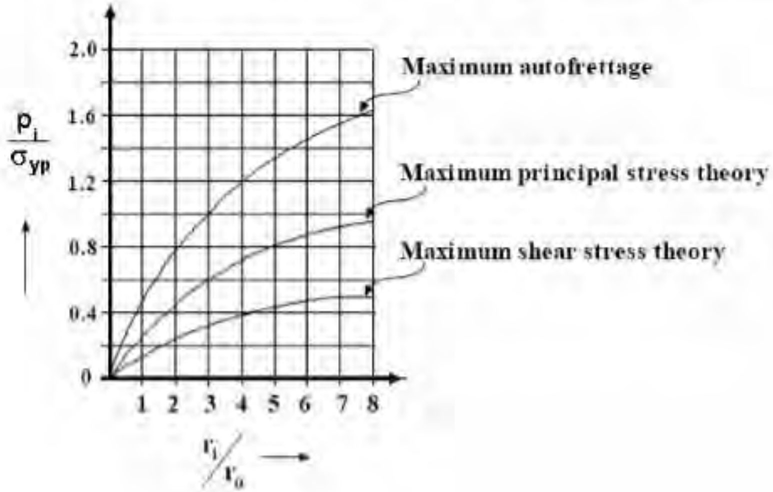
Since the basic equations are independent of whether the cylinders are open or closed ends, the expressions for σ_r and σ_θ apply to both the conditions. The stress distributions are shown in **figure- 9.3.1.5.3**.



9.3.1.5.3 - Stress distribution in a thick walled cylinder with autofrettage

If we roughly assume that $2\tau_{yp} = \sigma_{yp}$ we have
$$\frac{p_f}{\sigma_{yp}} = -\log\left(\frac{r_i}{r_o}\right) \quad (9.3.17)$$

The results of maximum principal stress theory and maximum shear stress theory along with the fully plastic results are replotted in **figure 9.3.1.5.4** where we may compare the relative merits of different failure criteria. It can be seen that cylinders with autofrettage may endure large internal pressure at relatively low wall thickness.



9.3.1.5.4. - Plots of p_i/σ_{yp} vs r_i/r_o for maximum shear stress theory, maximum principal stress theory and maximum **autofrettage.**

Finally it must be remembered that for true pressure vessel design it is essential to consult Boiler Codes for more complete information and guidelines. Pressure vessels can be extremely dangerous even at relatively low pressure and therefore the methodology stated here is a rough guide and should not be considered to be a complete design methodology.

9.3.2 Problems with Answers

Q.1: Determine the necessary thickness of the shell plates of 2.5m diameter boiler with the internal pressure of 1MPa. The material is mild steel with a tensile strength of 500MPa. Assuming an efficiency of the longitudinal welded joint to be 75% and a factor of safety of 5 find the stress in the perforated steel plate. In this case we use formulae for rivet joint.

A.1: Considering that the boiler design is based on thin cylinder principles the shell thickness is given by

$t = \frac{pr}{\sigma_{ty}\eta}$ where r is the boiler radius and η is the joint efficiency.

This gives

$$t = \frac{10^6 x1.25}{\left(\frac{500}{5}\right)x10^6 x0.75} = 0.0166\text{m} = 16.6\,\text{mm, say } 20\text{mm.}$$

The stress in the perforated plate is therefore given by

$$\sigma = \frac{pr}{t} \text{ i.e. } 62.5\text{MPa}$$

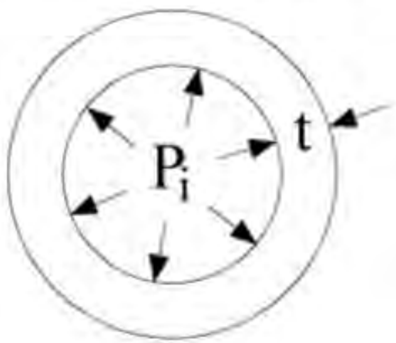
Q.2: A hydraulic cylinder with an internal diameter 250mm is subjected to an internal pressure of 10 MPa. Determine the wall thickness based on (a) Maximum яprincipal stress theory, b) Maximum shear stress theory and c) Maximum distortion energy theory of failure. Compare the results with wall thickness calculated based on thin cylinder assumption. Assume the yield stress of the cylinder material to be 60 MPa.

A.2: Considering that the hydraulic cylinders are normally designed on the thick cylinder assumption we have from section 9.3.1.1 for Maximum Principal stress Theory we have

$$t = r_i\left(\sqrt{\frac{1+\frac{p_i}{\sigma_{yp}}}{1-\frac{p_i}{\sigma_{yp}}}}-1\right)$$

$d_i = 250$ mm
$p_i = 10$ MPa
$\sigma_y = 60$ MPa
$t = ?$

Here $\frac{p_i}{\sigma_{yp}} = 10/60 = 0.167$ and r_i = 125 mm. This gives t = 22.9mm, say 23 mm



From section 9.3.1.2 for Maximum Shear Stress theory we have

$$t = r_i\left(\sqrt{\frac{1}{1-2\left(\frac{p_i}{\sigma_{yp}}\right)}}-1\right)$$

With $\frac{p_i}{\sigma_{yp}} \approx 0.167$ and r_i = 125 mm, t = 28.2 mm, say 29 mm.

From section 9.3.1.4 for maximum distortion energy theory we have

$$t = r_i\left(\frac{1}{\sqrt{1-\sqrt{3}\left(\frac{p_i}{\sigma_{yp}}\right)}}-1\right)$$

with $\frac{p_i}{\sigma_{yp}} \approx 0.167$ and r_i = 125mm t = 23.3 mm, say 24 mm.

Considering a thin cylinder $t = r_i\left(\frac{p_i}{\sigma_{yp}}\right)$ and this gives t = 20.875mm, say 21 mm.

The thin cylinder approach yields the lowest wall thickness and this is probably not safe. The largest wall thickness of 29mm predicted using the maximum shear stress theory is therefore adopted.

Q.3: A cylinder with external diameter 300mm and internal diameter 200mm is subjected to an internal pressure of 25 MPa. Compare the relative merits of a single thick walled cylinder and a composite cylinder with the inner cylinder whose internal and external diameters are 200mm and 250 mm respectively. A tube of 250 mm internal diameter and 300mm external diameter is shrunk on the main cylinder. The safe tensile yield stress of the material is 110 MPa and the stress set up at the junction due to shrinkage should not exceed 10 MPa.

A.3:

We first consider the stresses set up in a single cylinder and then in a composite cylinder.

Single cylinder

The boundary conditions are

at r = 150mm σ_r = 0 and at r = 100mm σ_r = - 20MPa

Using equation (10) in section 9.2.1

$$C_1 + \frac{C_2}{0.0225} = 0 \quad \text{and} \quad C_1 + \frac{C_2}{0.01} = -20$$

This gives C_1= 16 and C_2 = -0.36

The hoop stress at r = 100mm and r = 150 mm are **52 MPa** and **32 MPa** respectively.

Stress in the composite cylinder

The stresses in the cylinder due to shrinkage only can be found using the following boundary conditions

at r = 150mm $\sigma_r = 0$ and at r = 125mm σ_r = -10MPa

Following the above procedure the hoop stress at r = 150 mm and r = 125mm are **45.7MPa** and **55.75MPa** respectively.

The stress in the inner cylinder due to shrinkage only can be found using the following boundary conditions

at r = 100mm $\sigma_r = 0$ and at r = 125mm σ_r = -10MPa

This gives the hoop stress at r = 100mm and r = 125mm to be **- 55.55 MPa** and **– 45.55 MPa** respectively.

Considering the internal pressure only on the complete cylinder the boundary conditions are

at r = 150mm $\sigma_r = 0$ and at r = 100mm σ_r = -25 MPa

This gives

$(\sigma_\theta)_{r=150mm}$ = **40MPa** $(\sigma_\theta)_{r=125mm}$ = **49 MPa** $(\sigma_\theta)_{r=100mm}$ = **65MPa.**

Stress in the composite cylinder

The stresses in the cylinder due to shrinkage only can be found using the following boundary conditions

at r = 150mm σ_r = 0 and at r = 125mm σ_r = -10MPa

Following the above procedure the hoop stress at r = 150 mm and r = 125mm are **45.7MPa** and **55.75MPa** respectively.

The stress in the inner cylinder due to shrinkage only can be found using the following boundary conditions

at r = 100mm σ_r = 0 and at r = 125mm σ_r = -10MPa

This gives the hoop stress at r = 100mm and r = 125mm to be **- 55.55 MPa** and **– 45.55 MPa** respectively.

Considering the internal pressure only on the complete cylinder the boundary conditions are

at r = 150mm σ_r = 0 and at r = 100mm σ_r = -25 MPa

This gives

$(\sigma_\theta)_{r=150mm}$ = **40MPa** $(\sigma_\theta)_{r=125mm}$ = **49 MPa** $(\sigma_\theta)_{r=100mm}$ = **65MPa.**

Resultant stress due to both shrinkage and internal pressure

Outer cylinder

$(\sigma_\theta)_{r=150mm}$ = 40 +45.7 = **85.7 MPa**

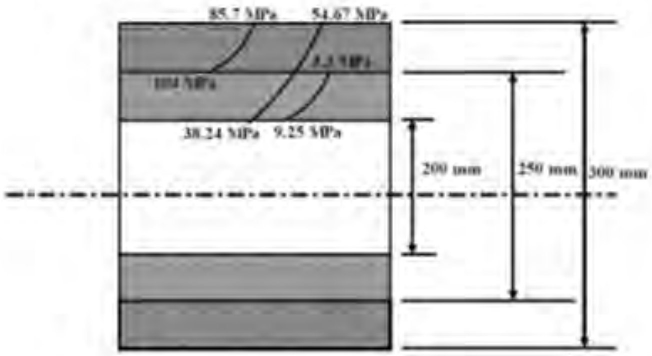
$(\sigma_\theta)_{r=125mm}$ = 49+55.75 = **104.75 MPa**

Inner cylinder

$(\sigma_\theta)_{r=125mm}$ = 49 -45.7 = **3.3 MPa**

$(\sigma_\theta)_{r=100mm}$ = 65 - 55.75 = **9.25 MPa**

The stresses in both the single cylinder and the composite are within the safe tensile strength of the material. However in the single cylinder the stress gradient is large across the wall thickness whereas in the composite cylinder the stress variation is gentle. These results are illustrated in **figure- 9.3.2.1**



9.3.2.1 - Stress gradients (circumferential) in the inner and outer cylinders as well as the gradient across the wall of a single cylinder.

9.3.3 Summary of this Lecture

The lesson initially discusses the application of different failure theories in thick walled pressure vessels. Failure criterion in terms of the ratio of wall thickness to the internal radius and the ratio of internal pressure to yield stress have been derived for different failure criterion. Failure criterion for prestressed composite cylinders and cylinders with autofrettage have also been derived. Finally comparisons of different failure criterion have been discussed.

Lecture

Theme 10

Design of Permanent Joints

10.1. Riveted Joints : Types and Uses

10.1.1. Rivets as permanent joints:

Often small machine components are joined together to form a larger machine part. Design of joints is as important as that of machine components because a weak joint may spoil the utility of a carefully designed machine part.

Mechanical joints are broadly classified into two classes viz., non-permanent **joints** and **permanent joints**.

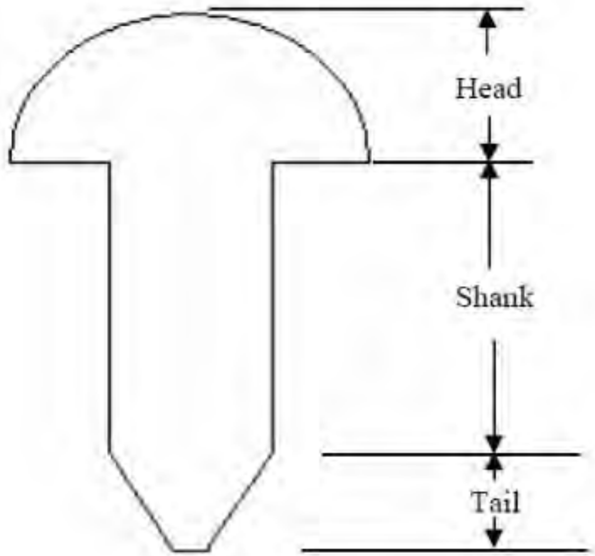
Non-permanent joints can be assembled and dissembled without damaging the components. Examples of such joints are threaded fasteners (like screw-joints), keys and couplings etc.

Permanent joints cannot be dissembled without damaging the components. These joints can be of two kinds depending upon the nature of force that holds the two parts. The force can be of mechanical origin, for example, riveted joints, joints formed by press or interference fit etc, where two components are joined by applying mechanical force. The components can also be joined by molecular force, for example, **welded joints, brazed joints, joints with adhesives etc**.

Not until long ago riveted joints were very often used to join structural members permanently. However, significant improvement in welding and bolted joints has curtained the use of these joints. Even then, rivets are used in structures, ship body, bridge, tanks and shells, where high joint strength is required.

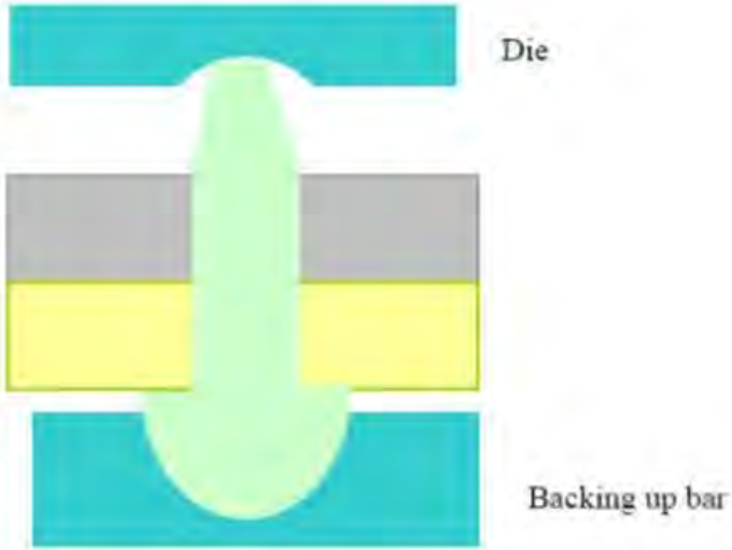
10.1.2. Rivets and Riveting:

A rivet is a short cylindrical rod having a head and a tapered tail. The main body of the rivet is called shank (see figure 10.1.1). According to Indian standard specifications rivet heads are of various types. Rivets heads for general purposes are specified by Indian standards IS: 2155-1982 (below 12 mm diameter) and IS: 1929-1982 (from 12 mm to 48 mm diameter). Rivet heads used for boiler works are specified by IS: 1928-1978. To get dimensions of the heads see any machine design handbook..



10.1.1: Rivet and its parts

Riveting is an operation whereby two plates are joined with the help of a rivet. Adequate mechanical force is applied to make the joint strong and leak proof. Smooth holes are drilled (or punched and reamed) in two plates to be joined and the rivet is inserted. Holding, then, the head by means of a backing up bar as shown in figure 10.1.2, necessary force is applied at the tail end with a die until the tail deforms plastically to the required shape. Depending upon whether the rivet is initially heated or not, the riveting operation can be of two types: (a) cold riveting riveting is done at ambient temperature and (b) hot riveting rivets are initially heated before applying force. After riveting is done, the joint is heat-treated by quenching and tempering. In order to ensure leak-proofness of the joints, when it is required, additional operation like caulking is done .



10.1.2: Riveting operation

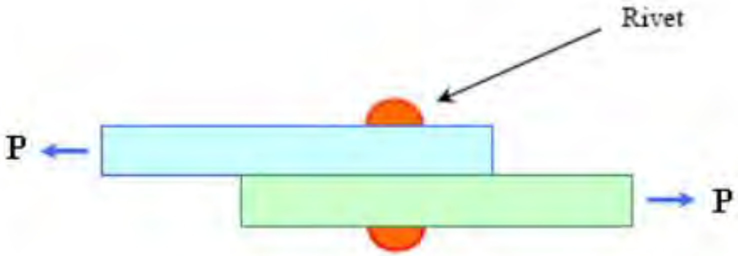
10.1.3. Types of riveted joints and joint efficiency:

Riveted joints are mainly of two types

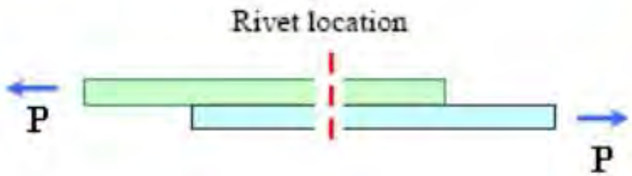
1. Lap joints
2. Butt joints

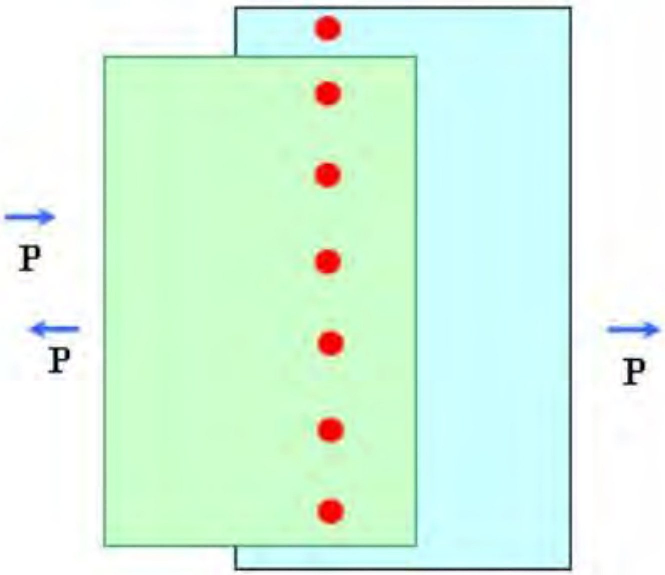
10.1.3.1 Lap Joints:

The plates that are to be joined are brought face to face such that an overlap exists, as shown in figure 10.1.3. Rivets are inserted on the overlapping portion. Single or multiple rows of rivets are used to give strength to the joint. Depending upon the number of rows the riveted joints may be classified as single riveted lap joint, double or triple riveted lap joint etc. When multiple joints are used, the arrangement of rivets between two neighbouring rows may be of two kinds. In chain riveting the adjacent rows have rivets in the same transverse line. In zig-zag riveting, on the other hand, the adjascent rows of rivets are staggered.

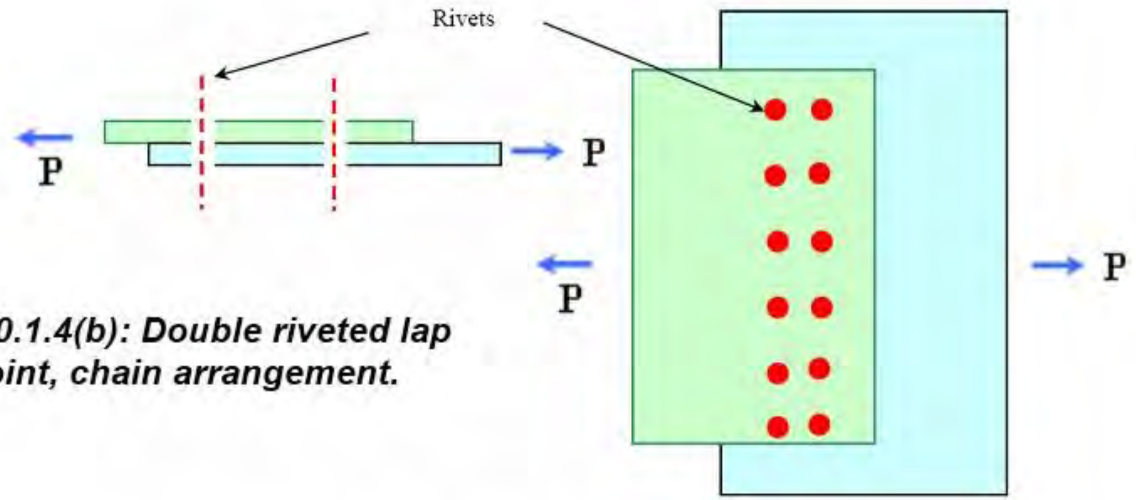


10.1.3: Lap joint

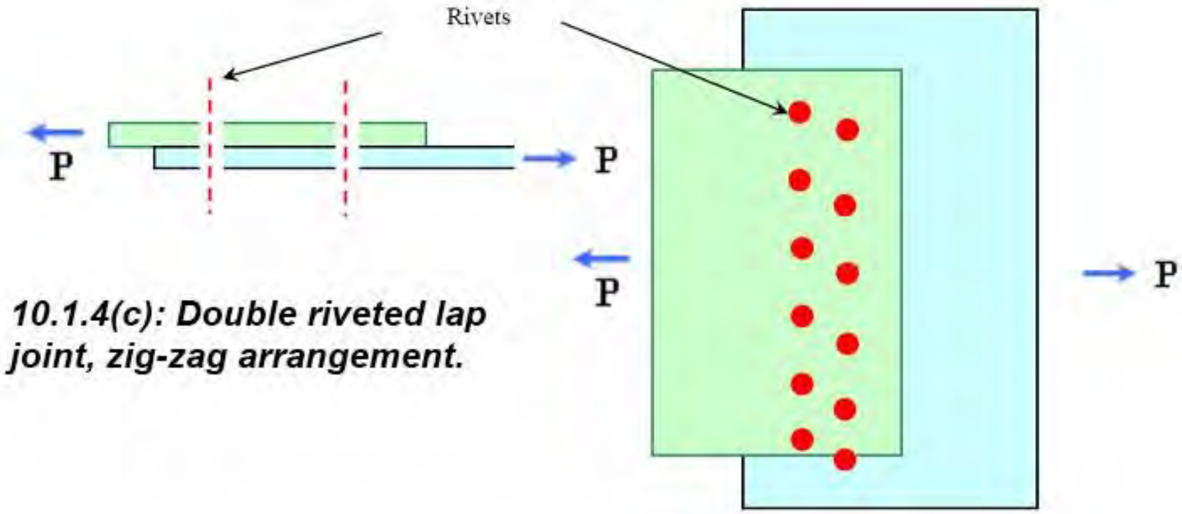




10.1.4(a): Single rivet lap joint



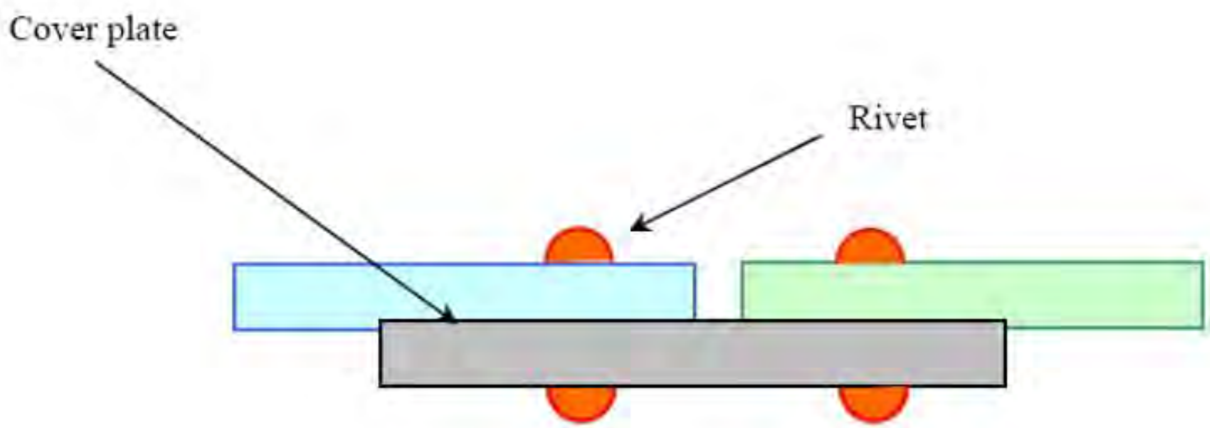
10.1.4(b): Double riveted lap joint, chain arrangement.



10.1.4(c): Double riveted lap joint, zig-zag arrangement.

10.1.3.2 Butt Joints

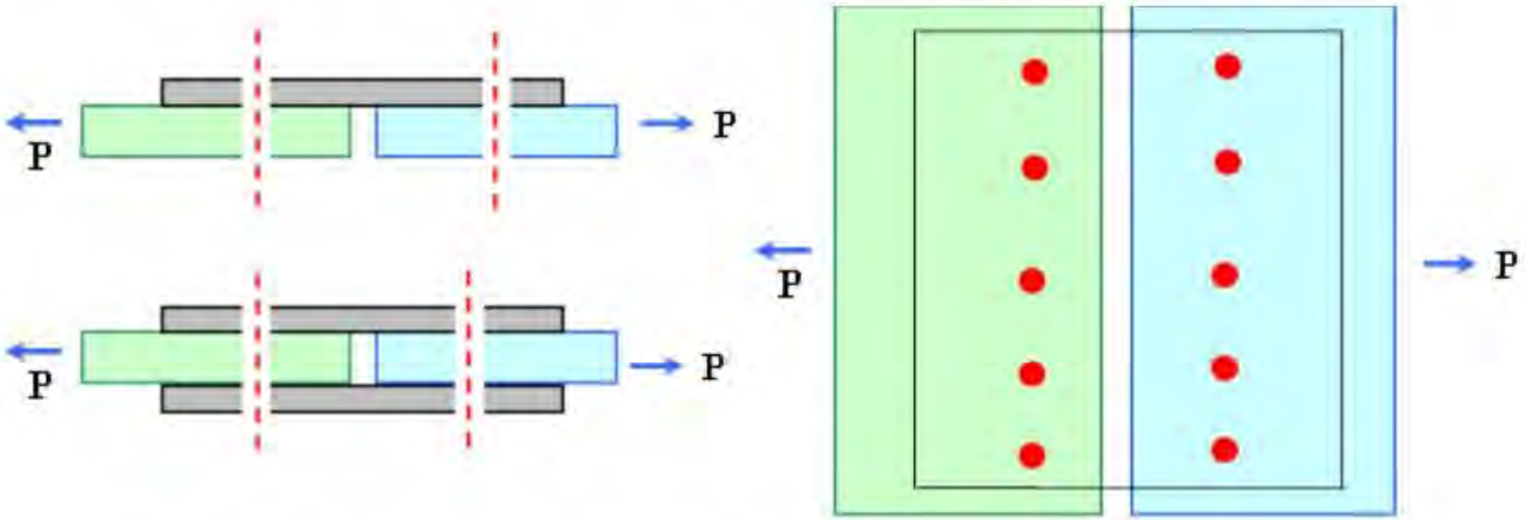
In this type of joint, the plates are brought to each other without forming any overlap. Riveted joints are formed between each of the plates and one or two cover plates. Depending upon the number of cover plates the butt joints may be single strap or double strap butt joints. A single strap butt joint is shown in figure 10.1.5.



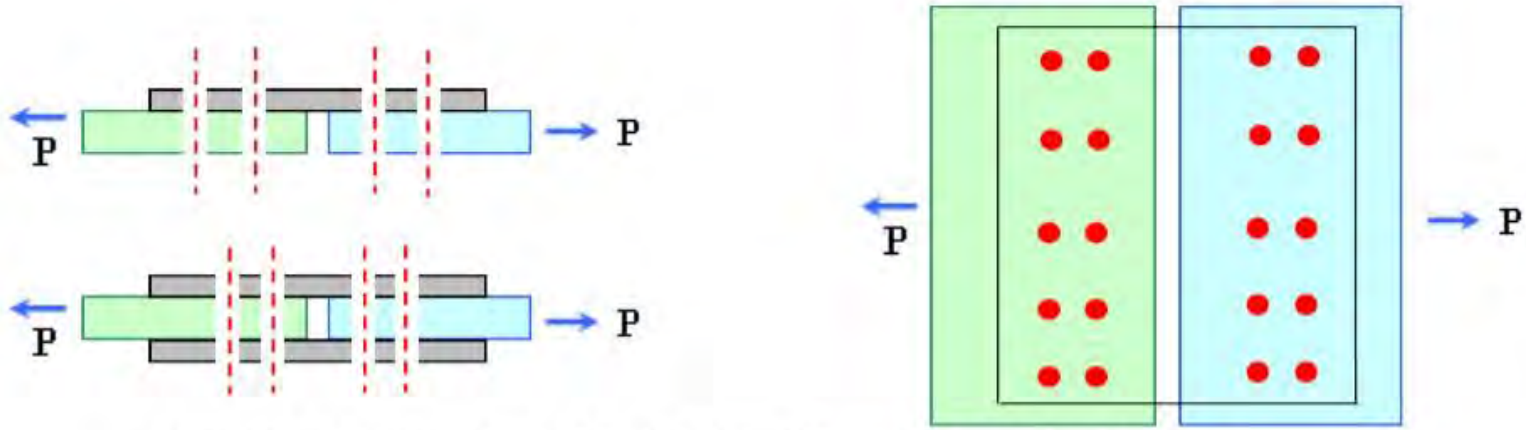
10.1.5: Butt joint with single strap.

Like lap joints, the arrangement of the rivets may be of various kinds, namely, single row, double or triple chain or zigzag. A few types of joints are shown in figure 10.1.6(a)-6(c).

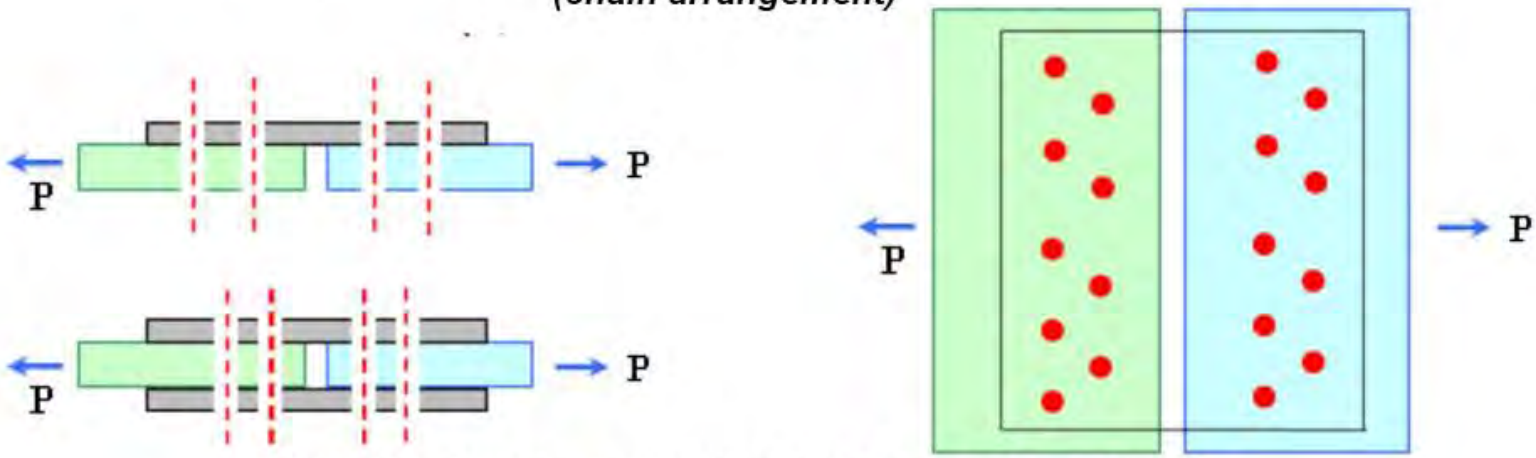
The strength of a rivet joint is measured by its efficiency. The efficiency of a joint is defined as the ratio between the strength of a riveted joint to the strength of an unrivetted joints or a solid plate. Obviously, the efficiency of the riveted joint not only depends upon the size and the strength of the individual rivets but also on the overall arrangement and the type of joints.



10.1.6(a): Single riveted butt joint with single and double straps



10.1.6(b): Double riveted butt joint with single and double straps (chain arrangement)



10.1.6(c): Double riveted butt joint with single and double straps (zig-zag arrangement)

Usual range of the efficiencies, expressed in percentiles of the commercial boiler joints are given in table-10.1.1.

Table 10.1.1: Efficiencies of riveted joints (in %)

Joints		*Efficiencies (in %)*
Lap	Single riveted	50-60
	Double riveted	60-72
	Triple riveted	72-80
Butt (double strap)	Single riveted	55-60
	Double riveted	76-84
	Triple riveted	80-88

4. Important terms used in riveted joints:

Few parameters, which are required to specify arrangement of rivets in a riveted joint are as follows:

a) *Pitch:* This is the distance between two centers of the consecutive rivets in a single row. (usual symbol *p*)

b) *b) Back Pitch:* This is the shortest distance between two successive rows in a multiple riveted joint. (usual symbol *tp*or *bp*)

c) *Diagonal pitch:* This is the distance between the centers of rivets in adjacent rows of zigzag riveted joint. (usual symbol *dp*)

d) *Margin or marginal pitch:* This is the distance between the centre of the rivet hole to the nearest edge of the plate. (usual symbol m)

These parameters are shown in figure 10.1.7.

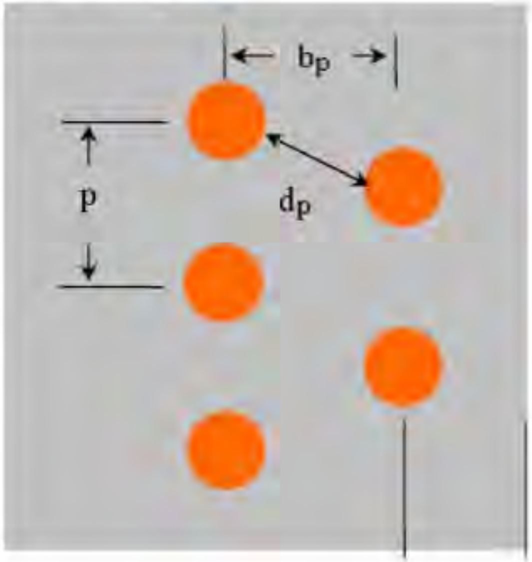


Figure 10.1.7: Important design parameters of riveted joint

Problens with Answers:

Q.1.What should be essential qualities of a rivet and its material?

A1: From the riveting procedure it is clear that a good rivet material must be tough and ductile. Steel (low carbon), coppers, brass are good candidates for rivets. According to Indian standard IS: 2998-1982 the material must have tensile strength of 40 MPa and elongation not less that 20 %. Further, the rivet shank must not be bent on itself through 1800 without cracking in cold condition. The same test must be done for rivet elevated to 6500 C and quenched.

Q.2.What are the uses of *snap headed, counter shank headed, conical headed* and *pan headed* rivets?

A2: Snap heads are used mainly for structural work and machine riveting. Counter shank heads are employed for ship building where flush surfaces are necessary. Conical heads are used where riveting is done by hand hammering. Pan heads are required where very high strength is needed since they have the maximum strength, but they are very difficult to shape.

Lecture

Theme 10

Design of Permanent Joints

10.2. Design of Riveted Joints

10.2.1. Strength of riveted joint:

Strength of a riveted joint is evaluated taking all possible failure paths in the joint into account. Since rivets are arranged in a periodic manner, the strength of joint is usually calculated considering one pitch length of the plate. There are four possible ways a single rivet joint may fail.

1) Tearing of the plate: If the force is too large, the plate may fail in tension along the row (see figure 10.2.1). The maximum force allowed in this case is

$$P_1 = s_t\,(p-d)t$$

where s_t = allowable tensile stress of the plate material, p = pitch, d = diameter of the rivet hole, t = thickness of the plate

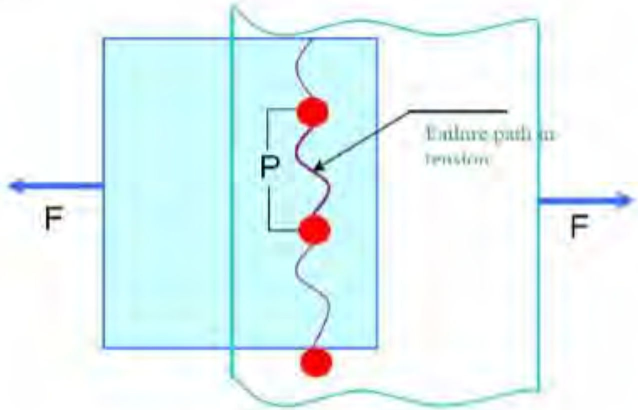


Figure 10.2.1: Failure of plate in tension (tearing)

2) *Shearing of the rivet:* The rivet may shear as shown in figure 10.2.2. The maximum force withstood by the joint to prevent this failure is

$$P_2 = s_s(\frac{\pi}{4}d^2) \quad \text{for lap joint, single strap butt joint}$$

$$R_2 = 2s_s(\frac{\pi}{4}d^2) \quad \text{for double strap butt joint}$$

where s_s=allowable shear stress of the rivet material.

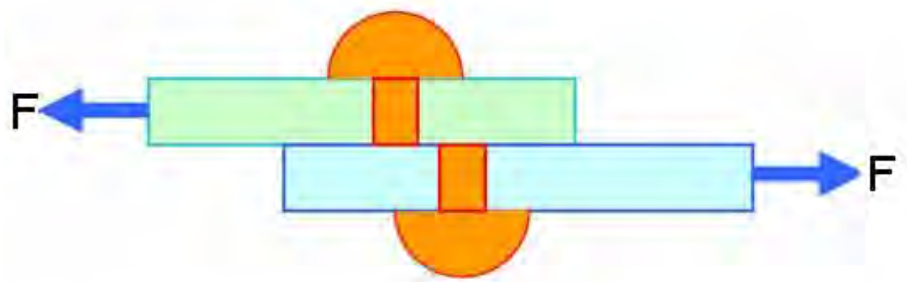


Figure 10.2.2: Failure of a rivet by shearing

3) Crushing of rivet: If the bearing stress on the rivet is too large the contact surface between the rivet and the plate may get damaged. (see figure 10.2.3). With a simple assumption of uniform contact stress the maximum force allowed is

$$P_3 = s_c dt$$

where s_c = allowable bearing stress between the rivet and plate material.

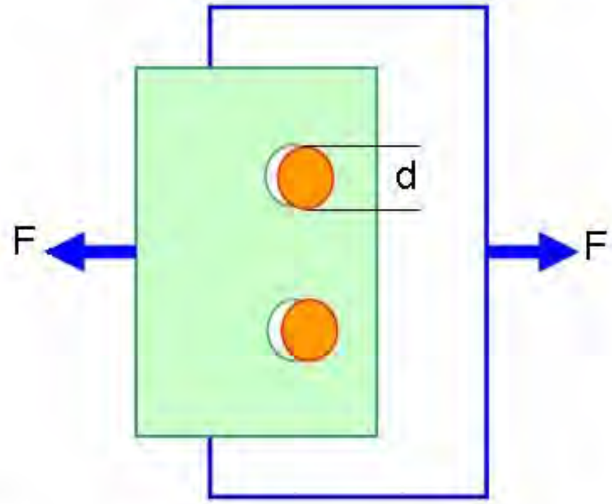


Figure 10.2.3: Failure of rivets by

4) Tearing of the plate at edge: If the margin is too small, the plate may fail as shown in figure 10.2.4. To prevent the failure a minimum margin of m=1.5d is usually provided.

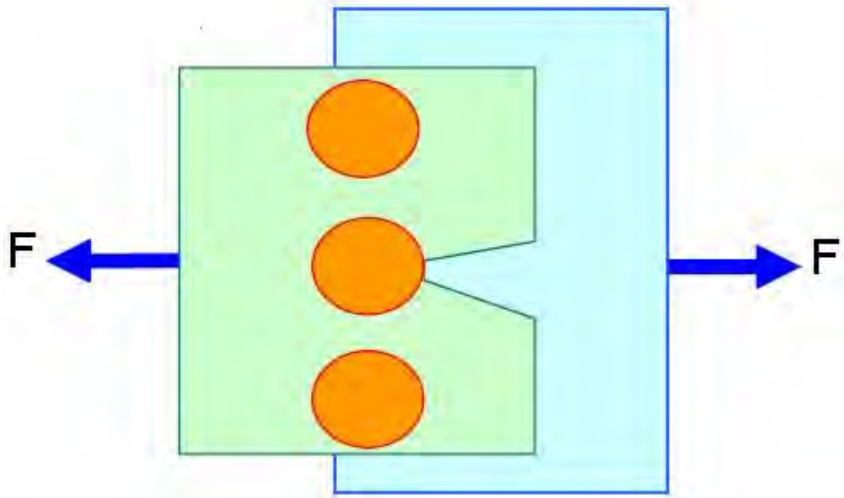


Figure 10.2.4: Tearing of the plate at the edge

10.2.2. Efficiency:

Efficiency of the single riveted joint can be obtained as ratio between the maximum of P_1, P_2, and P_3 and the load carried by a solid plate which is $s_t pt$. Thus efficiency

$$(\eta)=\frac{\min\{P_1,P_2,P_3\}}{s_t pt}$$

In a double or triple riveted joint the failure mechanisms may be more than those discussed above. The failure of plate along the outer row may occur in the same way as above. However, in addition the inner rows may fail. For example, in a double riveted joint, the plate may fail along the second row. But in order to do that the rivets in the first row must fail either by shear or by crushing. Thus the maximum allowable load such that the plate does not tear in the second row is $P_4 = s_t(p-d)t + \min\{P_2,P_3\}$.

Further, the joint may fail by

(1) shearing of rivets in both rows, (2) crushing of rivets in both rows

(3) shearing of rivet in one row and crushing in the other row.

The efficiency should be calculated taking all possible failure mechanism into consideration.

10.2.3. Design of rivet joints:

The design parameters in a riveted joints are *d, p* and *m*

Diameter of the hole (d): When thickness of the plate *(t)* is more than 8 mm, Unwin's formula is used, $d = 6\sqrt{t}$ mm.

Otherwise is obtained by equating crushing strength to the shear strength of the joint. In a double riveted zigzag joint, this implies

$$s_c t = \frac{\pi}{4} d\, s_s \quad \text{(valid for } t < 8 \text{ mm)}$$

However, should not be less than *t*, in any case. The standard size of is tabulated in code IS: 1928-1961. *dd*

Pitch (p): Pitch is designed by equating the tearing strength of the plate to the shear strength of the rivets. In a double riveted lap joint, this takes the following form.

$$s_t (p - d) t = s_s \times 2\left(\frac{\pi}{4} d^2\right)$$

But $p \geq 2d$ in order to accommodate heads of the rivets.

Margin (m): $m = 1.5d$.

In order to design boiler joints, (p_b: usually $0.33p+0.67d$ mm)

Questions and Answers:

Q.1. Two plates of 7 mm thickness are connected by a double riveted lap joint of zigzag pattern. Calculate rivet diameter, rivet pitch and distance between rows of rivets for the joint. Assume s_t = *90 MPa,* s_c= 120MPa.

A.1. Since t = 7mm<8mm, the diameter of the rivet hole is selected equating shear strength to the crushing strength, i.e.,

$$2\left(\frac{\pi}{4}d^2\right)s_s = 2dt\, s_c$$

yielding d=17.8mm. According to IS code, the standard size is d=19mm and the corresponding rivet diameter is 18 mm.

Pitch is obtained from the following

$s_t(p-d)t = 2s_s(\frac{\pi}{4}d^2)$, where $d = 19\,\text{mm}$ $\qquad p = 54 + 19 = 73\,\text{mm}$

[Note: If the joint is to comply with I.B.R. specification, then p_{max}=ct+41.28mm, where is a constant depending upon the type of joint and is tabulated in the code.] The distance between the two rivet rows is

$$p_d = \frac{p}{3} + \frac{2}{3}d = 37\,\text{mm} \ .$$

Q.2. A triple riveted butt joint with two unequal cover plates joins two 25 mm plates as shown in the figure below.

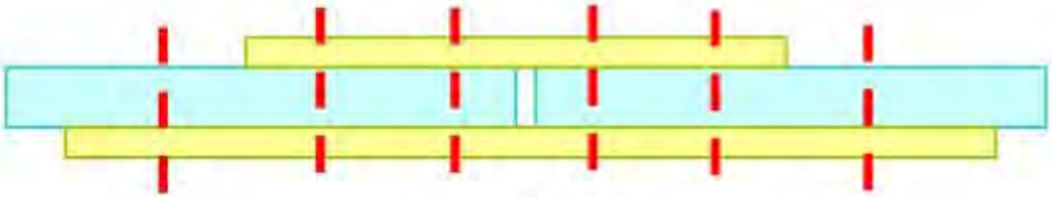


Figure: 10.2.5

The rivet arrangement is zigzag and the details are given below:

Pitch = 22 cm in outer row and 11 cm in inner rows,

Rivet diameter = 33 mm

Calculate the efficiency of the joint when the allowable stresses are 75 MPa, 60 MPa and 125 MPa in tension, shear and crushing, respectively.

A.2. From code it may be seen that the corresponding rivet hole diameter is 34.5 mm.

To find strength of the joint all possible failure mechanisms are to be considered separately.

(a) Tearing resistance of the plate in outer row:

$P_1 = (p - d_{hole})\, t\, s_T$ = (220-34.5) X 25 X 75 = 347.81 kN

(b) Shearing resistance of the rivet:

$$P_2 = 2 \times 4 \times \frac{\pi}{4} d^2 s_S + \frac{\pi}{4} d^2 s_S = 461.86 \text{ kN}$$

Note that within a pitch length of 22cm four rivets are in double shear while one rivet in single shear.

(c) Crushing resistance of the rivets

$$P_3 = 5 \times d\, t s_C = 515.62 \text{ kN}$$

(d) Shear failure of the outer row and tearing of the rivets in the second row

$$P_4 = (p - 2d_{hole}) t s_T + \frac{\pi}{4} d^2 s_S = 334.44 \text{ kN}$$

Note that in second row there are 2 rivets per pitch length and the rivets in outer row undergoes single shear.

There are other mechanisms of failure of the joint e.g. tearing along the innermost row and shearing or crushing of rivets in other two rows etc., but all of them will have higher resistance than those considered above. Hence the efficiency of the joint is

$$\eta = \frac{\min\{P_1, P_2, P_3, P_4\}}{p t s_T} = 0.8108$$

or when expressed in percentile 81.08 %.

Q.3. How is a rivet joint of uniform strength designed?

A.3. The procedure by which uniform strength in a riveted joint is obtained is known as *diamond riveting*, whereby the number of rivets is increased progressively from the outermost row to the innermost row (see figure below). A common joint, where this type of riveting is done, is Lozenge joint used for roof, bridge work etc.

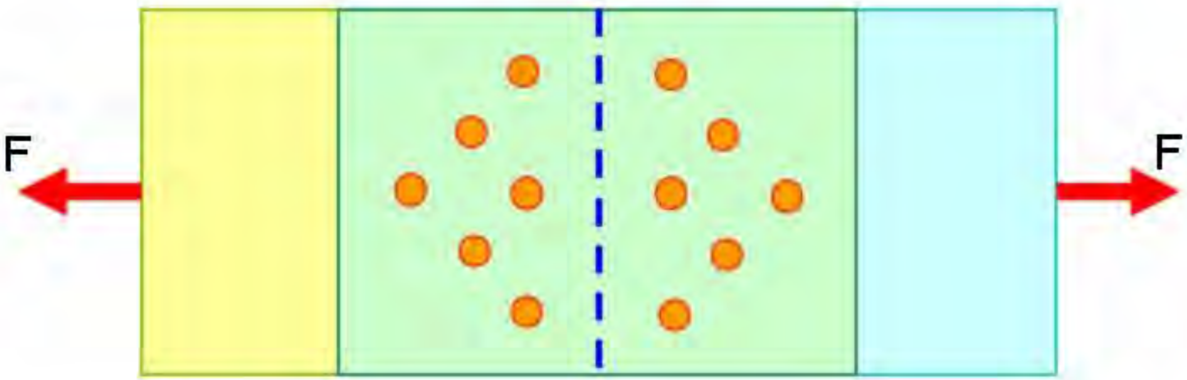


Figure 10.2.6: Diamond riveting in structural joint

Q.4. Two mild steel tie rods having width 200 mm and thickness 12.5 mm are to be connected by means of a butt joint with double cover plates. Find the number of rivets needed if the permissible stresses are 80 MPa in tension, 65 MPa in shear and 160 MPa in crushing.

A.4. As discussed earlier for a structural member Lozenge joint is used which has one rivet in the outer row.

The number of rivets can be obtained equating the tearing strength to the shear or crushing strength of the joint, i.e., from the equation

$$(b-d)ts_T = 2n_1\left(\frac{\pi}{4}d^2\right)s_s \quad \text{[Double shear]} \quad \text{or} \quad (b-d)ts_T = n_2(dt)s_c$$

where *b* and *t* are the width and thickness of the plates to be joined . In the problem *b*=200mm, *t*=12.5mm, s_T=80MPa, s_c=160MPa, s_s=65MPa and *d* is obtained from Unwin's formula $d = 6\sqrt{t}\ \text{mm} = 21.2\,\text{mm}$. According to IS code, the standard rivet hole diameter is 21.5 mm and corresponding rivet diameter is 20 mm. The number of rivets required is the minimum of the numbers calculated from the above two expressions. It may be checked that is found out to be 3.89 while n_2 is 4.216. Therefore, at least 5 rivets are needed.

Lecture

Theme 10

Design of Permanent Joints

10.3. Welded Joints: Types and Uses

10.3.1. Welded joints and their advantages:

Welding is a very commonly used permanent joining process. Thanks to great advancement in welding technology, it has secured a prominent place in manufacturing machine components. A welded joint has following advantages:

(1) Compared to other type of joints, the welded joint has higher efficiency. An efficiency > 95 % is easily possible.

(2) Since the added material is minimum, the joint has lighter weight.

(3) Welded joints have smooth appearances.

(4) Due to flexibility in the welding procedure, alteration and addition are possible.

(5) It is less expensive.

(6) Forming a joint in difficult locations is possible through welding.

The advantages have made welding suitable for joining components in various machines and structures. Some typically welded machine components are listed below.

1. Pressure vessels, steel structures.
2. Flanges welded to shafts and axles.
3. Crank shafts
4. Heavy hydraulic turbine shafts
5. Large gears, pulleys, flywheels
6. Gear housing
7. Machine frames and bases
8. Housing and mill-stands.

10.3.2. Basic types of welded processes:

Welding can be broadly classified in two groups

1) *Liquid state (fusion) welding* where heat is added to the base metals until they melt. Added metal (filler material) may also be supplied. Upon cooling strong joint is formed. Depending upon the method of heat addition this process can be further subdivided, namely

a. Electrical heating: Arc welding, Resistance welding,Induction welding

b. Chemical welding: Gas welding,Thermit welding

c. Laser welding

d. Electron beam welding

2) *Solid state welding*: Here mechanical force is applied until materials deform to plastic state. Bonds are then formed through molecular interaction. Solid state welding may be of various kinds, namely,

a. Cold welding

b. Diffusion welding

c. Hot forging

Descriptions of the individual welding processes are to be found in any standard textbook on welding.

10.3.3. Strength of welded joints:

Adequate care must be taken to enhance strength of the welded joint. It is seen that strength of a welded joint gets affected mainly by the following factors.

(1) *Crack initiation*: it is possible that cracks form while cooling a melted metal.

(2) *Residual stresses*: due to inhomogeneous heating of the base metals, residual stresses may exist upon cooling.

(3) *Metallurgical transformation*: in heat affected zone (HAZ) metallurgical properties may change leading to weakening of the joint.

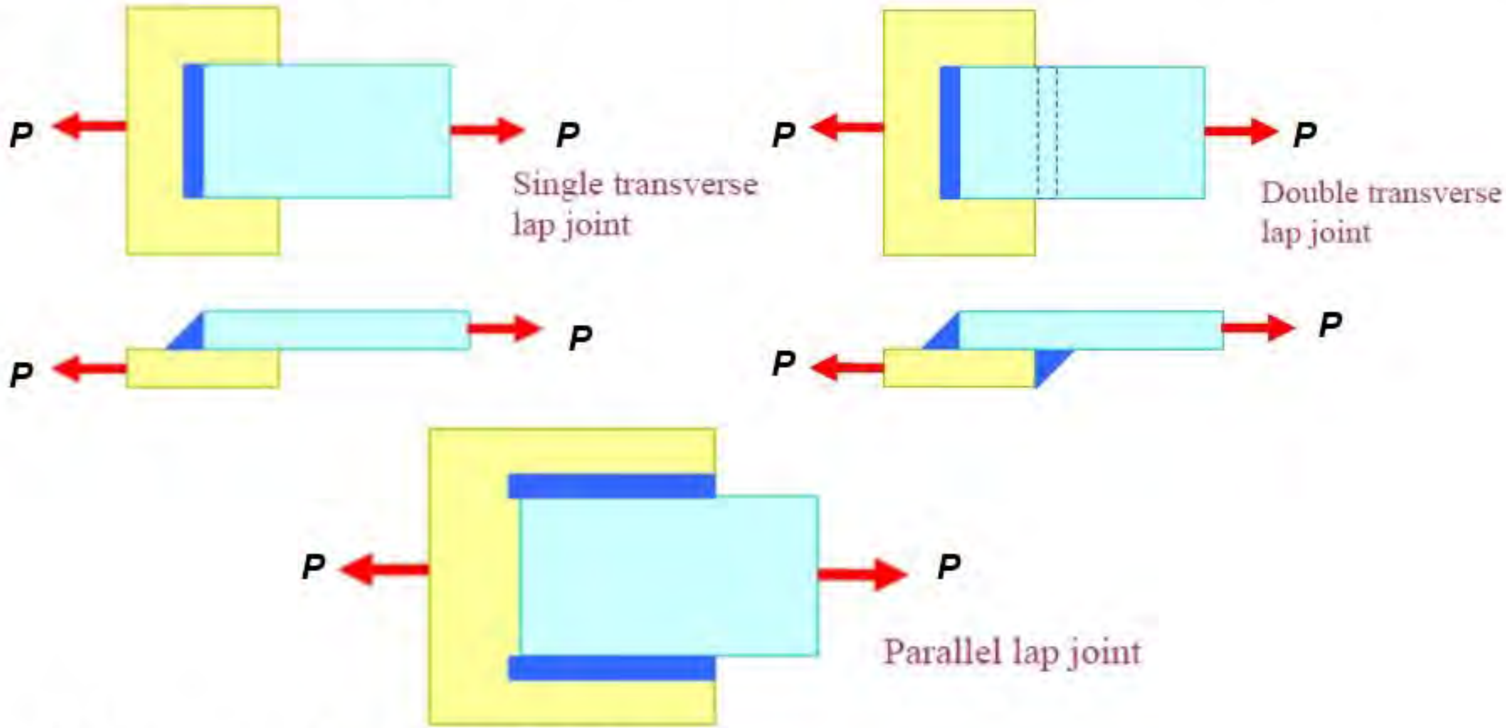
(4) *Defects*: of various kinds like incomplete penetration, porosity, slag inclusion which affect the strength of a welded joint.

(5) *Stress concentration*: abrupt change in the geometry after welding may introduce stress concentration in the structure.

10.3.4. Types of welded joints:

Welded joints are primarily of two kinds

1) *Lap or fillet joint*: obtained by overlapping the plates and welding their edges. The fillet joints may be single transverse fillet, double transverse fillet or parallel fillet joints (see figure 10.3.1).

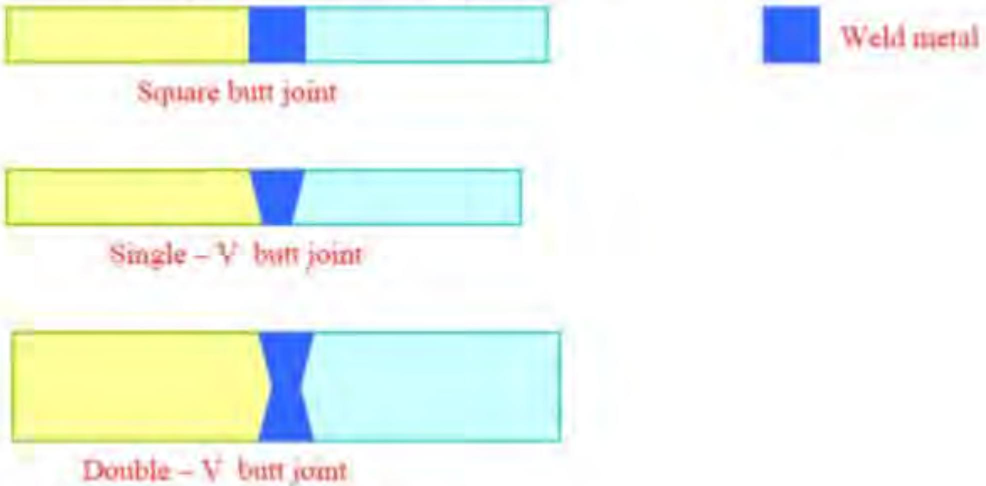


10.3.1: Different types of lap joints

2) *Butt joints*: formed by placing the plates edge to edge and welding them. Grooves are sometimes cut (for thick plates) on the edges before welding. According to the shape of the grooves, the butt joints may be of different types, e.g.,

a. Square butt joint
b. Single V-butt joint, double V-butt joint
c. Single U-butt joint, double U-butt joint
d. Single J-butt joint, double J-butt joint
e. Single bevel-butt joint, double bevel butt joint

These are schematically shown in figure 10.3.2.



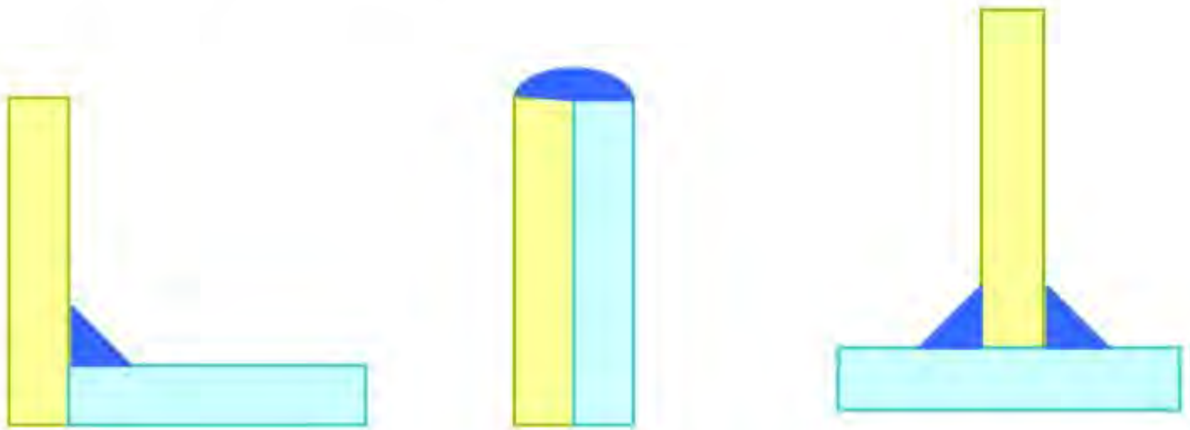
10.3.2: Different types of butt joints

There are other types of welded joints, for example,

a. Corner joint (see figure 10.3.3a)

b. Edge or seal joint (see figure 10.3.3b)

c. T-joint (see figure 10.3.3c)



(a) Corner joint (b) Edge joint (c) T - joint

10.3.3: Other types of welded joints

Each type of joint has its own symbol. The basic weld symbols are shown in Table-10.3.1.

Table10.3.1: Basic weld types and their symbols

Sl. No.	Type of weld	Symbol
1.	Fillet joint	
2.	Square butt joint	
3	Single V- butt joint	
4	Double V- butt joint	
5	Single U – butt joint	
6	Single bevel butt joint	

After welding is done the surface is properly finished. The contour of the welded joint may be flush, concave or convex and the surface finish may be grinding finish, machining finish or chipping finish. The symbols of the contour and the surface finish are shown in Table-10.3.2.

Table 10.3.2: Supplementary Weld Symbols

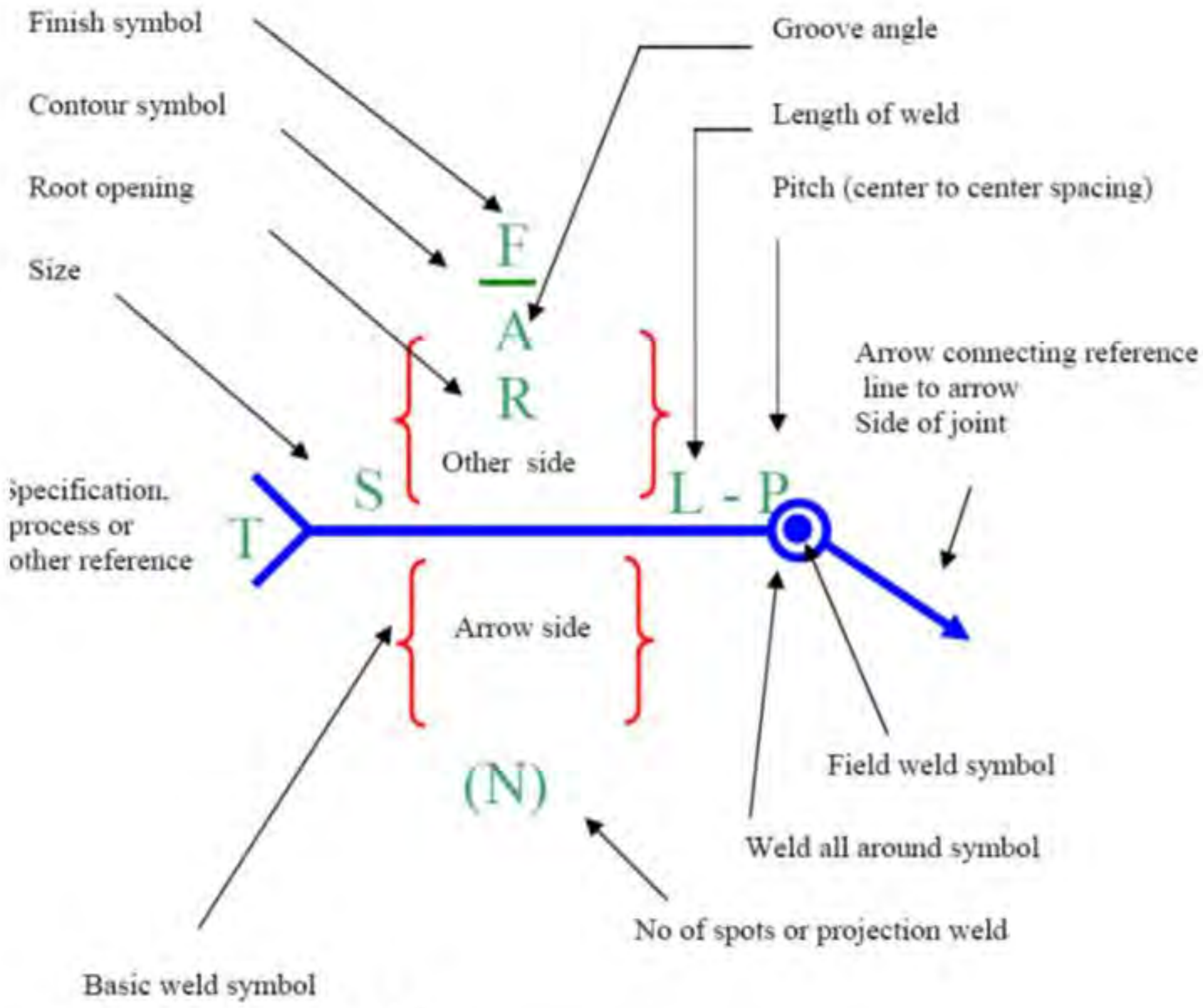
Sl No.	Particulars	Weld Symbol
1	Flush contour	
2	Convex contour	
3	Concave contour	
4	Grinding finish	G
5	Machining finish	M
6	Chipping finish	C

10.3.5. Welding symbol:

A welding symbol has following basic elements:

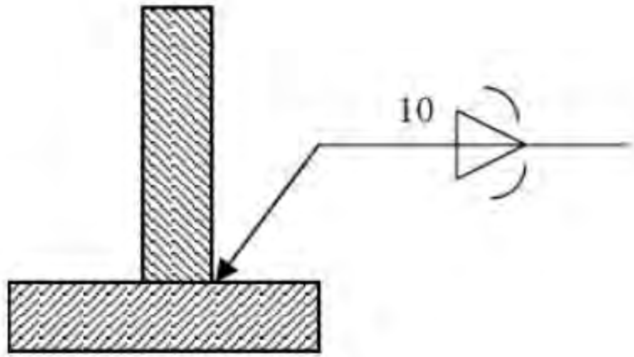
1. Reference line
2. Arrow
3. Basic weld symbols (like fillet, butt joints etc.)
4. Dimensions
5. Supplementary symbols
6. Finish symbols
7. Tail
8. Specification processes.

These welding symbols are placed in standard locations (see Figure 10.3.4)


Finish symbol
Contour symbol
Root opening
Size
Groove angle
Length of weld
Pitch (center to center spacing)
F
A
R
Other side
S
L - P
Arrow connecting reference
line to arrow
Side of joint
Specification,
process or
other reference
T
Arrow side
(N)
Field weld symbol
Weld all around symbol
No of spots or projection weld
Basic weld symbol

10.3.4

Example: If the desired weld is a fillet weld of size 10 mm to be done on each side of Tee joint with convex contour, the weld symbol will be as following. Fig. 10.3.5.



10.3.4

Lecture

Theme 10

Design of Permanent Joints

10.4. Design of Welded Joints

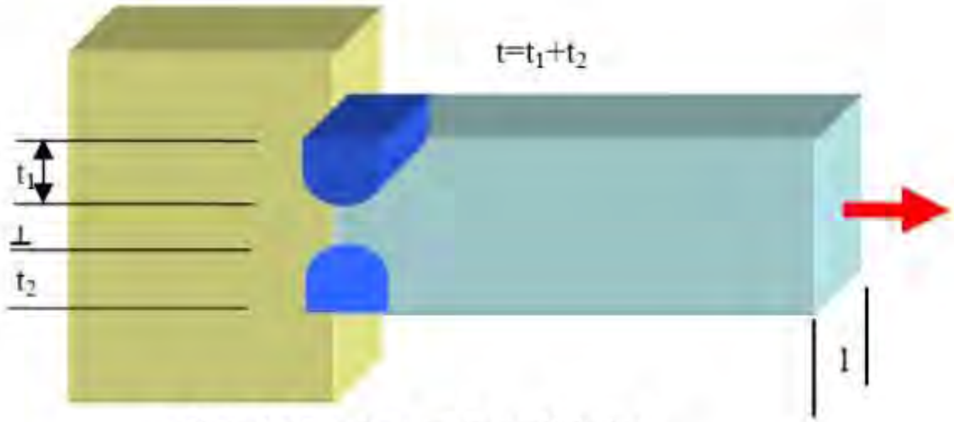
10.4.1. Design of a butt joint:

The main failure mechanism of welded butt joint is tensile failure. Therefore the strength of a butt joint is

$$\mathbf{P = s_t\, lt}$$

where s_T =allowable tensile strength of the weld material, t =thickness of the weld, l =length of the weld.

For a square butt joint is equal to the thickness of the plates. In general, this need not be so (see figure 1).



10.4.1: Design of a butt joint

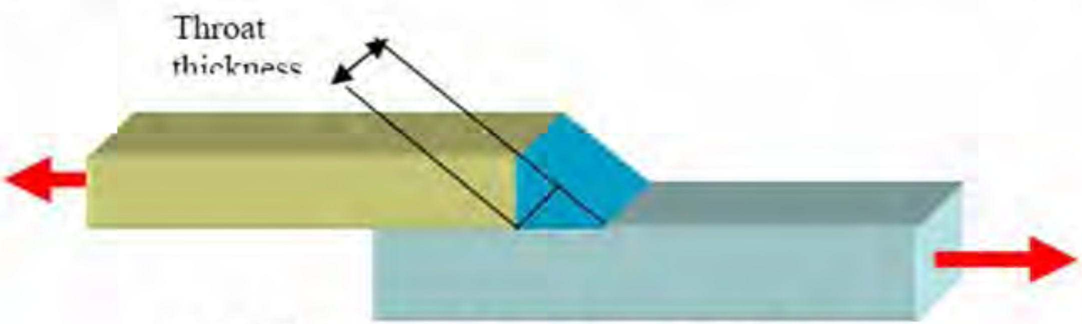
10.4.2. Design of transverse fillet joint:

Consider a single transverse joint as shown in figure 10.4.2. The general stress distribution in the weld metal is very complicated. In design, a simple procedure is used assuming that entire load *P* acts as shear force on the throat area, which is the smallest area of the cross section in a fillet weld. If the fillet weld has equal base and height, (*h*, say), then the cross section of the throat is easily seen to be $\frac{hl}{\sqrt{2}}$. With the above consideration the permissible load carried by a transverse fillet weld is

$$P = s_s A_{throat}$$

where s_s-allowable shear stress, A_{throat} =throat area.

For a double transverse fillet joint the allowable load is twice that of the single fillet joint.



10.4.2: Design of a single transverse fillet

10.4.3. Design of parallel fillet joint:

Consider a parallel fillet weld as shown in figure 10.4.3. Each weld carries a load *P/2*. It is easy to see from the strength of material approach that the maximum shear occurs along the throat area (try to prove it). The allowable load carried by each of the joint is $s_s A_t$ where the throat area $A_t = \frac{lh}{\sqrt{2}} = l \cdot h \cdot \sin 45^\circ$
The total allowable load is

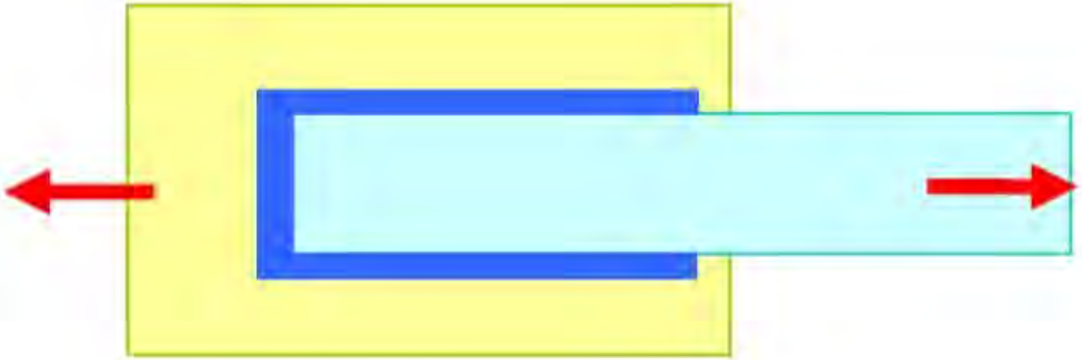
$$\mathbf{P = 2 s_s A_t}.$$

10.4.3: Design of a parallel fillet joint

In designing a weld joint the design variables are and . They can be selected based on the above design criteria. When a combination of transverse and parallel fillet joint is required (see figure-10.4.4) the allowable load is

$$\mathbf{P=2s_sA_t+s_sA_t'}$$

where A_t =throat area along the longitudinal direction, At'=throat area along the transverse direction.



10.4.4: Design of combined transverse and parallel fillet joint

10.4.4. Design of circular fillet weld subjected to torsion:

Consider a circular shaft connected to a plate by means of a fillet joint as shown in figure-10.4.5. If the shaft is subjected to a torque, shear stress develops in the weld in a similar way as in parallel fillet joint. Assuming that the weld thickness is very small compared to the diameter of the shaft, the maximum shear stress occurs in the throat area. Thus, for a given torque the maximum shear stress in the weld is

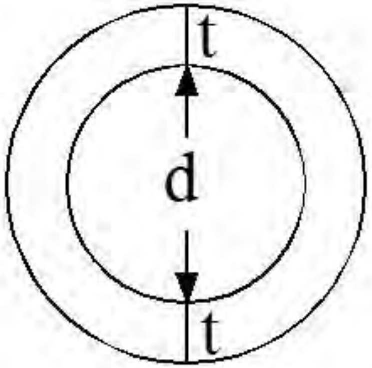
$$T = \frac{\tau I}{r}; \quad \tau = \frac{T \cdot r}{I} \qquad \tau_{max} = \frac{T\left(\frac{d}{2} + t_{throat}\right)}{I_p}$$

where T =torque applied, d =outer diameter of the shaft, t_{throat} = throat thickness, I_p=polar moment of area of the throat section.

$$I = \frac{\pi}{4} t_{throat} d^3, \quad \tau_{max} = \frac{\pi}{32}\left[(d + 2t_{throat})^4 - d^4\right]$$

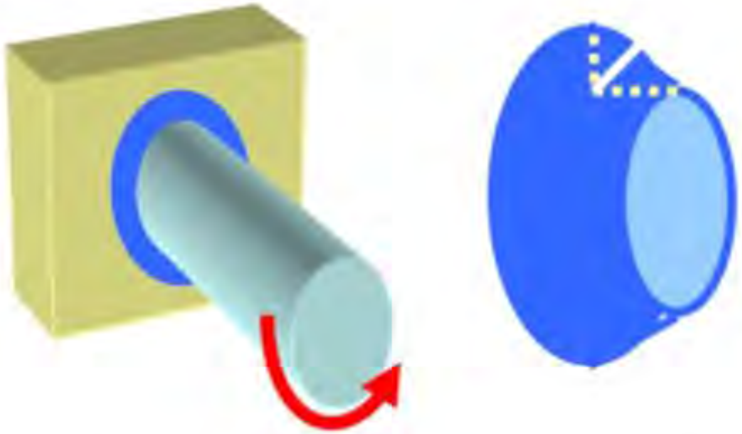
When

$$t_{throat} << d \ , \ \tau_{max} = \frac{T\frac{d}{2}}{\frac{\pi}{4} t_{throat} d^3} = \frac{2T}{\pi t_{throat} d^2}$$



The throat dimension and hence weld dimension can be selected from the equation

$$\frac{2T}{\pi t_{throat} d^2} = s_s$$



10.4.5: Design of a fillet weld for torsion

10.4.5. Design stresses of welds:

Determination of stresses in a welded joint is difficult because of

a. inhomogeneity of the weld joint metals
b. thermal stresses in the welds
c. changes of physical properties due to high rate of cooling etc.

The stresses in welded joints for joining ferrous material with MS electrode are tabulated below.

Table 10.4.5.1

Type of load		Bare electrodes (Static load)	Covered electrodes (Static load)
Butt weld	Tension (MPa)	91.5	112.5
	Compression (MPa)	105.4	126.5
	Shear (MPa)	56.2	70.3
Fillet weld	Shear (MPa)	79.5	98.5

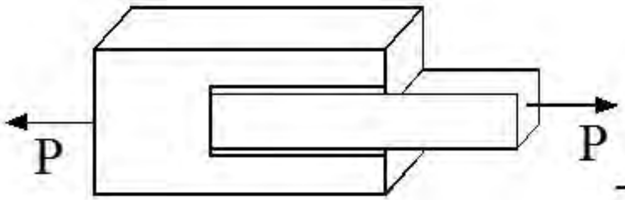
Welded joints are also subjected to eccentric loading as well as variable loading. These topics will be treated separately in later lessons.

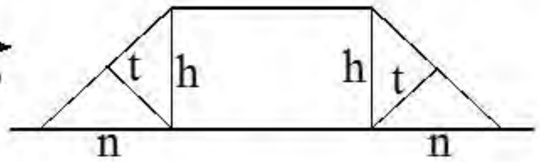
Problems with Answers:

Q.1. A plate 50 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 kN. Find the length of the weld. Assume allowable shear strength to be 56 MPa.

A.1. In a parallel fillet welding two lines of welding are to be provided. Each line shares a load of *P=50/2kN=25kN*. Maximum shear stress in the parallel fillet weld is *P/lt*, where t=throat length=$\frac{12.5}{\sqrt{2}}$ mm . Since $\frac{P}{lt} \le s_s = 56\times10^6$. Hence the minimum length of the weld is $\frac{25\times10^3\times\sqrt{2}}{56\times12.5\times10^3}$=50.5 mm However some extra length of the weld is to be provided as allowance for starting or stopping of the bead. An usual allowance of 12.5 mm is kept. (Note that the allowance has no connection with the plate thickness)

$t = h\cdot\sin 45^{\circ}$





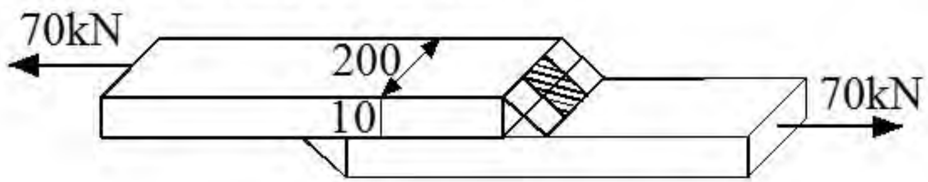
Q.2. Two plates 200 mm wide and 10 mm thick are to be welded by means of transverse welds at the ends. If the plates are subjected to a load of 70 kN, find the size of the weld assuming the allowable tensile stress 70 MPa.

A.2. According to the design principle of fillet (transverse) joint the weld is designed assuming maximum shear stress occurs along the throat area. Since tensile strength is specified the shear strength may be calculated as half of tensile strength, i.e., s_s=*35*MPa. Assuming there are two welds, each weld carries a load of 35 kN and the size of the weld is calculated from

$$35\times 10^3 = l\times(\frac{10\times 10^{-3}}{\sqrt{2}})\times 35\times 10^6 \qquad P = l\cdot t\sigma = s_s \qquad s_s = \frac{P}{lt}$$

or l =42.141 mm.

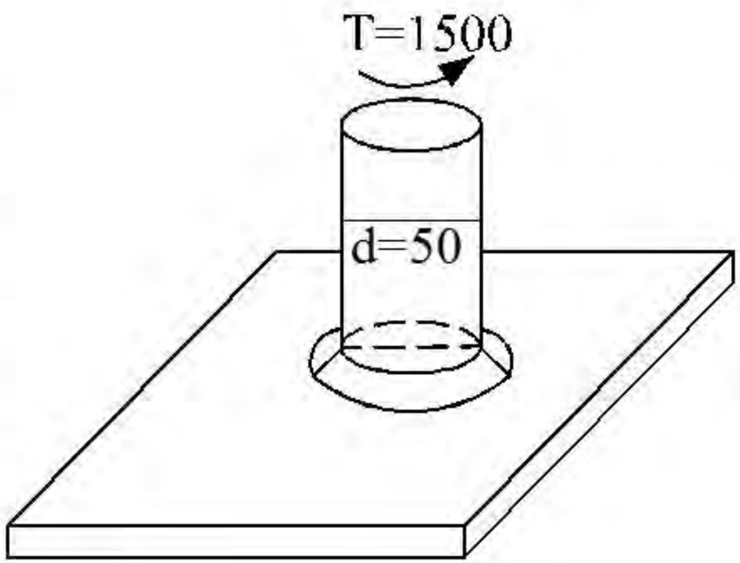
Adding an allowance of 12.5 mm for stopping and starting of the bead, the length of the weld should be 154 mm.



Q.3. A 50 mm diameter solid shaft is to be welded to a flat plate and is required to carry a torque of 1500 Nm. If fillet joint is used foe welding what will be the minimum size of the weld when working shear stress is 56 MPa.

A.3. According to the procedure for calculating strength in the weld joint,

$$\frac{2T}{\pi t_{throat} d^2} = s_s \, ,$$



where the symbols have usual significance. For given data, the throat thickness is 6.8 mm. Assuming equal base and height of the fillet the minimum size is 9.6 mm. Therefore a fillet weld of size 10 mm will have to be used.

$$t_u = \frac{2\tau}{\pi s_s d^2} = \frac{2 \times 1500 \times 10^3}{\pi (50)^2}$$

Lecture

Theme 10

Design of Permanent Joints

10.5. Design of Adhesive Joints

10.5.1. Adhesive joints and their advantages

If the load is not very large adhesive joints become very useful in joining metallic or non–metallic dissimilar materials. No special device is needed. But the disadvantage of this joint is that the joint gets weakened by moisture or heat and some adhesive needs meticulous surface preparation. In an adhesive joint, adhesive are applied between two plates known as adherend. The strength of the bond between the adhesive and adherend arise become of various reasons given below.

- The adhesive materials may penetrate into the adherend material and locks the two bodies.
- Long polymeric chain from the adhesive diffuse into the adherend body to form a strong bond.
- Electrostatic force may cause bonding of two surfaces.

The advantages of the adhesive joints are given below:

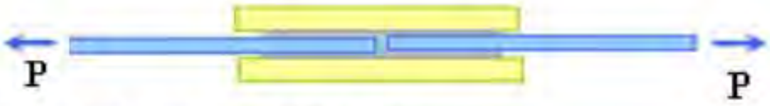
- The mechanism of adhesion helps to reduce stress concentration found in bolted, riveted and welded joints.
- Shock and impact characteristics of the joints are improved
- Dissimilar materials, such as metals, plastics, wood, ceramics can be joined.
- Adhesive joints allow sufficient mechanical compliance in parts subjected to thermal distortion.
- Adhesives can be contoured and formed in various fabrication processes.

10.5.2. Types of Adhesive Joints :

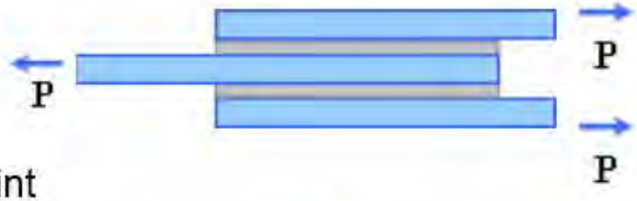
Common types of adhesive joints are shown in figure 10.5.1(a) – 1(d)

(a) Single lap (unsupported) joint.

(b) Balanced double lap adhesive joint



(c) Unbalanced double lap joint

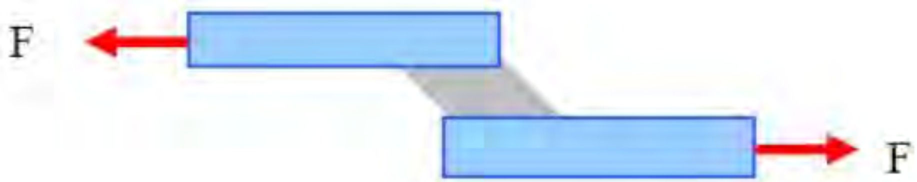


(d) Scarf Joint

10.5.1. Different types of adhesive joints

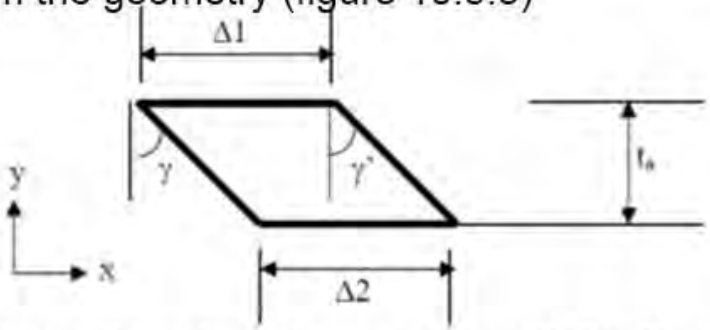
10.5.3. Stresses within adhesive :

Experimental evidence clearly indicates that the stress and strain in adhesive layer are nonlinear in nature. Consider a single lap joint pulled by a force such that the joint does not bend. If the force is too large the joint bends and the adherend gets separated from adhesive by a mechanism known as peeling. However, when bending does not take place, the adhesive deforms by shear (see figure 10.5.2).



10.5.2: Shear deformation of adhesive joint.

Consider a small section of adhesive after deformation. The following relation is at once obvious from the geometry (figure 10.5.3)



10.5.3: Deformation of an element of length Δx. (In the figure:

$$\Delta 1=(1+\varepsilon_{x1})\Delta x,\ \Delta 2=(1+\varepsilon_{x2})\Delta x,\ \gamma' = \gamma + \frac{d\gamma}{dx}\Delta x) \qquad (1+\varepsilon_{x_2})\Delta x + t_a\gamma = (1+\varepsilon_{x_1}) + t_a\left(\gamma + \frac{d\gamma}{dx}\right)4$$

$\sin\gamma = \gamma$ $\qquad \Delta 2 + t_a\gamma = \Delta 1 + t_a\gamma'$ $\quad$ or

$\sin\gamma' = \gamma'$ $\qquad (1+\varepsilon_{x_2}) - t_a\dfrac{d\gamma}{dx} = (1+\varepsilon_{x_1})$ $\quad$ or $\quad \varepsilon_{x_2} - \varepsilon_{x_1} = t_a\dfrac{d\gamma}{dx} = \dfrac{t_a}{G}\dfrac{d\tau}{dx}$

$$\varepsilon_{x_2} - \varepsilon_{x_1} = \frac{t_a}{G}\frac{d\tau}{dx} \qquad G\gamma = \tau,\ \ G = \frac{E}{2(1+\nu)} \qquad (10.5.1)$$

Where ε_{x1} = longitudinal strain of the top fiber, ε_{x2} = longitudinal strain of bottom fiber, τ = shear stress, G = Rigidity Modulus of adhesive = $E_a/2\,(1+\nu)$, t_a = thickness of adhesive

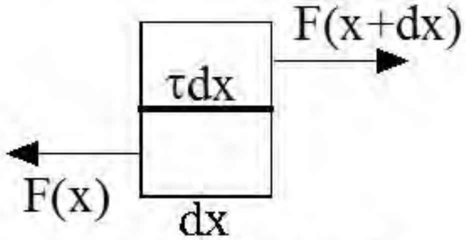
Assuming no slip (perfect bonding) between the adhered and adhesive ε_{xi} "s are then the longitudinal strains of the i-th plate i.e.

$$\varepsilon_{x_2} = \frac{F(x)}{E_2 t_2}, \; \varepsilon_{x_1} = -\frac{F(x)}{E_1 t_1} \tag{10.5.2}$$

Where, A = bt_i , t_i = thickness of the i-th plate, b = width assumed as unity.

In general F is a function of x, distance from the angle of the plate. Considering a small section of upper plate the following relation is obtained from equilibrium condition.

$$\frac{dF}{dx} = \tau' \qquad F(x+dx) - F(x) = \tau' dx \tag{10.5.3}$$



Since τ′=τ (continuity of stress), one gets ultimately

$$F\left(\frac{1}{E_2 t_2} + \frac{1}{E_1 t_1}\right) - \frac{t_a}{G}\frac{d^2F}{dx^2} = 0 \tag{10.5.4}$$

$$\varepsilon_{x_2} - \varepsilon_{x_1} = F\left(\frac{1}{E_2 t_2} + \frac{1}{E_1 t_1}\right) = \frac{t_a}{G}\frac{d^2F}{dx^2} = 0 \qquad \frac{d^2F}{dx^2} = F\frac{G}{t_a}\left(\frac{1}{E_2 t_2} + \frac{1}{E_1 t_1}\right)$$

or $$\frac{d^2F}{dx^2} - k^2F = 0 \qquad A = l^{\lambda x} \tag{10.5.5}$$

where $$k^2 = \frac{E_a}{2t_a(1+\nu)}\left(\frac{1}{E_1t_1} + \frac{1}{E_2t_2}\right) \tag{10.5.6}$$

which has solution τ= *ACoshkx+BSinhkx*. Noting that the shear stress is symmetric about the mid-section, τ=*ACoshkx*, which attains minimum value at x= 0.

Further $$\frac{\tau_{\max}}{\tau_{\min}} = Cosh\left(\frac{k}{2}\right). \tag{10.5.7}$$

If the force F is increased the stresses within adhesive go to plastic region and the joint fails as soon as entire adhesive becomes plastic.

The analysis done above is very crude. The adhesive joint may fail by peeling. The design procedure for this case is very complicated and not yet finalized. In the following a simple design procedure for a very common type of adhesive joint, namely, scarf joint is outlined.

10.5.4. Design of a scarf joint: As explained earlier an adhesive joint fails by shear, though a complicated peeling phenomenon may sometimes appear. The design of a scarf joint is very simple. The joint is based on shear failure theory assuming the shear to have uniform value along the adhesive-adherend interface. The effect of non-uniformity in the stress distribution is taken care by introducing a stress concentration factor. The shear stress experienced within the adhesive is very easily found out for a joint subjected to axial load (see figure 10.5.4a) and bending moment (Figure 10.5.4b) as shown below.

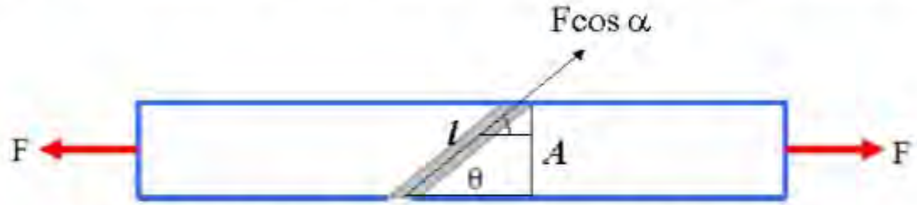
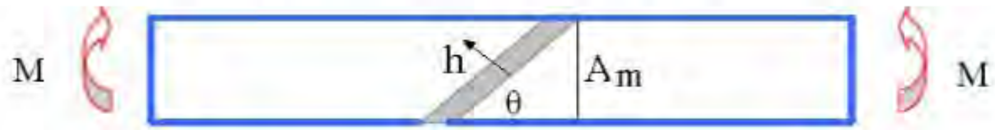


Figure 10.5.4a: A scarf joint with axial load



10.5.4b: A scarf joint with bending moment

A simple analysis shows that the shear stress in the adhesive is

$$\frac{A}{\sin\theta}=l,\ F\cdot\cos=F_{np.} \qquad \tau=\frac{F}{A}\sin\theta\cos\theta$$

where A = area of cross section of the bars, θ = angle of inclination of the adhesive with horizontal.

The joint is safe when $\tau \le \tau_{allow}/K$, where *K* is the stress concentration factor, usually 1.5 – 2. If the joint is subjected to bending moment *M* the maximum shear stress developed within adhesive is given by

$$\tau_{\max}=\sigma_{\max}\sin\theta\cos\theta=\frac{6M}{Ah}\sin\theta\cos\theta \qquad \frac{A}{\sin\theta}h=s$$

where *h* = depth of the adherend bar. Again, for a safe design this shear stress should not exceed a limiting value τ_{allow}/K.

10.5.5. Adhesive materials

In order to increase the joint efficiency the rheological properties of adhesive material should be quite similar to that of the adherends. When the adherends are dissimilar the elastic modulus of the adhesive should be equal to arithmetic average of the elastic moduli of the adherends. Common types of adhesives are epoxies, polyester resins, nitric rubber phenolics. Epoxies are extensively used for mechanical purposes because of their high internal strength in cohesion, low shrinkage stresses, low temperature cure and creep, insensitivity to moisture etc. Often fillers like aluminum oxides, boron fibers are used to improve mechanical strength. Polyester resins are widely used in commercial fields for various structural applications involving plastics operating at moderate temperature.

Lecture
Theme 11
Design of Joints for Special Loading

11.1. Design of Eccentrically Loaded Bolted/Riveted Joints

In many applications, a machine member is subjected to load such that a bending moment is developed in addition to direct normal or shear loading. Such type of loading is commonly known as eccentric loading. In this lesson design methodology will be discussed for three different types of joints subjected to eccentric loading

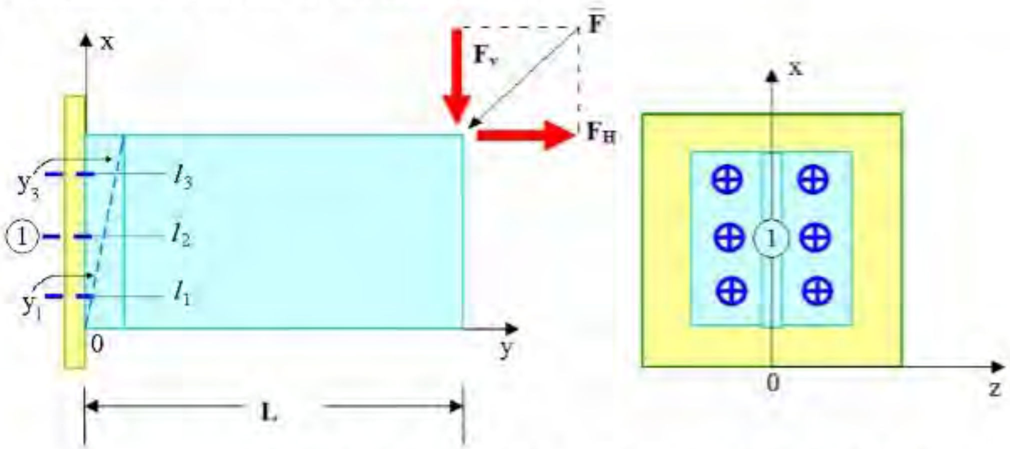
(1) Screw joint

(2) Riveted joint

(3) Welded joint

11.1.1. Eccentrically loaded screwed joint:

Consider a bracket fixed to the wall by means of three rows of screws having two in each row as shown in figure 11.1.1. An eccentric load F is applied to the extreme end of the bracket. The horizontal component, F_h, causes direct tension in the screws but the vertical component, F_v, is responsible for turning the bracket about the lowermost point in left (say point O), which in an indirect way introduces tension in the screws.



11.1.1: Eccentrically loaded bolted joint

It is easy to note that the tension in the screws cannot be obtained by equations of statics alone. Hence, additional equations must be formed to solve for the unknowns for this statically indeterminate problem. Since there is a tendency for the bracket to rotate about point O then, assuming the bracket to be rigid, the following equations are easily obtained.

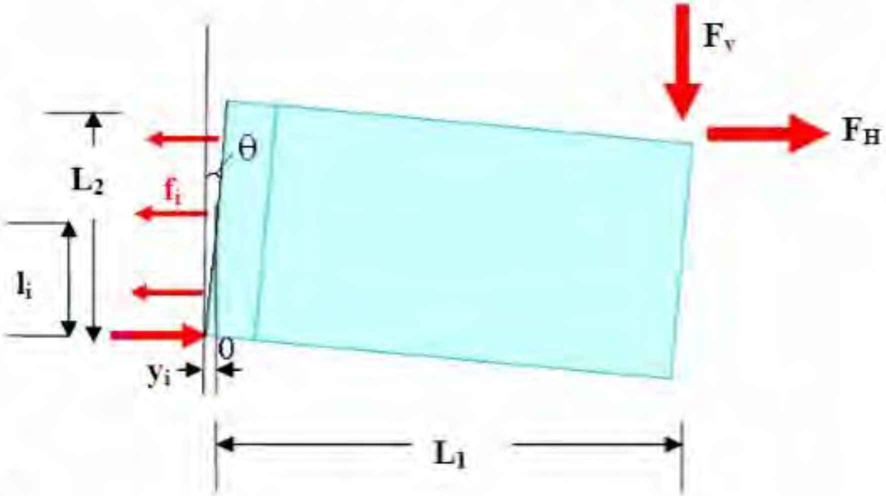
$$\theta \approx \tan\theta = \frac{y_1}{l_1} = \frac{y_2}{l_2} = \frac{y_3}{l_3} \qquad (11.1.1)$$

where y_i =elongation of the i-th bolt, l_i =distance of the axis of the i-th bolt from point O.

If the bolts are made of same material and have same dimension, then we can write. Hookes law

where *fi* =force in the i-th bolt, *k* =stiffness of the bolts. Thus *fi*∞*li* or *fi*=*ali* (*a* =proportionality constant).

$$f_i = ky_i \qquad (11.1.2)$$



11.1.2: Determination of forces in bolt

Using the moment balance equations about O, the lowermost point in the left side, the following equation is obtained.

$$2\sum f_i l_i = F_h L_1 + F_v L_2$$

$$\text{i.e., } \alpha = \frac{F_h L_1 + F_v L_2}{2\sum l_i^2}. \qquad (11.1.3)$$

The factor 2 appears because there are two bolts in a row.
Thus the force in the i-th screw is

$$f_i = \left[\frac{F_h L_1 + F_v L_2}{2\sum l_i^2}\right] l_i + \frac{F_h}{n}, \qquad (11.1.4)$$

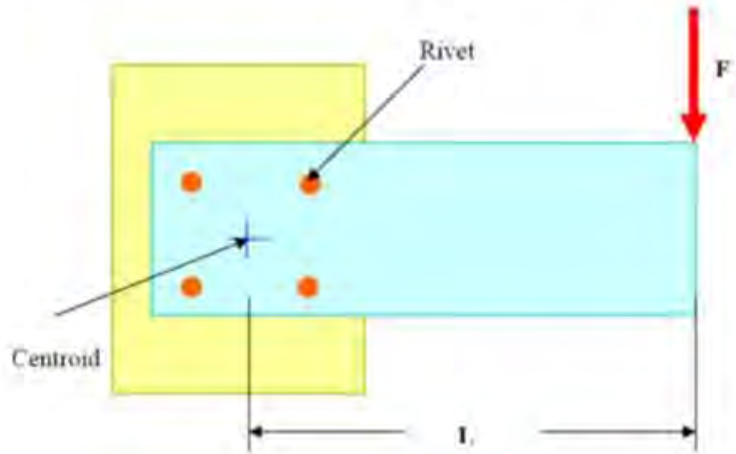
where n = total number of bolts.
For safe design of the joint it is therefore required that

$$\sigma = \max\left\{\frac{f_i}{A}\right\} \leq s_t \qquad (11.1.5)$$

where s_t =allowable tensile stress of the bolt.
Note that F_v causes also direct shear in the bolt. Its effect may be ignored for a preliminary design calculation.

2. Eccentrically loaded riveted joint:

Consider, now, a bracket, which carries a vertical load . The bracket, in this case, is connected to the wall by four rivets as shown in figure 11.1.3. The force,



11.1.3: Eccentrically loaded rivet joint

in addition to inducing direct shear of magnitude $F/4$ in each rivet, causes the whole assembly to rotate. Hence additional shear forces appear in the rivets.

Once again, the problem is a statically indeterminate one and additional assumptions are required. These are as following:

(1) magnitude of additional shear force is proportional to the distance between the rivet center and the centroid of the rivet assembly, whose co-ordinates are defined as

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i}, \ \bar{y} = \frac{\sum A_i y_i}{\sum A_i} \tag{11.1.6}$$

(A_i =area of the cross-section of the i-th rivet)

(2) directions of the force is perpendicular to the line joining centroid of the rivet group and the rivet center and the sense is governed by the rotation of the bracket.

Noting that for identical rivets the centroid is the geometric center of the rectangle, the force in the i-th rivet is

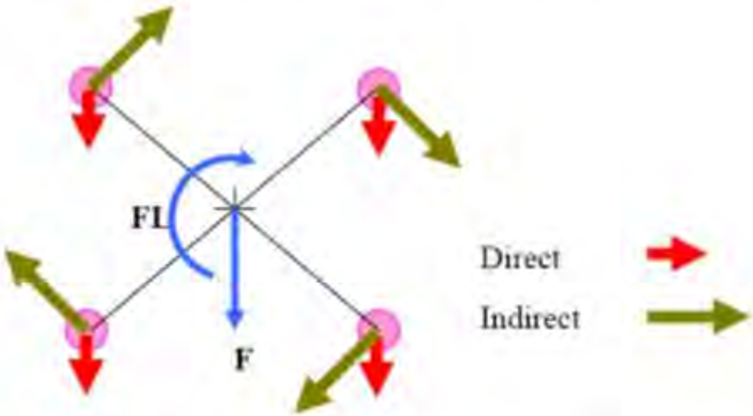
$$f_i = \alpha l_i \tag{11.1.7}$$

where α =proportional constant, l_i =distance of the i-th rivet from centroid.

Taking moment about the centroid

$$\sum_i f_i l_i = FL \qquad \text{or } \alpha = \frac{FL}{\sum_i l_i^2} \tag{11.1.8}$$

Thus, the additional force is given by the expession



$$f_i = \frac{FL}{\sum l_i^2} l_i . \qquad (11.1.9)$$

11.1.4: Forces on rivets due to

The net force in the i-th rivet is obtained by parallelogram law of vector addition as

$$f_i' = \sqrt{f_i^2 + \left(\frac{F}{4}\right)^2 + 2 \cdot \frac{F}{4} \cdot f_i \cos\theta_i} \qquad (11.1.10)$$

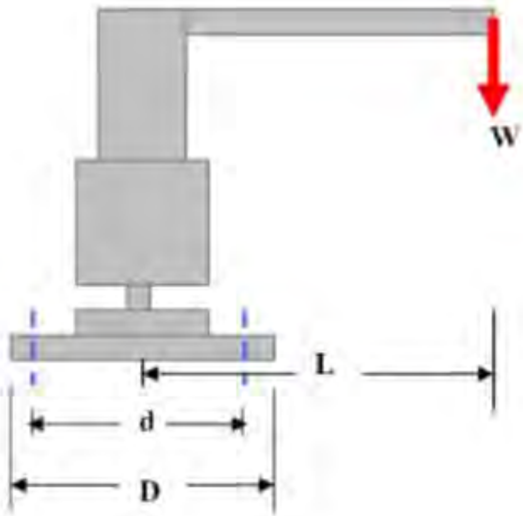
where θ_i =angle between the lines of action of the forces shown in the figure.

For safe designing we must have $\tau = \max\left(\frac{f_i'}{A}\right) \leq s_s$ (11.1.11)

where s_s=allowable shear stress of the rivet.

Problems with Answers:

Q.1. The base of a pillar crane is fastened to the foundation by *n* bolts equally placed on a bolt circle of diameter *d*. The diameter of the pillar is *D*. Determine the maximum load carried by any bolt when the crane carries a load *W* at a distance *L* from the center of the base.



A.1. In this case the pillar have a tendency to topple about the point on the outer diameter lying closest to the point of application of the load.

Choose the line joining the center of the base and the point of application of the load as the reference line. In this case y_i=distance of the i-th bolt from the tilting point $=\left(\frac{D}{2}\right)-\left(\frac{d}{2}\right)\cos\theta_i$

where θ_i=angular position of the i-th bolt.

Since there are *n* equally spaced bolts so

$$\theta_{i+1} - \theta_i = \frac{2\pi}{n}$$

Using the same considerations as done in section-1, the force in the i-th bolt is

$$f_i = \frac{W(L - D/2)}{\Sigma y_i^2}\left(\frac{D}{2} - \frac{d}{2}\cos\theta_i\right)$$

It is easy to see that

$$\Sigma y_i^2 = \frac{n}{2}\left(2\left(\frac{D}{2}\right)^2 + \left(\frac{d}{2}\right)^2\right).$$

Hence the maximum load occurs when $\theta_i = \pm\pi$ whereby

$$f_{\max} = \frac{W\left(L - \frac{D}{2}\right)\left(\frac{D}{2} + \frac{d}{2}\right)}{\frac{n}{2}\left(2\left(\frac{D}{2}\right)^2 + \left(\frac{d}{2}\right)^2\right)}.$$

Q.2. A bracket is supported by means of 4 rivets of same size as shown in figure 6. Determine the diameter of the rivet if the maximum shear stress is 140 MPa.

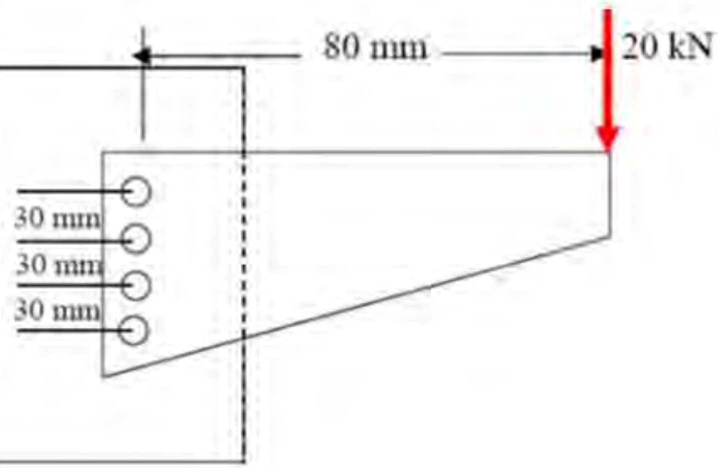
A.2. F_1 = The direct shear force =5 kN per rivet. The maximum indirect shear force occurs in the topmost or bottommost rivet and its magnitude is

$$F_2 = \frac{20 \times 80}{2 \times 15^2 + 2 \times 45^2} \times 45 \text{ kN}$$

and the direction is horizontal.

Therefore the maximum shear force on the rivet assembly is

$$F = \sqrt{F_1^2 + F_2^2}.$$



Hence

$\frac{\pi}{4} d^2 \times s_s = F$ which yields $d \approx 16$ mm.

Lecture

Theme 11

Design of Joints for Special Loading

11.2. Design of Eccentrically Loaded Welded Joints

There are many possible ways in which an eccentric loading can be imposed on a welded joint. A few cases are discussed below.

11.2.1. Eccentrically loaded transverse fillet joint:

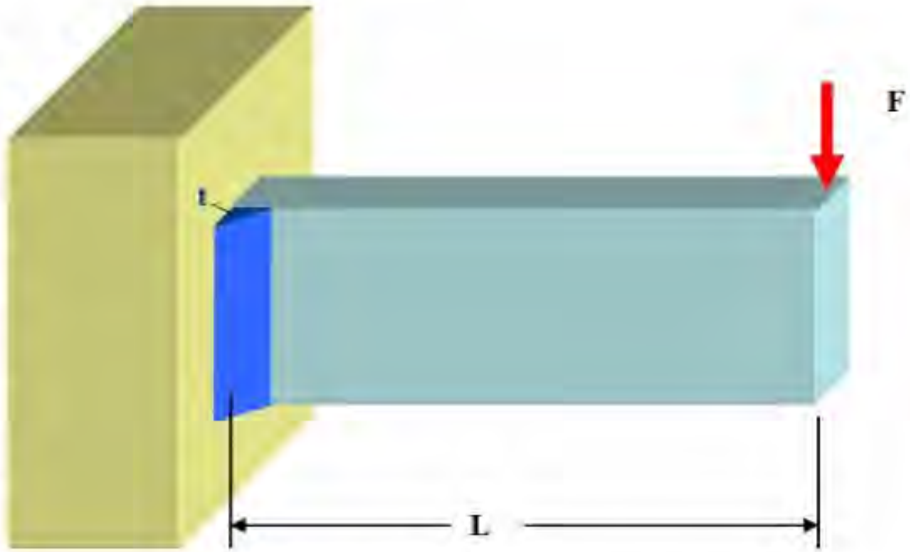
Consider a cantilever beam fixed to a wall by two transverse fillet joints as shown in figure 11.2.1. The beam is subjected to a transverse load of magnitude **F**.

Like any welded joint, the design is based upon the strength of the joint against failure due to shear force along the throat section. In this case any small section of the throat is subjected to

(a) direct shear stress of magnitude $F/2bt$

where b = length of the weld, t = thickness at the throat

and the factor 2 appears in the denominator for double weld.



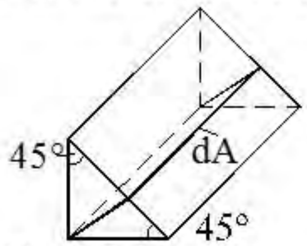
11.2.1. *A cantilever beam*

(b) Indirect shear stress due to bending of the beam, whose magnitude is calculated in the following manner and whose direction is perpendicular to that of the direct shear stress. Consider a small area **dA** in throat section lying at a distance *y* from the centerline, which is also the centroidal axis of the weld. An important assumption is made regarding the magnitude of the shear stress at a point within the area **dA**. It is assumed that the shear stress is proportional to the distance from the centroidal axis, that is *y* in this case, and directed along the horizontal. The proportionality constant is calculated using the moment equilibrium equation about centroid of the throat section. This gives,

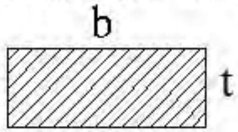
$$\tau(y)dy = dF, \qquad \oint \tau(y) y \, dA = FL \qquad \text{where } \tau(y) = cy. \tag{11.2.1}$$

Hence, $c = \dfrac{FL}{\oint y^2 \, dA}$. Therefore the magnitude of the shear stress is

$$\tau = \frac{FLy}{I_y} \qquad \tau = \frac{My}{I_y} \tag{11.2.2}$$



where the second moment of area of the throat section

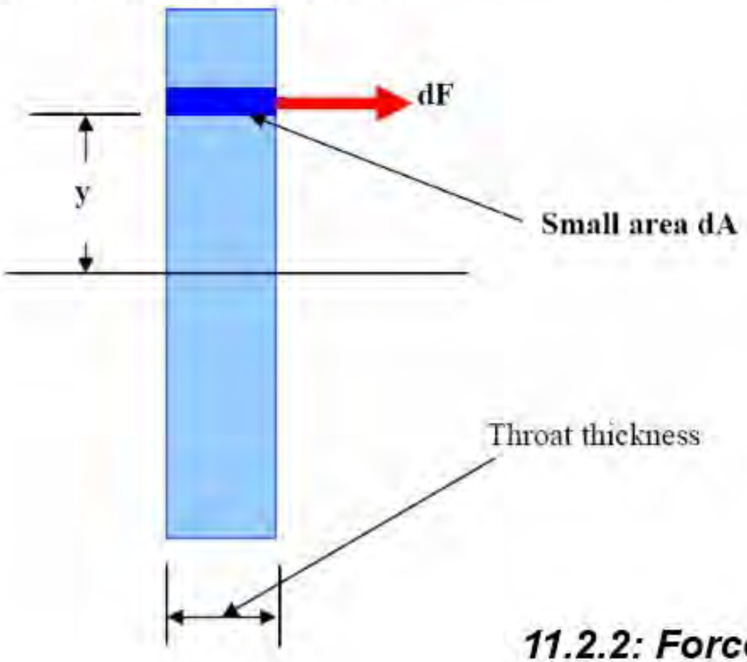


$$I_p = \oint y^2 \, dA = \frac{tb^3}{12}. \tag{11.2.3}$$

For an eccentrically loaded joint shown in figure 11.2.2 the maximum shear stress occurs at the extreme end and its magnitude is

$$\tau_{max} = \sqrt{(\tau_V)^2 + (\tau_H)^2} = \sqrt{\left(\frac{F}{2bt}\right)^2 + \left(\frac{3FL}{tb^2}\right)^2}.$$

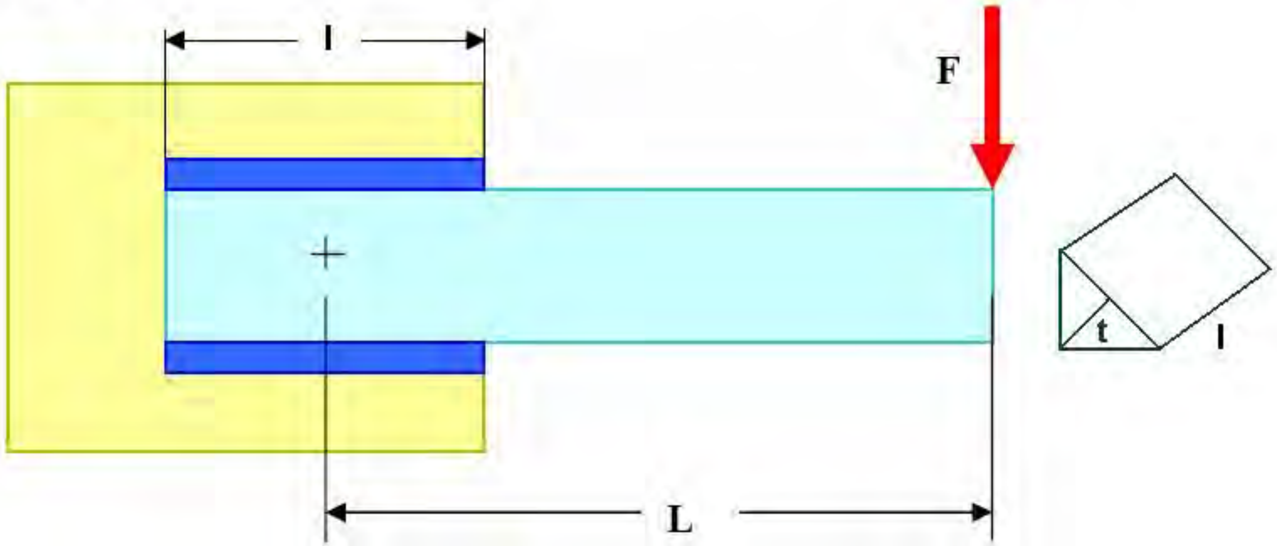
In order to design a safe welded joint $\tau_{max} \le S_S$, where S_s is the allowable shear stress of the weld material.



11.2.2: Forces on weld in bending

11.2.2. Eccentrically loaded parallel fillet joint:

Consider a cantilever beam connected to a wall by means of two parallel joints as shown in figure 11.2.3. The beam is required to carry a load **F** in transverse direction.



11.2.3: Eccentrically loaded parallel fillet joint

In order to select the size of the weld it is once again considered that the joint fails in shear along the throat section. For the given loading, the throat area is subjected to two shear stresses.

(a) Direct shear of magnitude *F / 2lt*

where *l* = length of the weld, *t* = thickness of the throat.

(b) Indirect shear stress owing to eccentricity of the loading. The magnitude and direction of the shear stresses are calculated using the similar assumption as in the last section. The magnitude of shear stress at any point is assumed to be proportional to its distance from the centroid of the throat area and the direction is perpendicular to the line joining the point and the centroid. The sense is the same as that of the rotation of the welded jont as a whole (if permitted). With this assumption the shear stress at a point at a distance *r* from the centroid is given by

$$\tau(r) = cr \qquad (11.2.4)$$

where the proportionality constant *c* is to calculated using the moment equilibrium equation. Taking moment about the centroid one finds

$$\oint \tau(r) r \, dA = FL \, , \qquad (11.2.5)$$

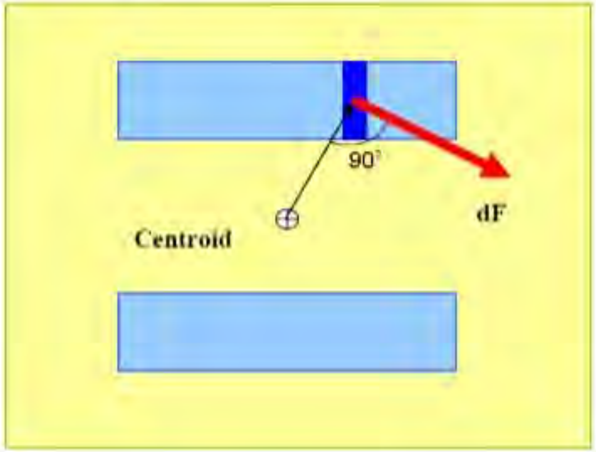
where *L* = distance of the line of action of **F** from centroid.

Thus,

$$c = \frac{FL}{J} \, , \qquad (11.2.6)$$

where $J = \oint r^2$ is the polar moment of the throat section about its centroid.

The net shear stress at a point is calculated by vector addition of the two kinds of shear stresses discussed above. (Note that the vector addition of stresses is in general not defined. In this case the resultant force at a point within an infinitesimal area is obtained using vector addition of forces calculated from the individual stress values. The resultant stress is the force divided by area. Since everywhere the same value of area is involved in calculation, the net stress is therefore the vector sum of the component stresses.) The weld size is designed such that the maximum shear stress does not exceed its allowable limiting value.



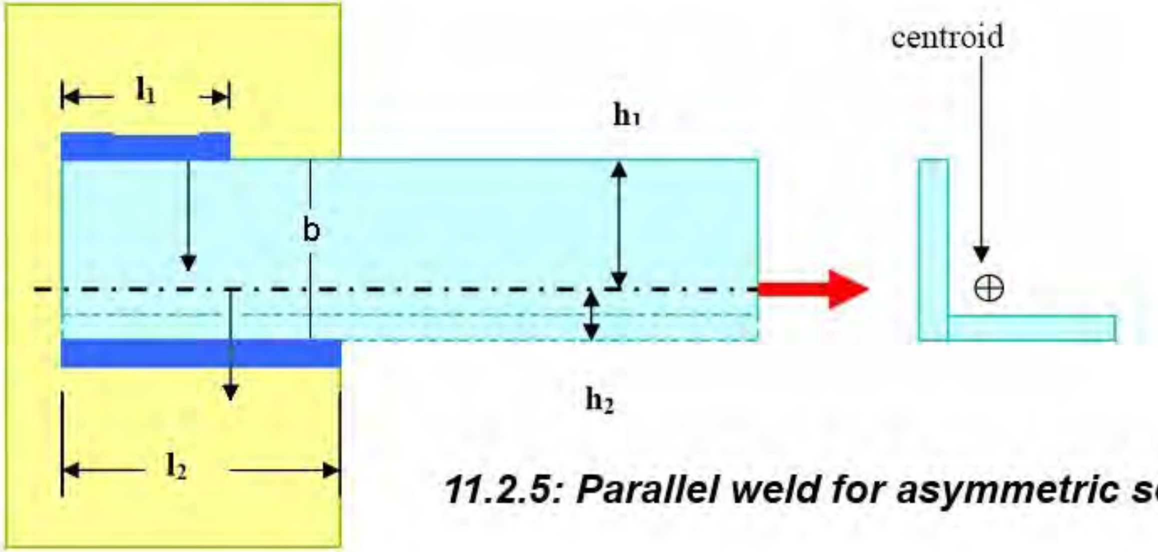
11..2.4: Forces on throat section due to torsion

11.2.3. Asymmetric Welded Section:

It is observed from section 1 and 2 that an eccentricity in loading causes extra shear stress in a welded joint. Thus it may be useful to reduce the eccentricity in loading. In some applications this is achieved by making the weld section asymmetric. Consider a plate subjected to an axial load **F** as shown in figure 11.2.5. The plate is connected to the wall by means of parallel fillet joint. Assume that the axial load is along the centroidal axis if the beam which is shown by dotted lines. If the welds are made of equal lengths in both sides, then the centroid of the welded section, being along the centerline of the beam will not lie on the cetroidal axis of the beam. Thus an eccentricity in loading is introduced. This situation may be avoided by making the two weld lengths unequal in such proportion that the eccentricity is removed. The relationship between and will be as following:

$$\frac{l_1}{l_2} = \frac{h_2}{h_1}, \qquad \begin{aligned} S_1 &= S_2 \\ l_1 h_1 &= l_2 h_2 \end{aligned} \qquad (11.2.7)$$

where l_1= length of the upper weld, l_2 = length of the lower weld, h_1 = distance of the upper weld from centroidal axis, h_2 = distance of the lower weld from centroidal axis.



11.2.5: Parallel weld for asymmetric section

The net length of the weld $l=l_1+l_2$ can be calculated from the strength consideration that is

$$\frac{F}{lt} \le S_S, \qquad \frac{F}{tS_S} = (l_1 + l_2),$$

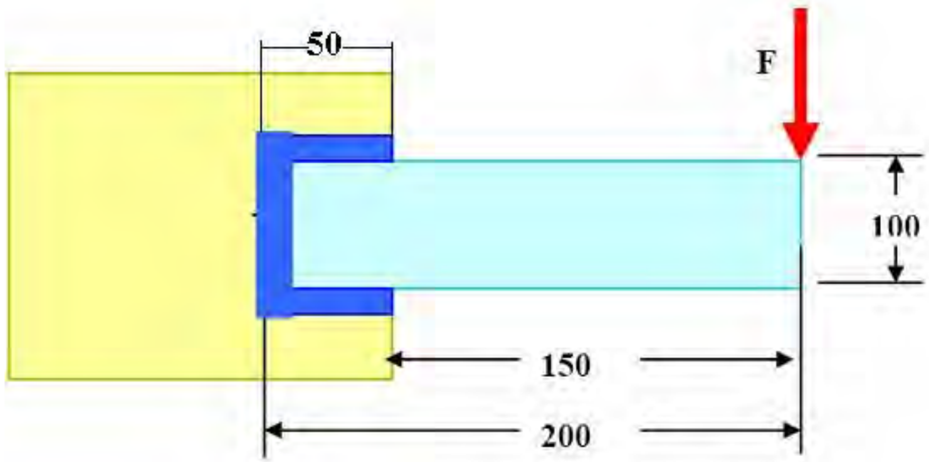
where t = thickness of the throat. Thus the individual lengths of the weld are as following:

$$l_1 = \left(\frac{h_2}{b}\right) l \qquad \text{and} \qquad l_2 = \left(\frac{h_1}{b}\right) l, \qquad h_1 + h_2 = b$$

where b= width of the plate.

Problems with Answers:

Q.1. A rectangular steel plate is welded as a cantilever to a vertical column and supports a single concentrated load of 60 kN as shown in figure below. Determine the weld size if the allowable shear stress in the weld material is 140 MPa.



A.1. The weld is subjected to two shear stresses

(1) Direct shear of magnitude 60,000/Area of the weld. The area of the throat section is easily found out to be 200 *t* where *t=0.707 h*. Thus direct shear stress is 424/*h* MPa. $2\times 50+100=200t=200h\cdot\sin 45^\circ$.

(2) The indirect shear stress as a point *r* distance away from the centroid of the throat section has magnitude

$$\tau=\frac{FLr}{J},$$

where *J* is the polar moment of area of the throat section and L is the eccentricity of the load. From the geometry of the throat section it may be calculated that the distance of centroid from left end = $x=\dfrac{l^2}{2l+b}=12.5$ mm

equation of moment equilisium or balance of moment

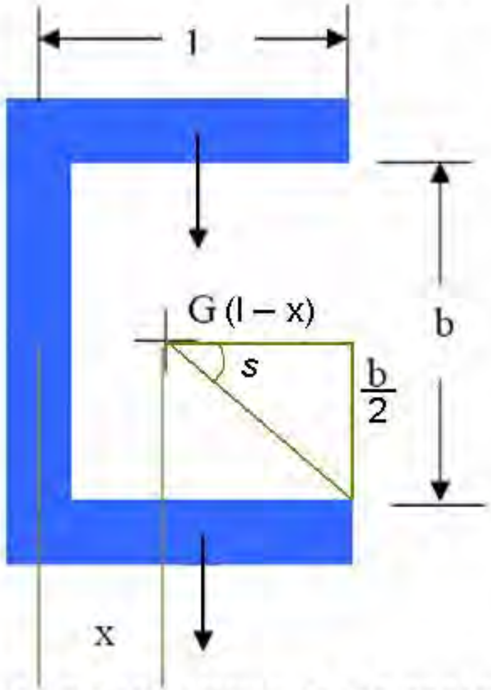
$$xb=(l-2x)l$$

$$x(b+2l)=l^2$$

(see figure below) and the polar moment about G is

$$J=\frac{h}{\sqrt{2}}\left[\frac{(b+2l)^3}{12}-\frac{l^2(b+l)^2}{b+2l}\right]$$ = 272530 *h* mm^4.

Thus the indirect shear stress has magnitude $\tau = \frac{41.28}{h} r$ MPa.



The maximum resultant shear stress depends on both the magnitude and direction of the indirect shear stress. It should be clear that the maximum shear stress appears at the extreme corner of the weld section which is at a distance $\sqrt{(\frac{b}{2})^2 + (l-x)^2}$ = 62.5 mm away from the centroid. Noticing that the included angle between the two shear forces as $\cos^{-1}\left(\frac{l-x}{r_{max}}\right) \approx 53.13^0$, the maximum value of the resultant shear stress is found out to be $\tau_{max} = \frac{2854.62}{h}$ MPa. Since this value should not exceed 140 MPa the minimum weld size must be *h*= 20.39 mm.

Lecture

Theme 11

Design of Joints for Special Loading

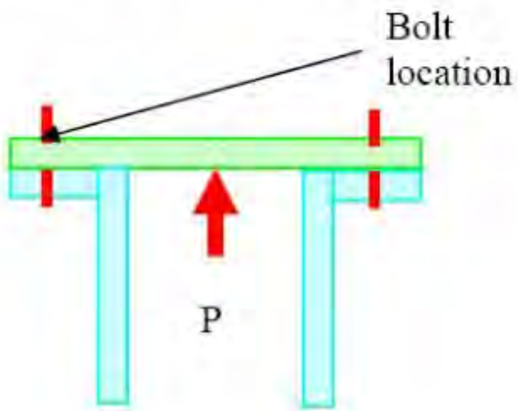
11.3. Design of Joints with Variable Loading

11.3.1. Variable loading in mechanical joints:

Machine parts are often subjected to variable loading. In many cases pulsating or intermittent loads are applied from outside, for example, in punching press forces of very large magnitude is applied for a short while (impulsive force), in crank shafts variable loads act due to nature of force arising from combustion cycle in cylinders. Often dynamic forces appear in the moving parts, e.g., inertia forces in machines and mechanisms, forces due to unbalance of the rotating components etc. Since these forces are to be withstood by the joints, care should be taken while designing a joint capable of resisting adequate load of variable magnitude. Design of two important mechanical joints is discussed below, namely, bolted and welded joints.

11.3.2. Bolted joints with variable loading:

Consider design of bolts to fasten a flat cover to a cylinder as shown in figure 11.3.1. In order to ensure leak proofness necessary pretension (usually 2840 d , in Newton while the nominal bolt diameter d is measured in millimeter) is applied. Depending upon operating condition the pressure inside the closed cylinder is likely to vary in somewhat periodic manner. Let the minimum and maximum value of the pressure be p_{min} and p_{max}, respectively.



11.3.1: Bolted cover plate

The pressure causes external force of magnitude

$$F = \frac{pA_c}{n}, \tag{11.3.1}$$

where n= number of equally spaced bolts on the bolt circle, n = area of cross section of the cylinder A_c, p = fluid pressure inside the cylinder.

It is known that only a fraction of external load is responsible for tensile stress within bolts, that is

$$F_b = F_i + CF \tag{11.3.2}$$

where F_i = initial tension in the bolt, C = factor that depends on the nature of joints. Some representative values of C's are tabulated in Table 1 below.

Table 11.3.1. Values of *C* for various types of joints

Type of joint	Value of *C*
Metal to metal joint with through bolt	0.00 – 0.10
Soft copper gasket with long bolts	0.5 – 0.7
Hard copper gasket with long bolt	0.25 – 0.5
Soft packing with through bolt	0.75 – 1.00
Soft packing with stud	1.0

Due to fluctuating external force the tensile load within each bolt takes minimum and maximum value of

$$F_{b,\min} = F_i + CF_{\min} \quad \text{and} \quad F_{b,\max} = F_i + CF_{\max} \tag{11.3.3}$$

respectively. The average and the fluctuating component of the normal stress are given by

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{F_i}{A_b} + C\frac{F_{\max} + F_{\min}}{2A_b} \qquad \sigma_{amp} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = C\frac{F_{\max} - F_{\min}}{2A_b} \tag{11.3.4}$$

respectively, where A_b is the root area of each bolt. The advantage of initial pretension is at once visible from the above expressions. The ratio σ_{amp}/σ_m gets drastically reduced, The safe size of the bolt can be calculated now from well-known Soderberg equation given below

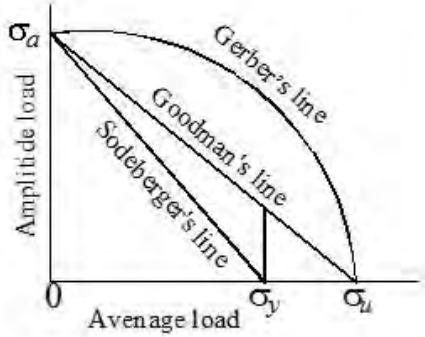
$$\frac{\sigma_{av}}{\sigma_Y} + \frac{k_f \sigma_{amp}}{S_E} = \frac{1}{N} \tag{11.3.5}$$

where σ_Y= Yield stress of the bolt material, S_E = Corrected endurance limit taking load-, size-, surface finish- factors,N = Factor of safety, k_f= fatigue stress concentration factor.

Alternatively, Goodman's equation or Gerber's line may be used to calculate the root area and hence the size of the bolts. The fatigue stress concentration factor plays an important role in the design. These are found by doing extensive experimentation. A few figures are shown in Table 2.

Table 11.3.2: Fatigue Stress Concentration Factor

Metric Grade	Fatigue stress Conc. factor
3.6 - 5.8	2.1 – 2.8
6.6 -10.9	2.3 – 3.8



11.3.3. Welded joints with variable loading:

Because of many intricacies involved in design of a welded joint, codes are extensively used to design such joint when it experiences variable loading. The value of the maximum fluctuating load is not allowed to exceed a limit specified in the code. This value depends on

a. type of the joint

b. type of stress experienced by the joint

c. a load factor K defined as the ratio of the minimum stress to the maximum stress. When the load is a steady one the factor takes unit value. For a complete reversal of stress the value of K = -1.

The design stress for completely reversing load is calculated using the formula

$$\sigma_{-1,d} = \frac{\sigma_{-1,a}}{k_{-1}} \qquad (11.3.6)$$

where $\sigma_{-1,d}$ = design stress for complete reversal of stress, $\sigma_{-1,a}$= allowable fatigue stress, k_{-1} = fatigue stress concentration factor tabulated below

Table 11.3.3: Fatigue stress concentration factor (k_{-1})

Type of weld	k_{-1}
Reinforced butt weld	**1.2**
T- butt joint with sharp corner	**2.0**
Toe of transverse fillet or normal fillet	**1.5**
Parallel fillet weld or longitudinal weld	**2.7**

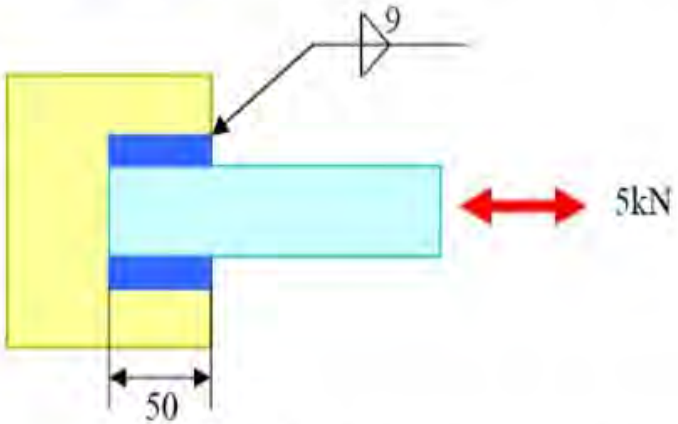
The values of the allowable fatigue stress ($\sigma_{-1,a}$) are also tabulated in the design code for various weld geometries. For example, the allowable fatigue stress for fillet weld is given (assuming the weld to be a line) as

$$\sigma_{-1,a} = \frac{358w}{1-K/2}, \text{ (in kgf/cm)} \qquad (11.3.7)$$

where *w* denotes the leg size of the fillet weld measured in centimeter. The design is found to be safe if the maximum value of the fluctuating stress is found to be lesser than the design stress.

Problems with Answers:

Q.1. A strap of mild steel is welded to a plate as shown in the following figure. Check whether the weld size is safe or not when the joint is subjected to completely reversed load of 5 kN.



A.1. As shown in the figure the joint is a parallel fillet joint with leg size as 9 mm and the welding is done on both sides of the strap. Hence the total weld length is 2(50) = 100 mm.

k_{-1} = 2.7 (parallel fillet joint, refer table 3)

w = 0.9 cm

K = -1 for completely reversed loading

The value of the allowable fatigue stress (assuming the weld to be a line) is then $\sigma_{-1} = \frac{358 \times 0.9}{1.5}$ = 214.8 kgf/cm = 214800 N/m (approx). The design stress is therefore $\sigma_{-1,d} = \frac{214800}{2.7}$ = 79556 N/m. Since the total length of the weld is 0.1 m, the maximum fluctuating load allowable for the joint is 7955.6 N. The joint is therefore safe.

Lecture

Theme 12

Design of brakes

12.1. Design of shoe brakes

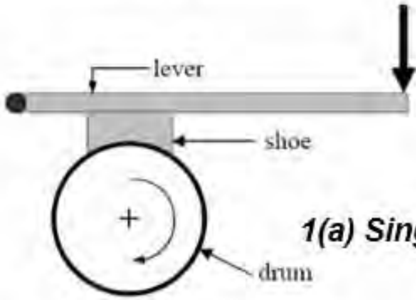
12.1.1. Types of brakes

Brakes are devices that dissipate kinetic energy of the moving parts of a machine. In mechanical brakes the dissipation is achieved through sliding friction between a stationary object and a rotating part. Depending upon the direction of application of braking force, the mechanical brakes are primarily of three types

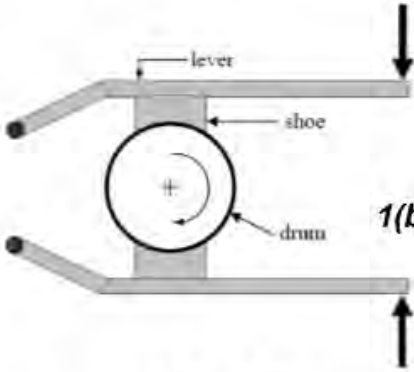
1. Shoe or block brakes – braking force applied radially
2. Band brakes – braking force applied tangentially.
3. Disc brake – braking force applied axially.

12.1.2. Shoe or block brake

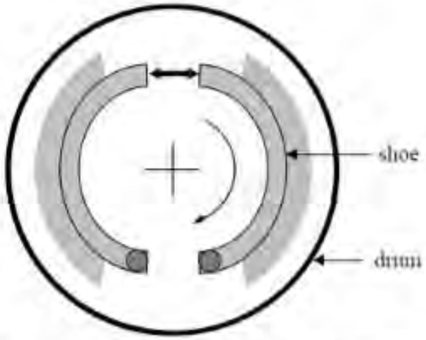
In a shoe brake the rotating drum is brought in contact with the shoe by suitable force. The contacting surface of the shoe is coated with friction material. Different types of shoe brakes are used, viz., single shoe brake, double shoe brake, internal expanding brake, external expanding brake. These are sketched in figure 12.1.1.



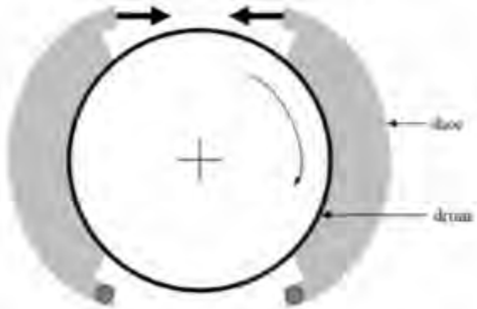
1(a) Single shoe brake



1(b) Double shoe brake



1(c): Internal expanding shoe brake

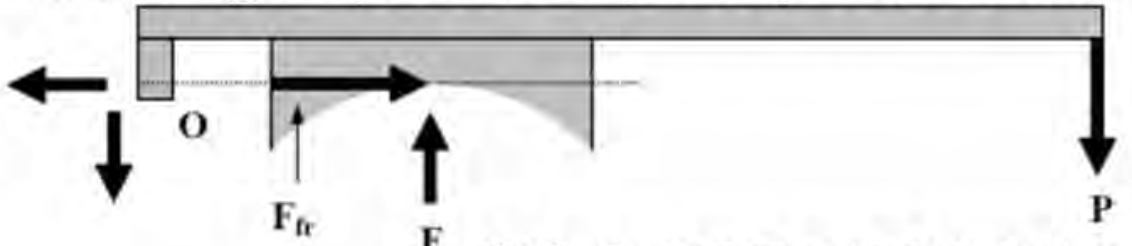


1(d): External expanding shoe brake

12.1.1: Different shoe brakes

12.1.2.1. Single Shoe brake

The force needed to secure contact is supplied by a lever. When a force F is applied to the shoe (see figure 12.1.2a) frictional force proportional to the applied force $F_{fr}=\mu' F$ develops, where μ' depends of friction material and the geometry of the shoe. A simplified analysis is done as discussed below.



12.1.2a: Free body diagram of a brake shoe

Though the exact nature of the contact pressure distribution is unknown, an approximation (based on wear considerations) is made as Where the angle is measured from the centerline of the shoe. If Coulomb's law of friction is assumed to hold good, then

$$p(\theta) = p_0 \cos\theta \qquad f_{fr}(\theta) = \mu p_0 \cos\theta \qquad (12.1.1)$$

Since the net normal force of the drum is F, one has

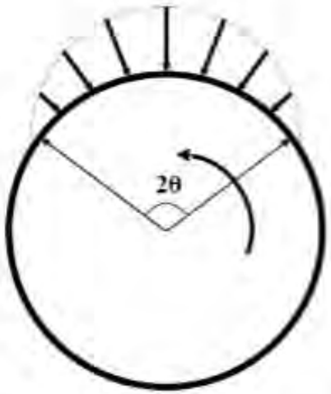
$$Rb \int_{-\theta_0}^{\theta_0} p(\theta)\cos\theta \, d\theta = F, \qquad (12.1.2)$$

Where R and b are the radius of the brake drum and width of the shoe respectively.

The total frictional torque is $$T = b\int_{-\theta_0}^{\theta_0} f_{fr}(\theta)R^2 d\theta \qquad (12.1.3)$$

If the total frictional force is assumed to be a concentrated one, then the equivalent force becomes $F_{fr} = \frac{T}{R}$. A simple calculation yields,

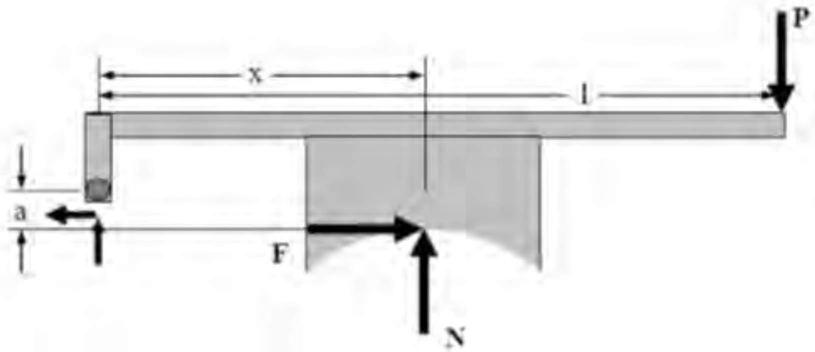
$$\mu' = \frac{4\mu \sin\theta_0}{2\theta_0 + \sin 2\theta_0} \qquad (12.1.4)$$



12.1.2(b): Pressure distribution on brake

It may be seen that for very small value of $\theta_0, \mu = \mu'$. Even when $\theta_0 = 30^0$, $\mu' = 1.0453\mu$. Usually if the contact angle is below 60^0, the two values of friction coefficient are taken to be equal.

Consider, now single shoe brakes as shown in figures 12.1.3(a) and 3(b). Suppose a force P is applied at the end of a lever arm with length l. The shoe placed at a distance x from the hinge experiences a normal force N and a friction force F, whose direction depends upon the sense of rotation of the drum. Drawing free body diagram of the lever and taking moment about the hinge one gets



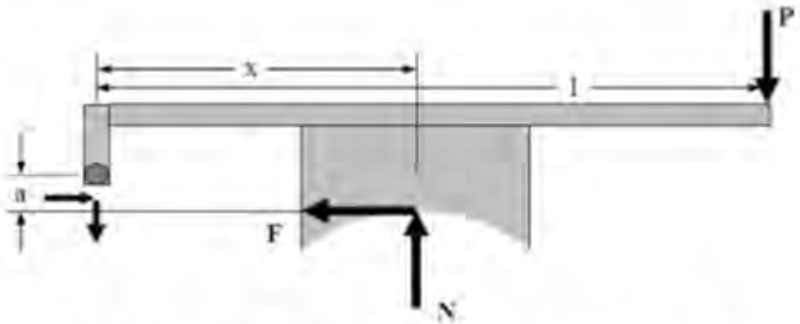
12.1.3(a): FBD of shoe (CW drum rotation)

(a) for clockwise rotation of the brake wheel,

$$Nx + Fa = Pl \qquad (12.1.5)$$

(b) for anticlockwise rotation of the brake wheel,

$$Nx - Fa = Pl. \qquad (12.1.6)$$



12.1.3(b): FBD of shoe (CCW drum rotation)

Where a is the distance between the hinge and the line of action of F and is measured positive when F acts below point O as shown in the figure.

Using Coulomb's law of friction the following results are obtained,

(a) for clockwise rotation $$F = \frac{\mu P l}{x + \mu a}, \tag{12.1.7}$$

(b) for anticlockwise rotation $$F = \frac{\mu P l}{x - \mu a}, \tag{12.1.8}$$

It may be noted that for anticlockwise rotating brake, if $\mu > \frac{x}{a}$, then the force *P* has negative value implying that a force is to applied in the opposite direction to bring the lever to equilibrium. Without any force the shoe will, in this case, draw the lever closer to the drum by itself. This kind of brake is known as 'self-locking, brake. Two points deserve attention.

(1) If a $<$ 0, the drum brake with clockwise rotation becomes self-energizing and if friction is large, may be self locking.

(2) If the brake is self locking for one direction, it is never self locking for the opposite direction. This makes the self locking brakes useful for 'back stop's of the rotors.

12.1.2.2. Double shoe brake

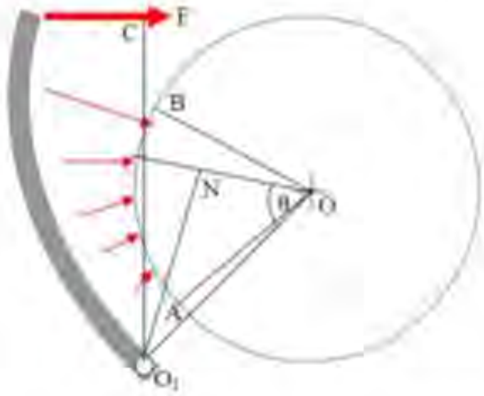
Since in a single shoe brake normal force introduces transverse loading on the shaft on which the brake drum is mounted two shoes are often used to provide braking torque. The opposite forces on two shoes minimize the transverse loading. The analysis of the double shoe brake is very similar to the single shoe brake.

12.1.2.3. External expanding shoe brake

An external expanding shoe brake consists of two symmetrically placed shoes having inner surfaces coated with frictional lining. Each shoe can rotate about respective fulcrum (say, O_1 and O_2). A schematic diagram with only one shoe is presented (figure 12.1.4) When the shoes are engaged, non-uniform pressure develops between the friction lining and the drum.

The pressure is assumed to be proportional to wear which is in turn proportional to the perpendicular distance from pivoting point (O_1N in figure 12.1.4). A simple geometrical consideration reveals that this distance is proportional to sine of the angle between the line joining the pivot and the center of the drum and the line joining the center and the chosen point. This means

$$p(\theta) = p_0 \sin\theta, \qquad (12.1.9)$$



12.1.4: Force distribution in externally expanding brake.

where the angle is measured from line OO_1 and is limited as $\theta_1 \le \theta \le \theta_2$.

Drawing the free body diagram of one of the shoes (left shoe, for example) and writing the moment equilibrium equation about O_1 (say) the following equation is resulted for clockwise rotation of the drum :

$$F_1 l = M_p - M_f. \tag{12.1.10}$$

Where F_1 is the force applied at the end of the shoe, and

$$M_p = \frac{1}{2} p_0 bR\delta \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2) \right]. \tag{12.1.11}$$

$$M_f = \frac{1}{2} \mu_0 bR\delta \left[R(\cos\theta_1 - \theta_2) - \frac{\delta}{4}(\cos 2\theta_1 - \cos 2\theta_2) \right]. \tag{12.1.12}$$

where δ is the distance between the center and the pivot (OO_1 in figure 12.1.4) and ℓ is the distance from the pivot to the line of action of the force F_1 (O_1C in the figure). In a similar manner the force to be applied at the other shoe can be obtained from the equation

$$F_2 l = M_p + M_f. \tag{12.1.13}$$

The net braking torque in this case is

$$T = \mu p_0 bR^2(\cos\theta_1 - \cos\theta_2). \tag{12.1.14}$$

12.1.2.4. Internal expanding shoe brake

Here the brake shoes are engaged with the internal surface of the drum. The analysis runs in the similar fashion as that of an external shoe brake. The forces required are

$$F_1 = (M_p + M_f)/l \qquad (12.1.15)$$

and

$$F_2 = (M_p - M_f)/l, \qquad (12.1.16)$$

respectively.

One of the important member of the expanding shoe brakes is the anchor pin. The size of the pin is to be properly selected depending upon the face acting on it during brake engagement.

Lecture

Theme 12

Design of brakes

12.2. Design of Band and Disc Brakes

12.2.1. Band brakes:

The operating principle of this type of brake is the following. A flexible band of leather or rope or steel with friction lining is wound round a drum. Frictional torque is generated when tension is applied to the band. It is known (see any text book on engineering mechanics) that the tensions in the two ends of the band are unequal because of friction and bear the following relationship:

$$\frac{T_1}{T_2} = e^{\mu\beta}, \tag{12.2.1}$$

where T_1 = tension in the taut side,

T_2 = tension in the slack side,

μ = coefficient of kinetic friction and

β = angle of wrap.

If the band is wound around a drum of radius R, then the braking torque is

$$T_{br} = (T_1 - T_2)R = T_1\left(1 - e^{-\mu\beta}\right)R \tag{12.2.2}$$

Depending upon the connection of the band to the lever arm, the member responsible for application of the tensions, the band brakes are of two types,

12.2.1.1. Simple band brake:

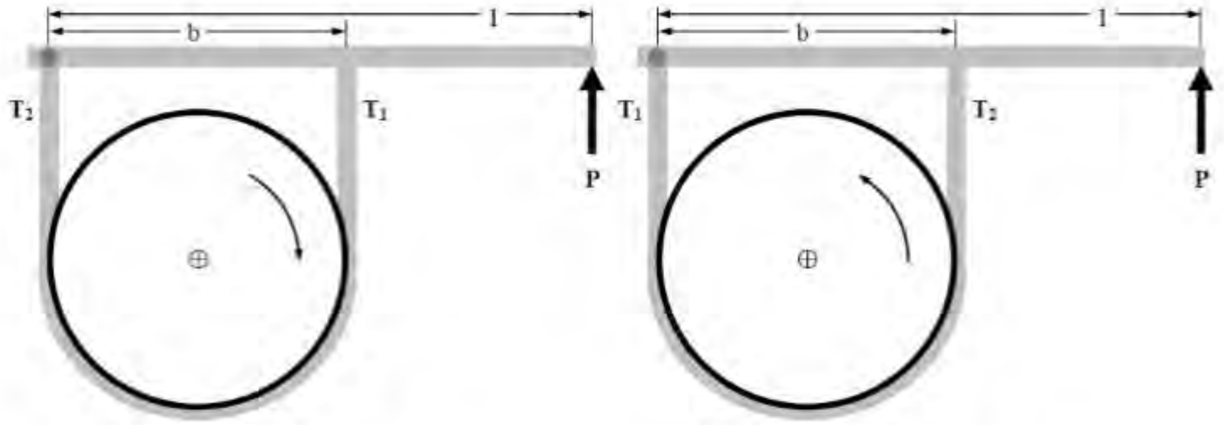
In simple band brake one end of the band is attached to the fulcrum of the lever arm (see figures 12.2.1(a) and 1(b)). The required force to be applied to the lever is

$$P = T_1 \frac{b}{l}$$ for clockwise rotation of the brake drum and

$$P = T_2 \frac{b}{l}$$ for anticlockwise rotation of the brake drum,

where *l* = length of the lever arm and

b = perpendicular distance from the fulcrum to the point of attachment of other end of the band.



1(a): Band brake with CW rotating drum 1(b): Band brake with CCW rotating drum

12.2.1: Band brakes

12.2.1. Differential band brake:

In this type of band brake, two ends of the band are attached to two points on the lever arm other than fulcrum (see figures 12.2.2(a) and 12.2.2(b)). Drawing the free body diagram of the lever arm and taking moment about the fulcrum it is found that

$P = T_2 \frac{a}{l} - T_1 \frac{b}{l}$, for clockwise rotation of the brake drum and

$P = T_1 \frac{a}{l} - T_2 \frac{b}{l}$, for anticlockwise rotation of the brake drum.

Hence, P is negative if

$$e^{\mu\beta} = \frac{T_1}{T_2} > \frac{a}{b} \text{ for clockwise rotation of the brake drum}$$

and $$e^{\mu\beta} = \frac{T_1}{T_2} < \frac{a}{b} \text{ for counterclockwise rotation of the brake drum.}$$ In these cases the force is to be applied on the lever arm in opposite direction to maintain equilibrium. The brakes are then self locking.

The important design variables of a band brake are the thickness and width of the band. Since the band is likely to fail in tension, the following relationship is to be satisfied for safe operation.

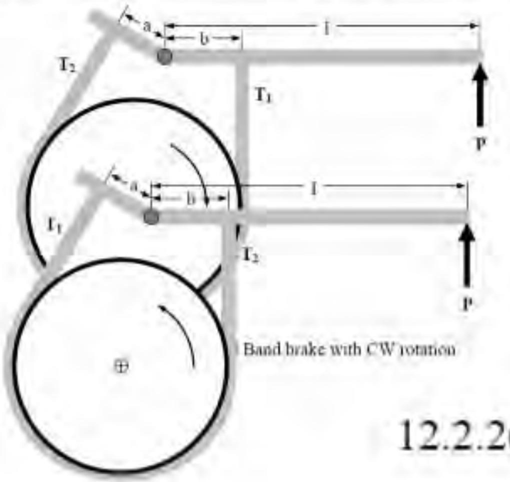
$$T_1 = wts_T \tag{12.2.3}$$

where w = width of the band,

t = thickness of the band and

s_T = allowable tensile stress of the band material. The steel bands of the following dimensions are normally used

w	25-40 mm	40-60 mm	80 mm	100 mm	140-200 mm
t	3 mm	3-4 mm	4-6 mm	4-7 mm	6-10 mm



12.2.2(b): Differential Band brake with CCW rotation

12.2.2. Band and block brakes:

Sometimes instead of applying continuous friction lining along the band, blocks of wood or other frictional materials are inserted between the band and the drum. In this case the tensions within the band at both sides of a block bear the relation

$$\frac{T_1}{T_1'} = \frac{1+\mu\tan\theta}{1-\mu\tan\theta}, \qquad (12.3.4)$$

where T_1 = tension at the taut side of any block

T_1' = tension at the slack side of the same block

2θ = angle subtended by each block at center.

If n number of blocks are used then the ratio between the tensions at taut side to slack side becomes

$$\frac{T_1}{T_2} = \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)^n. \qquad (12.3.5)$$

The braking torque is $T_{br} = (T_1 - T_2)R$

12.2.3. Disc brake:

In this type of brake two friction pads are pressed axially against a rotating disc to dissipate kinetic energy. The working principle is very similar to friction clutch. When the pads are new the pressure distribution at pad-disc interface is uniform, i.e. p=constant If F is the total axial force applied then

$$p = \frac{F}{A}, \qquad (12.3.6)$$

where *A* is the area of the pad.The frictional torque is given by

$$T_{braking} = \frac{\mu F}{A} \oint_A r\,dA \qquad (12.3.7)$$

where μ= coefficient of kinetic friction and *r* is the radial distance of an infinitesimal element of pad. After some time the pad gradually wears away. The wear becomes uniforms after sufficiently long time, when

$$Pr = constant = c(say) \qquad (12.3.8)$$

where

$F = \oint p\,dA = c\oint \frac{dA}{r}$. The braking torque is $$T_{braking}' = \mu \oint pr\,dA = \mu Ac = \frac{\mu AF}{\oint \frac{dA}{r}} \qquad (12.3.9)$$

It is clear that the total braking torque depends on the geometry of the pad. If the annular pad is used then

$$T_{br} = \frac{2}{3}\mu F\left(\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2}\right) \qquad T_{br}' = \mu F\left(\frac{R_1 + R_2}{2}\right) \qquad (12.3.10)$$

where R_1 and R_2 are the inner and outer radius of the pad.

12.2.4. Friction materials and their properties.

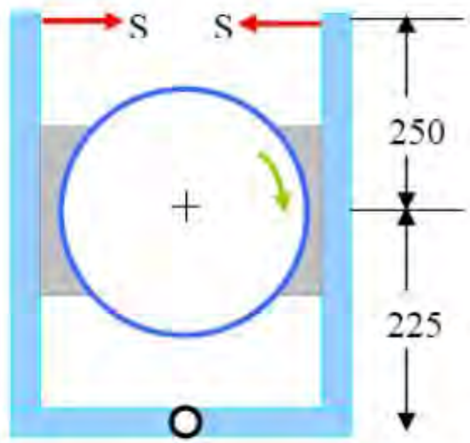
The most important member in a mechanical brake is the friction material. A good friction material is required to possess the following properties:

1. High and reproducible coefficient of friction.
2. Imperviousness to environmental conditions.
3. Ability to withstand high temperature (thermal stability)
4. High wear resistance.
5. Flexibility and conformability to any surface.

Some common friction materials are woven cotton lining, woven asbestos lining, molded asbestos lining, molded asbestos pad, Sintered metal pads etc.

Problems with Answers:

Q.1. A double shoe brake has diameter of brake drum 300mm and contact angle of each shoe 90 degrees, as shown in figure below. If the coefficient of friction for the brake lining and the drum is 0.4, find the spring force necessary to transmit a torque of 30 Nm. Also determine the width of the brake shoe if the braking pressure on the lining material is not to exceed 0.28 MPa.



12.2.1:

A.1. The friction force required to produce the given torque is

$$F_1 + F_2 = \frac{30}{0.150} = 200(N)$$

The normal forces on the shoes are $N_1 = \frac{F_1}{\mu'}$, $N_2 = \frac{F_2}{\mu'}$, where

$\mu' = \frac{4\mu \sin\theta_0}{2\theta_0 + \sin 2\theta_0}(\theta_0 = \frac{\pi}{4}) = 0.44$. Writing the moment equilibrium equations about the pivot points of individual shoes (draw correct FBDs and verify)

$$-Sl + N_1 x + F_1 a = 0 \Rightarrow F_1 = \frac{Sl}{a + \frac{x}{\mu'}} = 0.718\, S, \text{ and } Sl - N_2 x + F_2 a = 0 \Rightarrow F_2 = \frac{Sl}{\frac{x}{\mu'} - a} = 1.1314\, S$$

This yields S = 98.4(N).

Width of the friction lining :

According to the pressure distribution assumed for a shoe brake, the maximum bearing pressure occurs at the centerline of the shoe. The width of the brake lining must be selected from the higher values of the normal forces, in this case N_2. Noting that $N_2 = Rbp_{max} \int_{-\pi/4}^{\pi/4} \cos^2\theta \, d\theta$,

Where R = 0.150, $p_{max} = 0.28\, X\, 10^6$, $N_2 = 1.314 \times 98.4 / 0.44$, the value of b is calculated to be 5.4 mm or 6 mm (approx.).

Q2. A differential band brake has brake drum of diameter 500mm and the maximum torque on the drum is 1000 N-m. The brake embraces 2/3rd of the circumference. If the band brake is lined with asbestos fabric having a coefficient of friction 0.3, then design the steel band. The permissible stress is 70 MPa in tesnion. The bearing pressure for the brake lining should not exceed 0.2 MPa.

A.2. The design of belt is to be carried out when the braking torque is maximum i.e. T_{br} = 1000 N-m. According to the principle of band brake

$$T_{br} = T_1(1-e^{-\mu\beta})R = T_1\left(1-e^{-0.3\times\frac{4\pi}{3}}\right)\times 0.25$$

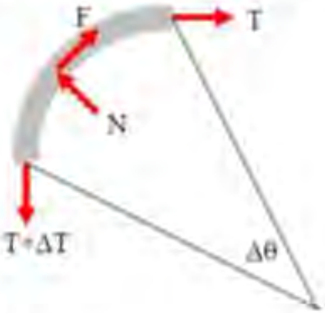
Which yield $T_1 = 5587N,\ \mathrm{T}_2 = e^{-\mu\beta}\ \mathrm{T}_1 = 1587N.$ In order to find the pressure on the band, consider an infinitesimal element. Fig. 12.2.4 The force balance along the radial direction yields

$$N = T\Delta\theta$$

Since $N = p\,b\,R\,\Delta\theta$ so $p = \frac{T}{bR}$.

The maximum pressure is $p_{max} = \frac{T_1}{bR}$.

Hence $b = \frac{5587}{0.25\times 0.2\times 10^6}$ = 0.112 m (approx.)



12.2.4

The thickness t of the band is calculated from the relation

$$S_t\,bt = T_1$$

Which yields $t = \frac{5587}{70\times 10^6\times 0.1117}$ = 0.0007145 m or 1 mm (approx.).

Lecture

Theme 13

Belt drives

13.1. Introduction to Belt drives

13.1.1 Flexible Machine Elements

Belt drives are called flexible machine elements. Flexible machine elements are used for a large number of industrial applications, some of them are as follows.

1. Used in conveying systems

Transportation of coal, mineral ores etc. over a long distance

2. Used for transmission of power.

    Mainly used for running of various industrial appliances using prime movers like electric motors, I.C. Engine etc.

3. Replacement of rigid type power transmission system.

A gear drive may be replaced by a belt transmission system

Flexible machine elements has got an inherent advantage that, it can absorb a good amount of shock and vibration. It can take care of some degree of misalignment between the driven and the driver machines and long distance power transmission, in comparison to other transmission systems, is possible. For all the above reasons flexible machine elements are widely used in industrial application.

Although we have some other flexible drives like rope drive, roller chain drives etc. we will only discuss about belt drives.

13.1.2 Typical belt drives

Two types of belt drives, an open belt drive, (Fig. 13.1.1) and a crossed belt drive (Fig. 13.1.2) are shown. In both the drives, a belt is wrapped around the pulleys. Let us consider the smaller pulley to be the driving pulley. This pulley will transmit motion to the belt and the motion of the belt in turn will give a rotation to the larger driven pulley. In open belt drive system the rotation of both the pulleys is in the same direction, whereas, for crossed belt drive system, opposite direction of rotation is observed.

13.1.3 Nomenclature of Open Belt Drive

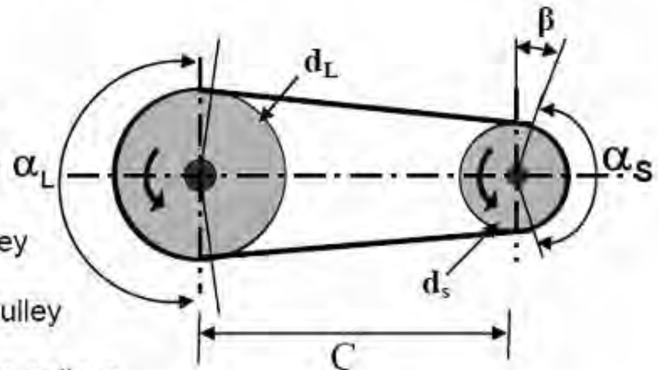
d_L - Diameter of the larger pulley

d_S – Diameter of the smaller pulley

α_L- Angle of wrap of the larger pulley

α_S – Angle of wrap of the smaller pulley

C- Center distance between the two pulleys



Basic Formulae

$\alpha_L = 180^\circ + 2\beta$ $\alpha_S = 180^\circ - 2\beta$

Where angle β is, $\beta = \sin^{-1}\left(\frac{d_L - d_S}{2C}\right)$ (13.1.1)

L_0 = Length of open belt $L_o = \frac{\pi}{2}(d_L + d_S) + 2C + \frac{1}{4C}(d_L - d_S)^2$

13.1.4 Nomenclature of Cross Belt Drive

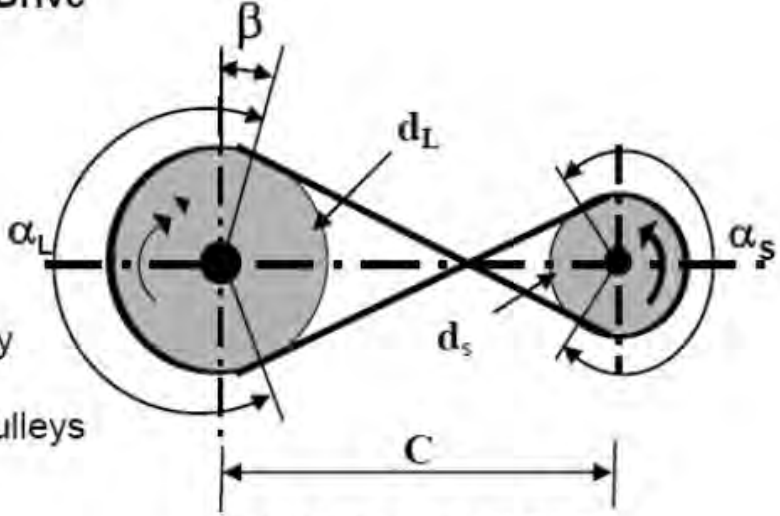
d_L - Diameter of the larger pulley

d_S – Diameter of the smaller pulley

α_L- Angle of wrap of the larger pulley

α_S – Angle of wrap of the smaller pulley

C- Center distance between the two pulleys



13.1.2 Cross belt drive

Basic Formulae

$\alpha_L = \alpha_S = 180^\circ + 2\beta$

Where angle β is,

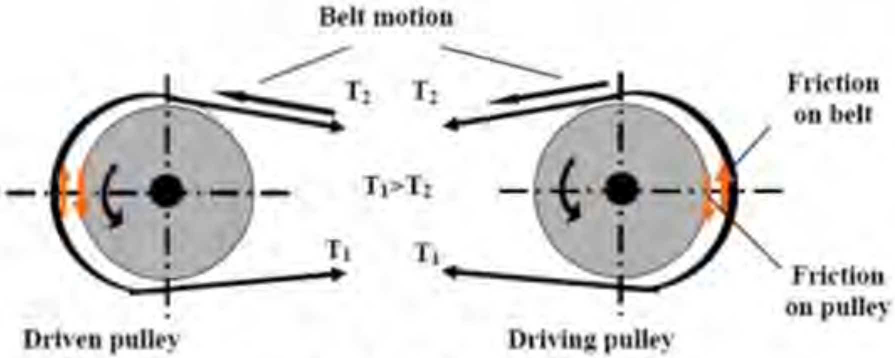
$$\beta = \sin^{-1}\left(\frac{d_L - d_S}{2C}\right) \quad (13.1.2)$$

Length of cross belt

$$L_c = \frac{\pi}{2}(d_L + d_S) + 2C + \frac{1}{4C}(d_L + d_S)^2$$

13.1.5 Belt tensions

The belt drives primarily operate on the friction principle. i.e. the friction between the belt and the pulley is responsible for transmitting power from one pulley to the other. In other words the driving pulley will give a motion to the belt and the motion of the belt will be transmitted to the driven pulley. Due to the presence of friction between the pulley and the belt surfaces, tensions on both the sides of the belt are not equal. So it is important that one has to identify the higher tension side and the lower tension side, which is shown in Fig. 13.1.3.



13.1.3 Belt tensions

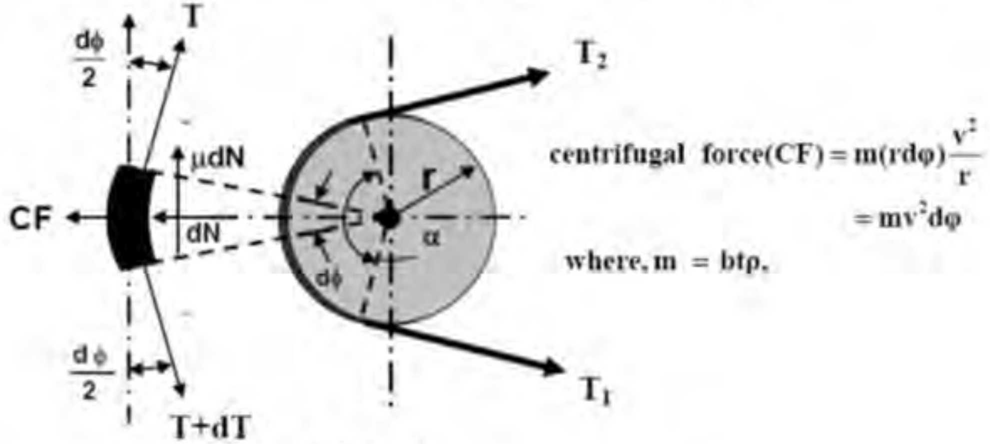
When the driving pulley rotates (in this case, anti-clock wise), from the fundamental concept of friction, we know that the belt will oppose the motion of the pulley. Thereby, the friction, **f** on the belt will be opposite to the motion of the pulley. Friction in the belt acts in the direction, as shown in Fig. 13.1.3, and will impart a motion on the belt in the same direction. The friction **f** acts in the same direction as T_2. Equilibrium of the belt segment suggests that T_1 is higher than T_2. Here, we will refer T_1 as the tight side and T_2 as the slack side, ie, T_1 is higher tension side and T_2 is lower tension side.

Continuing the discussion on belt tension, the figures though they are continuous, are represented as two figures for the purpose of explanation. The driven pulley in the initial stages is not rotating. The basic nature of friction again suggests that the driven pulley opposes the motion of the belt. The directions of friction on the belt and the driven pulley are shown the figure. The frictional force on the driven pulley will create a motion in the direction shown in the figure. Equilibrium of the belt segment for driven pulley again suggests that T_1 is higher than T_2.

It is observed that the slack side of the belt is in the upper side and the tight side of the belt is in the lower side. The slack side of the belt, due to self weight, will not be in a straight line but will sag and the angle of contact will increase. However, the tight side will not sag to that extent. Hence, the net effect will be an increase of the angle of contact or angle of wrap. It will be shown later that due to the increase in angle of contact, the power transmission capacity of the drive system will increase. On the other hand, if it is other way round, that is, if the slack side is on the lower side and the tight side is on the upper side, for the same reason as above, the angle of wrap will decrease and the power transmission capacity will also decrease. Hence, in case of horizontal drive system the tight side is on the lower side and the slack side is always on the upper side.

13.1.6 Derivation of relationship between belt tensions

The Fig.13.1.4 shows the free body diagram of a belt segment.



13.1.4

The belt segment subtends an angle **dφ** at the center. Hence, the length of the belt segment,

$$\mathbf{dl = r\,d\varphi} \qquad (13.1.3)$$

At the impending condition, ie., when the belt is in just in motion with respect to the pulley, the forces acting on the belt segment are shown in Fig.13.1.4. This belt segment is subjected to a normal force acting from the pulley on the belt segment and due to the impending motion the frictional force will be acting in the direction as shown in the figure.

$$\mathbf{f = \mu dl} \qquad (13.1.4)$$

where **μ** is the coefficient of friction between the belt and the pulley.

The centrifugal force due to the motion of the belt acting on the belt segment is denoted as CF and its magnitude is,

$$\mathbf{CF = [m(rd\varphi)x\ v^2]/r = mv^2d\varphi} \qquad (13.1.5)$$

Where, **v** is the peripheral velocity of the pulley **m** is the mass of the belt of unit length,

$$\mathbf{m = bt\rho} \qquad (13.1.6)$$

where, **b** is the width, **t** is the thickness and **ρ** is the density of the belt material.

From the equation of equilibrium in the tangential and normal direction,

$$\Sigma F_t = 0$$

$$T\cos\frac{d\phi}{2} - (T + dT)\cos\frac{d\phi}{2} + \mu dN = 0 \qquad (13.1.7)$$

$$\Sigma F_n = 0$$

$$mv^2 d\phi + dN + T\sin\frac{d\phi}{2} - (T + dT)\left(\sin\frac{d\phi}{2}\right) = 0 \qquad (13.1.8)$$

For small angle, dφ,

$$\cos\frac{d\phi}{2} \approx 1 \quad \text{and} \quad \sin\frac{d\phi}{2} \approx \frac{d\phi}{2} \qquad (13.1.9)$$

Therefore, simplified form of (13.1.**7**) is,

$$dN = \frac{dT}{\mu} \tag{13.1.10}$$

From (13.1.**8**) and using (13.1.**9**),

$$mv^2 d\phi + \frac{dT}{\mu} - T d\phi = 0 \qquad \frac{dT}{T - mv^2} = \mu d\phi \tag{13.1.11}$$

Considering entire angle of wrap,

$$\int_{T_1}^{T_2} \frac{dT}{T - mv^2} = \int_0^{\alpha} \mu d\phi \tag{13.1.12}$$

The final equation for determination of relationship between belt tensions is,

$$\frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\alpha} \tag{13.1.13}$$

It is important to realize that the pulley, driven or driver, for which the product, $\mu\alpha$ of (13.1.**13**) is the least, should be considered to determine the tension ratio. Here, α should be expressed in radians.

13.1.7 Elastic Creep and Initial Tension

Presence of friction between pulley and belt causes differential tension in the belt. This differential tension causes the belt to elongate or contract and create a relative motion between the belt and the pulley surface. This relative motion between the belt and the pulley surface is created due to the phenomena known as elastic creep. The belt always has an initial tension when installed over the pulleys. This initial tension is same throughout the belt length when there is no motion. During rotation of the drive, tight side tension is higher than the initial tension and slack side tension is lower than the initial tension. When the belt enters the driving pulley it is elongated and while it leaves the pulley it contracts. Hence, the driving pulley receives a larger length of belt than it delivers. The average belt velocity on the driving pulley is slightly lower than the speed of the pulley surface. On the other hand, driven pulley receives a shorter belt length than it delivers. The average belt velocity on the driven pulley is slightly higher than the speed of the pulley surface.

Let us determine the magnitude of the initial tension in the belt.

Tight side elongation $\propto$ $(\mathbf{T_1 - T_i})$

Slack side contraction $\propto$ $(\mathbf{T_i - T_2})$

Where, $\mathbf{T_i}$ is the initial belt tension .

Since, belt length remains the same, ie, the elongation is same as the contraction,

$$T_i = \frac{T_1 + T_2}{2} \tag{13.1.14}$$

It is to be noted that with the increase in initial tension power transmission can be increased. If initial tension is gradually increased then $\mathbf{T_1}$ will also increase and at the same time $\mathbf{T_2}$ will decrease. Thus, if it happens that $\mathbf{T_2}$ is equal to zero, then $\mathbf{T_1 = 2T_i}$ and one can achieve maximum power transmission.

13.1.8 Velocity ratio of belt drive

Velocity ratio of belt drive is defined as,

$$\frac{N_L}{N_S} = \frac{d_s + t}{d_L + t}(1 - s) \tag{13.1.15}$$

where, N_L and N_S are the rotational speeds of the large and the small pulley respectively, **s** is the belt slip and **t** is the belt thickness.

13.1.9 Power transmission of belt drive

Power transmission of a belt drive is expressed as,

$$P = (T_1 - T_2)v \tag{13.1.16}$$

where, **P** is the power transmission in Watt and **v** is the belt velocity in m/s.

Problems with Answers:

Q.1. A pump is driven by an electric motor through a open type flat belt drive. Determine the belt specifications for the following data.

Motor pulley diameter(d_S) = 300 mm, Pump pulley diameter(d_L) = 600 mm
Coefficient of friction (μ_S) for motor pulley = 0.25
Coefficient of friction (μ_L) for pump pulley = 0.20
Center distance between the pulleys=1000 mm; Rotational speed of the motor=1440 rpm;
Power transmission = 20kW; density of belt material (ρ)= 1000 kg/m^3 ; allowable stress for the belt material (σ) = 2 MPa; thickness of the belt = 5mm.

A.1.

Determination of angle of wrap

$$\beta = \sin^{-1}\left(\frac{d_L - d_s}{2C}\right) = 8.63^\circ$$

$$\alpha_L = 180 + 2\beta = 197.25^\circ = 3.44\text{rad}$$

$$\alpha_s = 180 - 2\beta = 162.75^\circ = 2.84\text{rad}$$

Length of open belt

$$L_O = \frac{\pi}{2}(d_L + d_s) + 2C + \frac{1}{4C}(d_L - d_s)^2 =$$

$$= \frac{\pi}{2}(600 + 300) + 2000 + \frac{1}{4000}(600 - 300)^2 = 3436\text{mm}$$

$$\nu = \frac{\pi \times 300 \times 1440}{60 \times 1000} = 22.62 m/s$$

$$m = bt\rho = \frac{b}{10^3} \times \frac{5}{10^3} \times 10^3 = 0.005 kg/m$$

$$m\nu^2 = 2.56 \times bN$$

Now,

$$\mu_s \alpha_s = 0.25 \times 2.84 = 0.71$$

$$\mu_L \alpha_L = 0.20 \times 3.44 = 0.688$$

$\therefore$ larger pulley governs the design

$$\frac{T_1 - 2.56b}{T_2 - 2.56b} = e^{0.688} = 1.99 \qquad (1)$$

power equation

$$P = (T_1 - T_2) \times \nu$$

$\therefore$ putting data,

$$(T_1 - T_2) = 884.17N \qquad (2)$$

again, $T_1 = 2 \times b \times 5N$

$= 10bN$ (from permissible stress) (3)

From (1), (2) and (3), solving for b,

$b \approx 240mm$

Hence, the required belt dimensions are,

Length $= 3436mm$; breadth $= 240mm$ and thickness $= 5mm$

Q2. What are the advantages of a belt drive?

A2. The advantages of a belt drive are that, it can absorb a good amount of shock and vibration. It can take care of some degree of misalignment between the driven and the driver machines and long distance power transmission, in comparison to other transmission systems, is possible.

Q3. Why the slack side of the belt of a horizontal belt drive is preferable to place on the top side?

A3. The slack side of the belt is preferably placed on the top side because, the slack side of the belt, due to its self weight, will sag. For this reason the angle of contact between the belt and the pulleys will increase. However, the tight side will not sag to that extent. Hence, the net effect will be an increase in the angle of contact or angle of wrap. Thus, due to the increase in angle of contact, the power transmission capacity of the drive system will increase.

Q4. Which one should be the governing pulley to calculate tension ratio?

A4. The pulley, driven or driver, for which the product, $\mu\alpha$ of equation for belt tension is the least, should be considered to determine the tension ratio.

Lecture

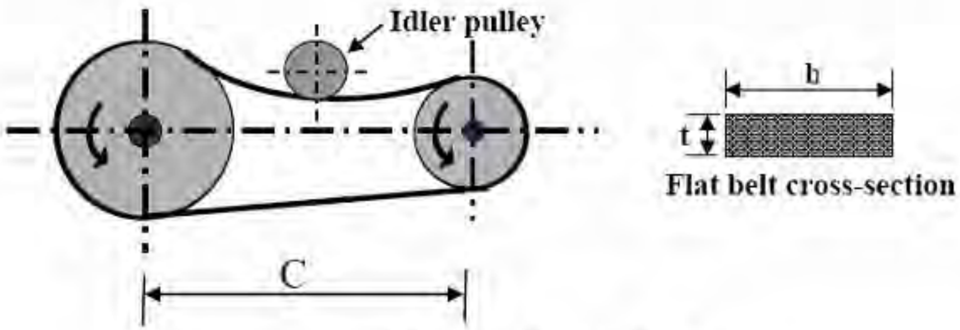
Theme 13

Belt drives

13.2. Design of Flat Belt drives

13.2.1 Flat belt drives

Flat belts drives can be used for large amount of power transmission and there is no upper limit of distance between the two pulleys. Belt conveyer system is one such example. These drives are efficient at high speeds and they offer quite running. A typical flat belt drive with idler pulley is shown in Fig. 13.2.1. Idler pulleys are used to guide a flat belt in various manners, but do not contribute to power transmission. A view of the flat belt cross section is also shown in the figure.



13.2.1 Belt drive with idler

The flat belts are marketed in the form of coils. Flat belts are available for a wide range of width, thickness, weight and material. Depending upon the requirement one has to cut the required belt length from the coil and join the ends together. The fixing of the joint must be done properly because the belt normally gets snapped from the improper joints. The best way is to use a cemented belt from the factory itself or with care one can join these belts with various types of clips that are available in the market.

13.2.2 Belt Material

Leather

Oak tanned or chrome tanned.

Rubber

Canvas or cotton duck impregnated with rubber. For greater tensile strength, the rubber belts are reinforced with steel cords or nylon cords.

Plastics

Thin plastic sheets with rubber layers

Fabric

Canvas or woven cotton ducks

The belt thickness can be built up with a number of layers. The number of layers is known as **ply.**

The belt material is chosen depending on the use and application. Leather oak tanned belts and rubber belts are the most commonly used but the plastic belts have a very good strength almost twice the strength of leather belt. Fabric belts are used for temporary or short period operations.

13.2.3 Typical Belt drive specifications

Belts are specified on the following parameters

Material

The decision of the material to be used depends on the type of service.

No. of ply and Thickness

Ply is the number of layers as indicated earlier. Therefore, the number of ply is decided depending upon the belt tensile strength required for a given power transmission.

Maximum belt stress per unit width

The belts are subjected to tensile load only. Therefore, the allowable tensile load depends on the allowable stress on the belt and its cross sectional area. It is customary to provide the belt stress value for a given belt thickness and per unit belt width. Hence, a designer has to select a belt thickness and then calculate the required belt width. Otherwise, one can calculate the belt cross sectional area and then adjust the belt thickness and the width from the standards.

The maximum belt stress is also dependent on the belt speed. Hence, the maximum belt stress (for a given belt thickness and per unit belt width) is provided either for different belt speeds or for a specified speed.

Density of Belt material

Density of Belt material is provided as, per unit length per unit cross section. Density of Belt material is required to calculate the centrifugal force on the belt.

Coefficient of friction of the belt material

Coefficient of friction for a pair of belt material and pulley material is provided in design data book.

13.2.4 Modification of Belt stress

When Maximum belt stress/ unit width is given for a specified speed, a speed correction factor (C_{SPD}) is required to modify the belt stress when the drive is operating at a speed other than the specified one.

When angle of wrap is less than 180^0 :

The maximum stress values are given for an angle of wrap is 180^o for both the pulleys, ie, pulleys are of same diameter. Reduction of belt stress is to be considered for angle of wrap less than 180^o . The belt stress is to be reduced by 3% for each ten degree lesser angle of wrap or as specified in a handbook. For e.g., if the angle of wrap is 160^o , then the belt stress is to be reduced by 6%. This factor is given as C_W

13.2.5 Design considerations for flat belt drives

Transmission ratio of flat belt drives is normally limited to 1:5
Centre distance is dependent on the available space. In the case of flat belt drives there is not much limitation of centre distance. Generally the centre distance is taken as more than twice the sum of the pulley diameters. If the centre distance is too small then rapid flexing of the belt takes place and some amount of belt life will be lost.
Depending on the driving and driven shaft speeds, pulley diameters are to be calculated and selected from available standard sizes.
Belt speed is recommended to be within 15-25 m/s.
A belt drive is designed based on the design power, which is the modified required power. The modification factor is called the service factor. The service factor depends on hours of running, type of shock load expected and nature of duty.
Hence,

Design Power (**P** $_{dcs}$) = service factor (**C** $_{sev}$)* Required Power (**P**) (13.2.1)

C_{sev} = 1.1 to 1.8 for light to heavy shock.

From the basic equations for belt drive, it can be shown that,

$$P_{des} = bt\left(\sigma' - \rho v^2\right)\left(1 - \frac{1}{e^{\mu\alpha}}\right)v \quad \text{where,} \quad \sigma' = \sigma_{max} \times C_{SPD} \times C_W \qquad (13.2.2)$$

Finally, the calculated belt length is normally kept 1% short to account for correct initial tension.

Problems with Answers:

A.1. Design a flat belt drive for the following data:

Drive: AC motor, operating speed is 1440 rpm and operates for over 10 hours. The equipment driven is a compressor, which runs at 900 rpm and the required power transmission is 20 kW.

Q.1. Let us consider the belt speed to be 20 m/s, which is within the recommended range.

The given speed ratio = 1440/900 =1.6

Let the belt material be leather, which is quite common.

Now,

$$20 = \frac{\pi \times d_s \times 1440}{60 \times 1000}$$

$$\therefore d_s = 265.3\ \text{mm}$$

$$\therefore d_L = 1.6 \times 265.3 = 424.5\text{mm}$$

From the standard sizes available, d_S=280 mm and d_L = 450 mm.

Recalculated speed ratio.

$$\frac{d_L}{d_s} = \frac{450}{280} = 1.607 \approx 1.61$$

Therefore, the choice of both the pulley diameters is acceptable.

$$\text{Center distance, } C > 2(d_L + d_L)$$

$$\therefore C > 1460\text{mm}$$

$$\text{Hence, let } C = 1500\ \text{mm (it is assumed that space is available)}$$

Considering an open belt drive, the belt length,

$$L_O = \frac{\pi}{2}(d_L + d_S) + 2C + \frac{1}{4C}(d_L - d_s)^2 =$$

$$= \frac{\pi}{2}(450 + 280) + 3000 + \frac{1}{6000}(450 - 280)^2 \approx 4151\text{mm}$$

As a guideline, to take into consideration the initial tension, the belt length is shortened by 1%. Hence, the required belt length,

L_O = 4110 mm.

Determi nation of angle of wrap

$$\beta = \sin^{-1}\left(\frac{d_L - d_S}{2C}\right) = 3.25^\circ$$

$$\alpha_L = 180 + 2\beta = 186.5^\circ = 3.26\,\text{rad}$$

$$\alpha_S = 180 - 2\beta = 173.5^\circ = 3.03\,\text{rad}$$

For the leather belt, the co-efficient of friction, μ may be taken as 0.4.
In this design, both the pulley materials are assumed to be the same, hence, angle of wrap for the smaller pulley being lower, smaller pulley governs the design and the angle of wrap is 3.03 radian.

$$\text{Design power, } P_{des} = \text{service factor } (C_{sev}) \times \text{required power } (P)$$
$$= 1.3 \times 20 \text{ kW} = 26 \text{ kW}$$

The value 1.3 is selected from design data book for the given service condition.

For the design stress in the belt, $\sigma' = \sigma_{max} \times C_{SPD} \times C_W$

However, design stress, σ', for leather belt may be considered as 2 MPa. Similarly, density of leather belt is 1000 kg/m^3.

$$P_{des} = bt\left(\sigma' - \rho v^2\right)\left(1 - \frac{1}{e^{\mu\alpha}}\right)v$$

$$26 \times 10^3 = bt\left(2 - \frac{10^3 \times 20^2}{10^6}\right)\left(1 - \frac{1}{3.36}\right) \times 20$$

$$bt = 1156.78 \text{ mm}^2$$

Let us choose standard belt thickness, t =6.5 mm
Therefore standard belt width, b = 180mm

A leather belt of 6.5 mm thickness, 180 mm width and 4110 mm length will satisfy the design conditions.

Q.2. Name some of the common flat belt materials.

A.2. Leather, rubber, plastics and fabrics are some of the common flat belt materials.

Q.3. What is the correction factors used to modify belt maximum stress?

A.3. Correction factor for speed and angle of wrap are used to modify the belt maximum stress. This correction is required because stress value is given for a specified drive speed and angle of wrap of 1800. Therefore, when a drive has different speed than the specified and angle of wrap is also different from 1800 , then above mentioned corrections are required.

Q.4. What is the recommended center distance and belt speed for a flat belt drive?

A.4. The recommendations are; the center distance should be greater than twice the sum of pulley diameters and the belt speed range should be within 15-25 m/s.

Lecture

Theme 13

Belt drives

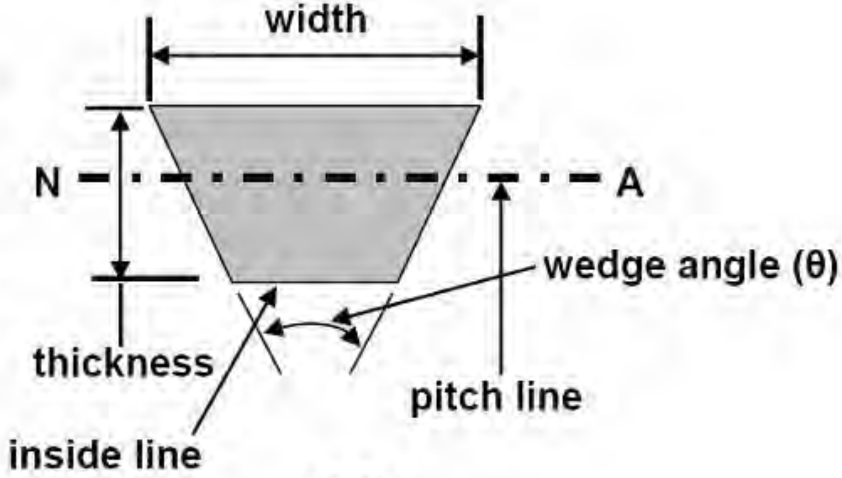
13.3. Design of V- Belt drives

13.3.1 V - Belt Drives

Among flexible machine elements, perhaps V-belt drives have widest industrial application. These belts have trapezoidal cross section and do not have any joints. Therefore, these belts are manufactured only for certain standard lengths. To accommodate these belts the pulleys have V shaped grooves which makes them relatively costlier. Multiple groove pulleys are available to accommodate number of belts, when large power transmission is required. V-belt drives are most recommended for shorter center distances. In comparison to flat belt drives, these drives are slightly less efficient. V belt can have transmission ratio up to 1:15 and belt slip is very small. As the belts are endless type, V-belt drives do not suffer from any joint failure and are quiet in operation. V-belts constitute fabric and cords of cotton, nylon etc. and impregnated with rubber.

13.3.2 Nomenclature of V-belt

A typical V-belt section is shown in Fig.13.3.1. The geometrical features of the belt section are indicated in the figure. The pitch line, which is also marked as N-A, is the neutral axis of the belt section. The design calculations for V-belt drives are based on the pitch line or the neutral axis. These belts are available in various sections depending upon power rating.



13.3.1

13.3.3 Standard V-belt sections

The standard V-belt sections are A, B, C, D and E. The table below contains design parameters for all the sections of V-belt. The kW rating given for a particular section indicates that, belt section selection depends solely on the power transmission required, irrespective of number of belts. If the required power transmission falls in the overlapping zone, then one has to justify the selection from the economic view point also.

Section	kW range	Minimum pulley pitch diameter (mm)	Width (mm)	Thickness (mm)
A	0.4 - 4	125	13	8
B	1.5 -15	200	17	11
C	10 -70	300	22	14
D	35-150	500	32	19
E	70-260	630	38	23

As for example, a single belt of B section may be sufficient to transmit the power, instead of two belts of A section. This may increase the cost as well as weight of the pulley, as two-grooved pulley is required. In general, it is better to choose that section for which the required power transmission falls in the lower side of the given range.

Another restriction of choice of belt section arises from the view point of minimum pulley diameter. If a belt of higher thickness (higher section) is used with a relatively smaller pulley, then the bending stress on the belt will increase, thereby shortening the belt life.

13.3.4 Designation of V belt

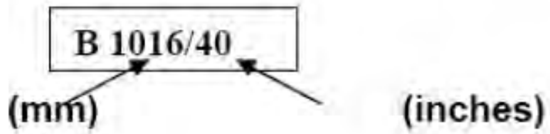
It has been mentioned that, the calculations for V-belt drives are based on pitch diameter. However, V-belts are designated with nominal inside length (this is easily measurable compared to pitch length). Therefore, to arrive at the inside length, the following relationship is useful.

$$\textbf{Inside length + X=Pitch Length} \qquad (13.3.1)$$

Value Of X

	A	B	C	D
X (mm)	36	43	56	79

For example, a B- section belt with nominal inside length of 1016 mm or 40 inches (nearest value obtained from belt catalogue) is required for a V-belt drive. Then this belt is designated as,



13.3.5 V- belt Equation

V-belts have additional friction grip due to the presence of wedge. Therefore, modification is needed in the equation for belt tension. The equation is modified as,

$$\frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\alpha / \sin\frac{\theta}{2}} \qquad (13.3.2)$$

13.3.6 V-belt power rating

Each type of belt section has a power rating. The power rating is given for different pitch diameter of the pulley and different pulley speeds for an angle of wrap of 180°. A typical nature of the chart is shown below. Here, for example, for pitch diameter of D_1, power rating of the A section belt is kW_1, kW_2, kW_3, kW_4 for belt speeds of N_1, N_2, N_3, N_4 respectively. Similar tables are available for the belts of other sections.

kW rating of V-belts for different belt speeds ($\alpha = 180^\circ$)

Belt Section	Pitch Diameter	N_1	N_2	N_3	N_4
A	D_1	kW_1	kW_2	kW_3	kW_4
	D_2				
	D_3				

13.3.7 V belt design factors

Service Factor

A belt drive is designed based on the design power, which is the modified required power. The modification factor is called the service factor. The service factor depends on hours of running, type of shock load expected and nature of duty.

Hence,

Design Power (P_{dcs}) = service factor (C_{sev})* Required Power (**P**) (13.3.3)

C_{sev} = 1.1 to 1.8 for light to heavy shock.

Modification of kW rating

Power rating of a typical V-belt section requires modification, since, the ratings are given for the conditions other than operating conditions. The factors are as follows,

Equivalent smaller pulley diameter

In a belt drive, both the pulleys are not identical, hence to consider severity of flexing, equivalent smaller pulley diameter is calculated based on speed ratio. The power rating of V-belt is then estimated based on the equivalent smaller pulley diameter (d_{es}).

$$\mathbf{d_{ES} = C_{SR} d_S} \quad (13.3.4)$$

where, C_{sr} is a factor dependent on the speed ratio.

Angle of wrap correction factor

The power rating of V-belts are based on angle of wrap, **α =**1800 . Hence, Angle of wrap correction factor ($\mathbf{C_{vw}}$) is incorporated when α is not equal to 180° .

Belt length correction factor

There is an optimum belt length for which the power rating of a V-belt is given. Let, the belt length be small then, in a given time it is stressed more than that for the optimum belt length. Depending upon the amount of flexing in the belt in a given time a belt length correction factor (C_{vL}) is used in modifying power rating. Therefore, incorporating the correction factors,

Modified power rating of a belt (kW)

= Power rating of a belt (kW) x C_{vw} x C_{vl} (13.3.5)

13.3.8 Selection of V- belt

The transmission ratio of V belt drive is chosen within a range of 1:15

Depending on the power to be transmitted a convenient V-belt section is selected.

The belt speed of a V-belt drive should be around 20m/s to 25 m/s, but should not exceed 30 m/s.

From the speed ratio, and chosen belt speed, pulley diameters are to be selected from the standard sizes available.

Depending on available space the center distance is selected, however, as a guideline,

$$d_L < C < 3(d_L + d_S) \qquad (13.3.6)$$

The belt pitch length can be calculated if C, d_L and d_S are known. Corresponding inside length then can be obtained from the given belt geometry. Nearest standard length, selected from the design table, is the required belt length.

From section (13.3.7) above, the design power and modified power rating of a belt can be obtained. Therefore,

$$\text{Number of belts} = \frac{\text{Design Power}}{\text{Modified power rating of the belt}} \qquad (13.3.7)$$

Problems with Answers:

Q.1. Design a flat belt drive for the following data:

Drive: AC motor, operating speed is 1440 rpm and operates for over 10 hours. The equipment driven is a compressor, which runs at 900 rpm and the required power transmission is 20 kW.

A.1. Since it is a V belt drive, let us consider belt speed, v = 25 m/sec.

$$\text{Design power, } P_{des} = \text{service factor } (C_{sev}) \times \text{required power (P)}$$
$$= 1.3 \times 20\text{ kW} = 26\text{ kW}$$

The value 1.3 is selected from design data book for the given service condition.
Hence, obvious choice for belt section is **C**

Now, $25 = \dfrac{\pi \times d_s \times 1440}{60 \times 1000}$ $\quad \therefore d_S = 331.6\text{ mm} \quad \therefore d_L = 1.6 \times 331.6 = 530.6\text{mm}$

standard sizes are,

d_S=315 mm and d_L=530 mm $\qquad$ d_S=355 mm and d_L = 560 mm.

First combination gives the speed ratio to be 1.68
Second combination gives the speed ratio to be 1.58.
So, it is better to choose the second combination because it is very near to the given speed ratio.
Therefore, selected pulley diameters are d_S=355 mm and d_L= 560 mm.
Center distance, C should be such that, $d_L < C < 3(d_L + d_S)$

Let us consider, C = 1500 mm, this value satisfies the above condition.
Considering an open belt drive, the belt length,

$$L_O = \frac{\pi}{2}(d_L + d_S) + 2C + \frac{1}{4C}(d_L - d_s)^2 =$$

$$= \frac{\pi}{2}(560 + 355) + 3000 + \frac{1}{6000}(560 - 355)^2 \approx 4444 \text{ mm}$$

Inside length of belt = 4444 – 56 = 4388 mm from (13.3.1)
The nearest value of belt length for C-section is 4394 mm (from design data book)
Therefore, the belt designation is **C: 4394/173**
Power rating (kW) of one C-section belt
Equivalent small pulley diameter is,

$$d_{ES} = C_{SR} d_s = 355 \times 1.12 = 398 \text{ mm}$$

$$C_{SR} = 1.12 \text{ is obtained from the hand book}$$

For the belt speed of 25 m/sec, the given power rating (kW) = 12.1 kW

For the obtained belt length, the length correction factor C_{vl}=1.04

Determi nation of angle of wrap

$$\beta = \sin^{-1}\left(\frac{d_L - d_S}{2C}\right) = 3.92^\circ$$

$$\alpha_L = 180 + 2\beta = 187.84^\circ = 3.28\,\text{rad}$$

$$\alpha_S = 180 - 2\beta = 172.16^\circ = 3.00\,\text{rad}$$

For the angle of wrap of 3.00 radian (smaller pulley), the angle of wrap factor, C_{vw} is found to 0.98.for a C section belt.

Therefore, incorporating the correction factors,

Modified power rating of a belt (kW) = Power rating of a belt (kW) x C_{vw} x C_{vl}
= 12.1 x 0.98 x 1.04 = 12.33 kW

$$\text{Number of belts} = \frac{26}{12.33} = 2.1 \approx 2$$

2 numbers of C 4394/173 belts are required for the transmission of 20 kW.

Questions with Answers

Q.1. How a V-belt section is selected?

A.1. From the given table, depending upon the required power transmission, a belt section is chosen. However, the smaller pulley diameter should be less than the pulley diameter as mentioned for the chosen belt section.

Q.2. Why angle of wrap correction factor and belt length correction factor is required to modify power rating of a belt?

A.2. The power rating of V-belts are based on angle of wrap, $\alpha = 180^0$. Hence, for any angle of wrap, other than 180^0, a correction factor is required. Similarly, if the belt length is different from optimum belt length for which the power rating is given, then belt length correction factor is used, because, amount of flexing in the belt in a given time is different from that in optimum belt length.

Q.3. How a V-belt is designated?

A.3. Let a V-belt of section A has inside length of 3012 mm. Then its designation will be **A 3012/118.** Where, 118 is the corresponding length in inches.

Lecture

Theme 14

Brief overview of bearings

14.1. Fluid Film bearings

14.1.1 Brief overview of bearings

Bearings are broadly categorized into two types, fluid film and rolling contact type.

1. Fluid Film bearings

In fluid film bearing the entire load of the shaft is carried by a thin film of fluid present between the rotating and non-rotating elements. The types of fluid film bearings are as follows:

1.1. Sliding contact type

1.2. Journal bearing

1.3. Thrust bearing

1.4 Slider bearing

2. Rolling contact bearings

In rolling contact bearings, the rotating shaft load is carried by a series of balls or rollers placed between rotating and non-rotating elements. The rolling contact type bearings are of two types, namely,

2.1. Ball bearing

2.2. Roller bearing

14.1.2 Comparison of bearing frictions

The Fig. 14.1.1 shows a plot of Friction vs. Shaft speed for three bearings. It is observed 1) that for the lower shaft speeds the journal bearing have more friction than roller and ball bearing and ball bearing friction being the lowest. For this reason, the ball bearings and roller bearings are also called as anti friction bearings. 2) However, with the increase of shaft speed the friction in the ball and roller bearing phenomenally increases but the journal bearing friction is relatively lower than both of them. 3) Hence, it is advantageous to use ball bearing and roller

bearing at low speeds. Journal bearings are mostly suited for high speeds and high loads.

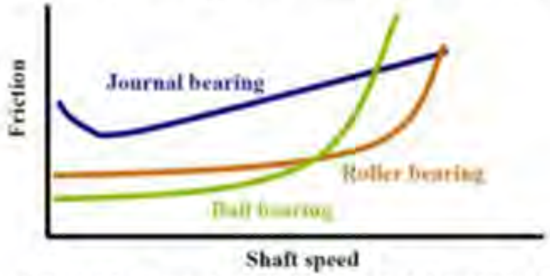
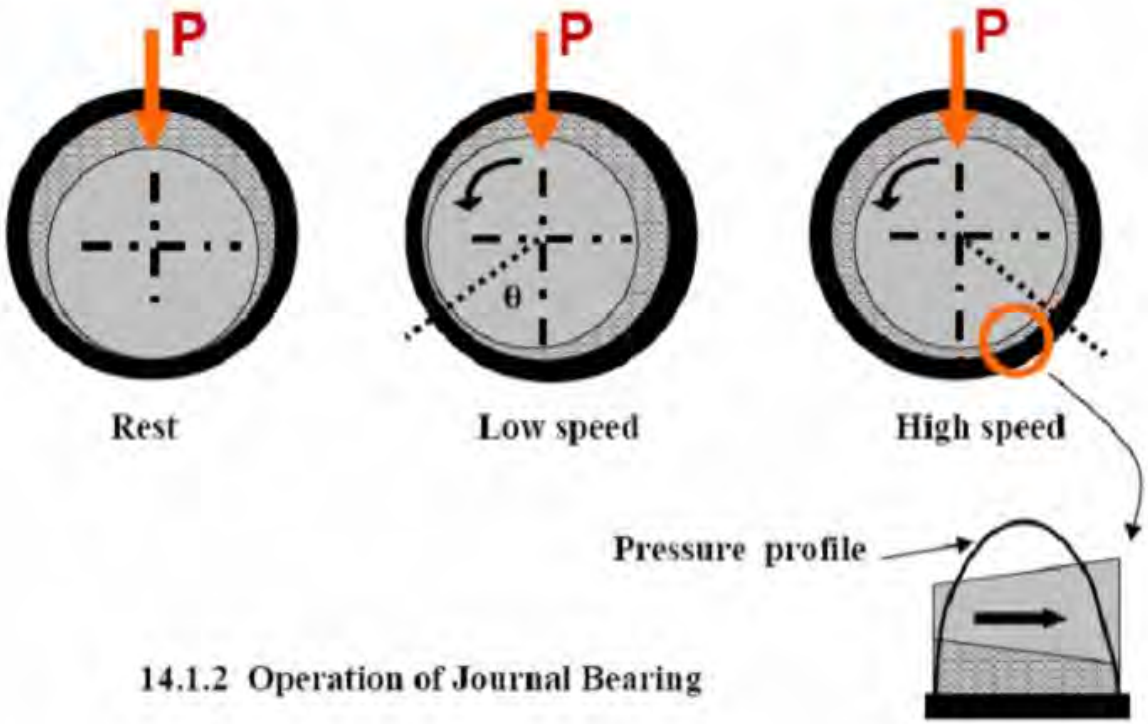


Fig. 14.1.1 Comparison of friction for different bearings

4) The ball and roller bearings require less axial space but more diametrical space during installation and low maintenance cost compared to journal bearings. Ball bearings and roller bearing are relatively costly compared to a journal bearing. 5) The reliability of journal bearing is more compared to that of ball and roller bearings.

Here, we will discuss only about journal, ball and roller bearings, being most commonly used in design.

14.1.3 Journal Bearing



14.1.2 Operation of Journal Bearing

Fig. 14.1.2 describes the operation of a journal bearing. The black annulus represents the bush and grey circle represents the shaft placed within an oil film shown by the shaded region. The shaft, called journal, carries a load P on it. The journal being smaller in diameter than the bush, it will always rotate with an eccentricity.

1) When the journal is at rest, it is seen from the figure that due to bearing load P, the journal is in contact with the bush at the lower most position and there is no oil film between the bush and the journal. 2) Now when the journal starts rotating, then at low speed condition, with the load P acting, it has a tendency to shift to its sides as shown in the figure. At this equilibrium position, the frictional force will balance the component of bearing load. In order to achieve the equilibrium, the journal orients itself with respect to the bush as shown in figure. The angle θ, shown for low speed condition, is the angle of friction. Normally at this condition either a metal to metal contact or an almost negligible oil film thickness will prevail. 3) At the higher speed, the equilibrium position shifts and a continuous oil film will be created as indicated in the third figure above. This continuous fluid film has a converging zone, which is shown in the magnified view. It has been established that due to presence of the converging zone or wedge, the fluid film is capable of carrying huge load. If a wedge is taken in isolation, the pressure profile generated due to wedge action will be as shown in the magnified view.

4) Hence, to build-up a positive pressure in a continuous fluid film, to support a load, a converging zone is necessary. Moreover, simultaneous presence of the converging and diverging zones ensures a fluid film continuity and flow of fluid. The journal bearings operate as per the above stated principle.

14.1.4. The background of hydrodynamic theory of lubrication

Petroff (1883) carried out extensive experimental investigation and showed the dependence of friction on viscosity of lubricant, load and dimensions of the journal bearing. Tower (1883 and later) also conducted experimental investigation on bearing friction and bearing film pressure. The experimental investigations by Petroff and Tower form the background of the hydrodynamic theory. Later on Osborne Reynolds conducted experiments and published the findings in the form of present day hydrodynamic theory of lubrication and the corresponding mathematical equation is known as Reynolds' equation.

The Reynolds' equation (simplified form)

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial z}\right)=\rho\frac{U}{2}\frac{\partial h}{\partial x}+\ldots \qquad (14.1.1)$$

where,

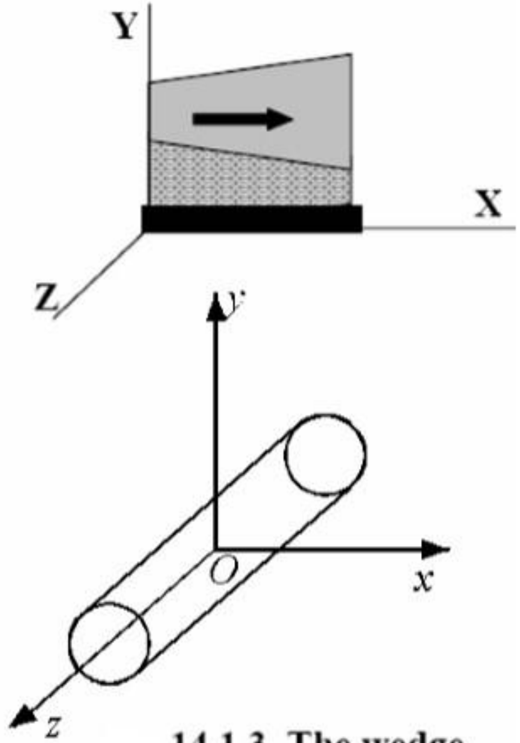
U : surface speed of the wedge, in x-direction

p : pressure at any point(x,z) in the film

μ : Absolute viscosity of the lubricant

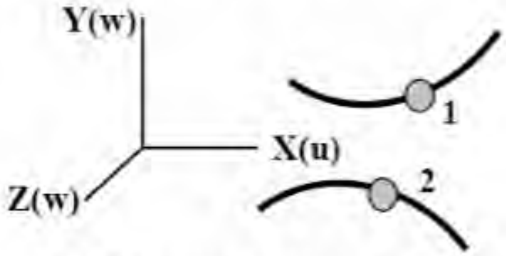
h : film thickness

1)The left hand side of the equation represents flow under the pressure gradient. 2)The corresponding right hand side represents a pressure generation mechanism. 3) In this equation it has been assumed that the lubricant is incompressible and Newtonian. The wedge shape, that was discussed earlier,4)is assumed to be a straight profile as shown in Fig.14.1.3. The bearing is very long in the Z direction and the variation of pressure is in the X and Y direction.



14.1.3 The wedge

Let us have a look at the right hand term in details.



14.1.4 Pressure generation mechanism

$$...=\frac{\partial}{\partial x}\left(\rho\frac{u_1+u_2}{2}h\right)+\frac{\partial}{\partial y}\left(\rho\frac{v_1+v_2}{2}h\right)+\rho\frac{\partial h}{\partial t}+h\frac{\partial \rho}{\partial t} \qquad (14.1.2)$$

squeeze film **compression**

$$...=\rho\frac{U}{2}\frac{\partial h}{\partial x}+\frac{1}{2}\rho h\frac{\partial U}{\partial x}+\frac{1}{2}Uh\frac{\partial \rho}{\partial x}+... \qquad (14.1.3)$$

Physical wedge **stretch**

There are two moving surfaces 1 and 2 as indicated in Fig. 14.1.4. 1) For 1 the velocities are u1, v1 and w1 along the three coordinate axes X, Y and Z respectively. 2) For 2, similarly the velocities are u2, v2 and w2 respectively. Equation (14.1.2) represents the full form of the right hand side of Reynolds' equation. For the purpose of explanation, partial derivative of only the first term of equation (14.1.2) is written in equation (14.1.3). Here u1+ u2 have been replaced by U.

1)The first term of (14.1.3), $\rho\frac{U}{2}\frac{\partial h}{\partial x}$, represents a physical wedge. 2)The second term $\frac{1}{2}(\rho h)\frac{\partial U}{\partial x}$ is known as the stretch. All the three terms of (14.1.3) contribute in pressure generation mechanism.

The term, $\rho\frac{\partial h}{\partial t}$ in equation (14.1.2) is called squeeze film; with respect to time how the film thickness is changing is given by this term.

The last term, $h\frac{\partial \rho}{\partial t}$ is the compressibility of the fluid with time and it is termed as compression.

The simplified form of the Reynolds's equation, (14.1.1), has only the physical wedge term, $\rho\frac{U}{2}\frac{\partial h}{\partial x}$.

14.1.5 Design parameters of journal bearing

The first step for journal bearing design is determination of bearing pressure for the given design parameters,

1) Operating conditions (temperature, speed and load)
2) Geometrical parameters (length and diameter)
3) Type of lubricant (viscosity)

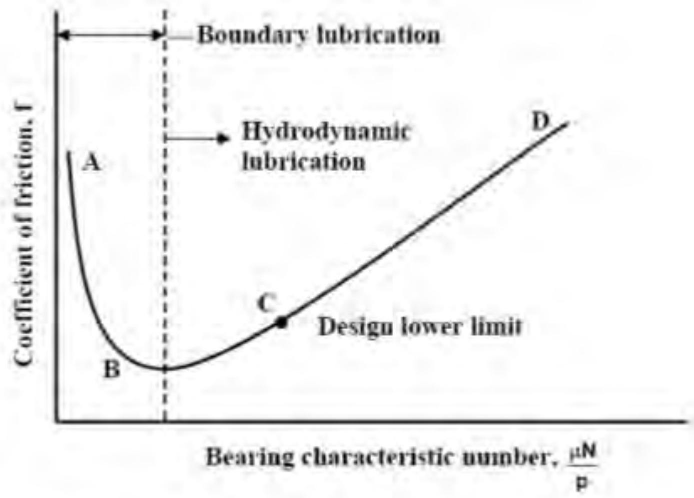
The design parameters, mentioned above, are to be selected for initiation of the design. The bearing pressure is known from the given load capacity and preliminary choice of bearing dimensions. After the bearing pressure is determined, a check for proper selection of *design zone* is required. The selection of design zone is explained below.

Selection of design zone

The Fig. 14.1.5 shows the results of test of friction by McKee brothers. Figure shows a plot of variation of coefficient of friction with bearing characteristic number. Bearing characteristic number is defined as,

$$\text{Bearing characteristic number} = \frac{\mu N}{p}$$

It is a non-dimensional number, where μ is the viscosity, N is the speed of the bearing and p is the pressure given by $p = \frac{P}{dl}$,d and l being diameter and length of the journal respectively.



14.1.5 Results of test of friction (McKee brothers)

The plot shows 1) that from B with the increase in bearing characteristic number the friction increases and from B to A with reduction in bearing characteristic number the friction again increases. So B is the limit and the zone between A to B is known as boundary lubrication or sometimes termed as imperfect lubrication. Imperfect lubrication means that metal – metal contact is possible or some form of oiliness will be present. 2) The portion from B to D is known as the hydrodynamic lubrication .The calculated value of bearing characteristic number should be somewhere in the zone of C to D. This zone is characterized as design zone.

For any operating point between C and D due to fluid friction certain amount of temperature generation takes place. Due to the rise in temperature the viscosity of the lubricant will decrease, thereby, the bearing characteristic number also

decreases. Hence, the operating point will shift towards C, resulting in lowering of the friction and the temperature. As a consequence, the viscosity will again increase and will pull the bearing characteristic number towards the initial operating point. Thus a self control phenomenon always exists. For this reason the design zone is considered between C and D. 3) The lower limit of design zone is roughly five times the value at B. On the contrary, if the bearing characteristic number decreases beyond B then friction goes on increasing and temperature also increases and the operation becomes unstable.

4) Therefore, it is observed that, bearing characteristic number controls the design of journal bearing and it is dependent of design parameters like, operating conditions (temperature, speed and load), geometrical parameters (length and diameter) and viscosity of the lubricant.

14.1.6 Methods for journal bearing design

Broadly there are two methods for journal bearing design, they are,

First Method: developed by M. D. Hersey

Second Method: developed by A. A. Raimondi and J. Boyd

1) Method developed by M. D. Hersey

This method is based on dimensional analysis, applied to an infinitely long bearing. Analysis incorporates a side-flow correction factor obtained from the experiment of S. A. McKee and T. R. McKee (McKee Brothers).

McKee equation for coefficient of friction, for full bearing is given by,

$$\text{Coefficient of friction, } f = K_1 \frac{\mu N}{p} \frac{d}{c} + K_2 \qquad (14.1.4)$$

Where, p : pressure on bearing (projected area) = $\frac{P}{Ld}$

- L : length of bearing
- d : diameter of journal
- N : speed of the journal
- μ : absolute viscosity of the lubricant
- c : difference bush and journal diameter
- K_2 : side-flow factor = 0.002 for (L/d) 0.75-2.8

The constant is dependent on the system of units. For example, $K_1 = \frac{473}{10^{10}}$,

when μ is in centipoise, p is in psi , N is in rpm and d and c in inches.
The steps to be followed are,
Basic design parameters are provided by the designer from the operating conditions. These are,
Bearing load (P)   Journal diameter (d)   Journal speed (N)
Depending upon type of application, selected design parameters are obtained from a design handbook, these are,
L/d ratio   Bearing pressure(p)   c/d ratio
Proper lubricant and an operating temperature
The heat generation in the bearing is given by,

$H_g = fPv$ where, v is the rubbing velocity

The heat dissipation is given by,

$$H_d = KA(t_b - t_a)$$

where,
A=projected bearing area   K=heat dissipation coefficient
t_b=bearing surface temperature t_a=temperature of the surrounding

Next steps are as follows,

Value of $\frac{\mu N}{p}$ should be within the design zone

Equation (14.1.7) is used to compute *f*

Heat generation and heat dissipation are computed to check for thermal equilibrium.

Iteration with selected parameters is required if thermal equilibrium is not established.

Provision for external cooling is required if it is difficult to achieve thermal equilibrium.

The method described here is relatively old. The second method is more popular and is described below.

2. Method developed by A. A. Raimondi and J. Boyd

This method is based on hydrodynamic theory. The Reynolds equation (14.1.1) does not have any general solution. Assuming no side flow, Sommerfeld (1904) proposed a solution and defined a parameter, known as Sommerfeld number, given as,

$$\frac{r}{c} f = \varphi\left[\left(\frac{r}{c}\right)^2 \frac{\mu N}{p}\right] \tag{14.1.5}$$

where, φ = A functional relationship, for different types of bearings

$\left[\left(\frac{r}{c}\right)^2 \frac{\mu N}{p}\right]$ = Sommerfeld number, S (dimensionless)

The Sommerfeld number is helpful to the designers, because it includes design parameters; bearing dimensions r and c , friction f , viscosity μ, speed of rotation N and bearing pressure p. But it does not include the bearing arc. Therefore the functional relationship can be obtained for bearings with different arcs, say 360^o , 60^o etc.

Raimondi and Boyd (1958) gave a methodology for computer–aided solution of Reynolds equation using an iterative technique. For L/d ratios of 1, 1:2 and 1:4 and for bearing angles of 360^0 to 60^0 extensive design data are available.

Charts have been prepared by Raimondi and Boyd for various design parameters, in dimensionless form, are plotted with respect to Sommerfeld number.

All these charts are for $360^0 - 60^0$ bearings.

The design parameters which are given by Raimondi and Boyd are as follows,

Design parameters

h_0/c : Minimum film thickness

$(r/c)f$: Coefficient of friction

$Q/(rcNL)$: Flow

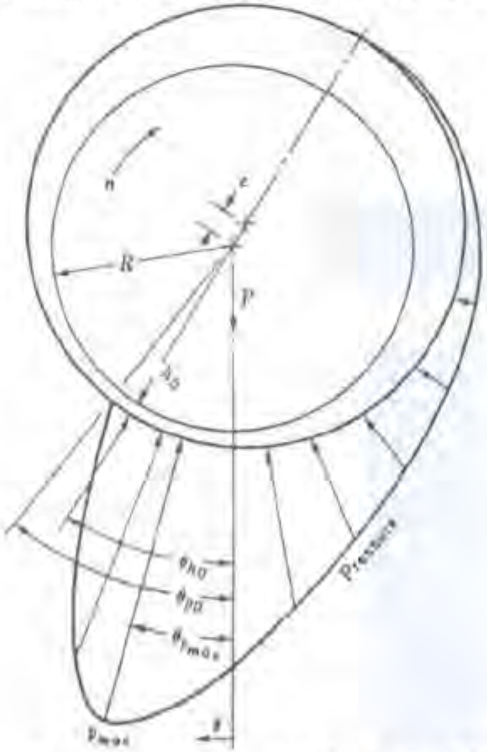
Q_s/Q : Flow ratio

p/p_{max} : Maximum film pressure ratio

θ_{p_0}, deg : Terminating position of film

θ_{h_0}, deg : Minimum film thickness position

The above design parameters are defined in the Fig. 14.1.6. The pressure profile shown is only for the positive part of the bearing where the converging zone is present. Negative part has not been shown because it is not of use.



14.1.6 Nomenclature of a journal bearing

14.1.7 Materials for bearing

The common materials used for bearings are listed below.

Lead based babbits : around 85 % Lead; rest are tin, antimony and copper
(pressure rating not exceeding 14MPa)
Tin based babbits : around 90% tin; rest are copper, antimony and lead
(pressure rating not exceeding 14MPa)
Phosphor bronze : major composition copper; rest is tin, lead, phosphorus
(pressure rating not exceeding 14MPa)
Gun metal : major composition copper; rest is tin and zinc
(pressure rating not exceeding 10MPa)

Cast iron : pressure rating not exceeding 3.5 MPa

Other materials commonly used are, silver, carbon-graphite, teflon etc.

Problems with answers

Q1. Broadly, what are the types of bearings?

A1. Broadly bearings are of two types; Fluid Film bearings, where the entire load of the shaft is carried by a thin film of fluid present between the rotating and non-rotating elements and Rolling contact bearings, where the rotating shaft load is carried by a series of balls or rollers placed between rotating and non-rotating elements.

Q2. Highlight friction characteristics of bearings.

A2. For the lower shaft speeds the journal bearing have more friction than roller and ball bearing and ball bearing friction being the lowest. However, with the increase of shaft speed the friction in the ball and roller bearing phenomenally increases but the journal bearing friction is relatively lower than both of them.

Q3. Can a block moving over a constant height fluid film carry load?

A3. In this case the block can not carry any load. It can be shown mathematically that a wedge shaped fluid film can only generate pressure, thereby can withstand load.

Q4. What is Sommerfeld number? What importance it has in context of journal bearing design?

A4. Sommerfeld number is given by, $S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{p}$ with usual notations. This number includes design parameters; bearing dimensions r and c , friction f , viscosity μ, speed of rotation N and bearing pressure p. Only it does not include the bearing arc. Therefore for a given bearing arc, the Sommerfeld number indicates the operational state of a fluid film bearing.

Lecture

Theme 14

Brief overview of bearings

14.2. Rolling contact bearings

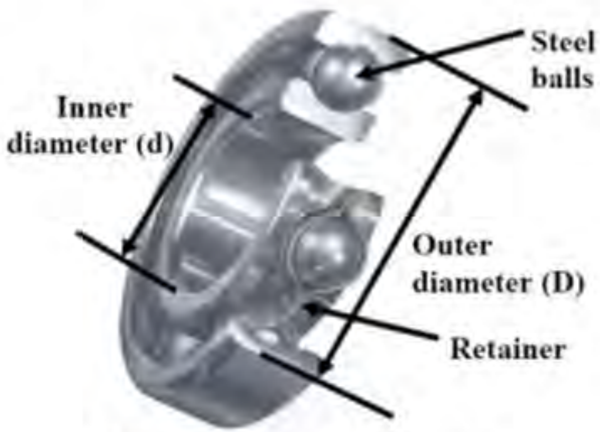
14.2.1 Rolling contact bearing

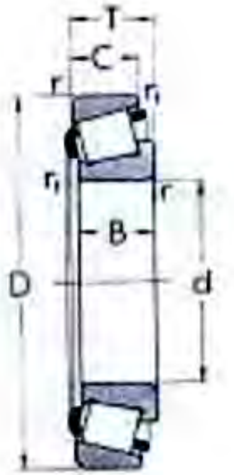
Rolling contact bearings are also called anti-friction bearing due to its low friction characteristics. These bearings are used for radial load, thrust load and combination of thrust and radial load. These bearings are extensively used due to its relatively lower price, being almost maintenance free and for its operational ease. However, friction increases at high speeds for rolling contact bearings and it may be noisy while running. These bearings are of two types,

Ball bearing and *Roller bearing*

14.2.2 Ball bearing

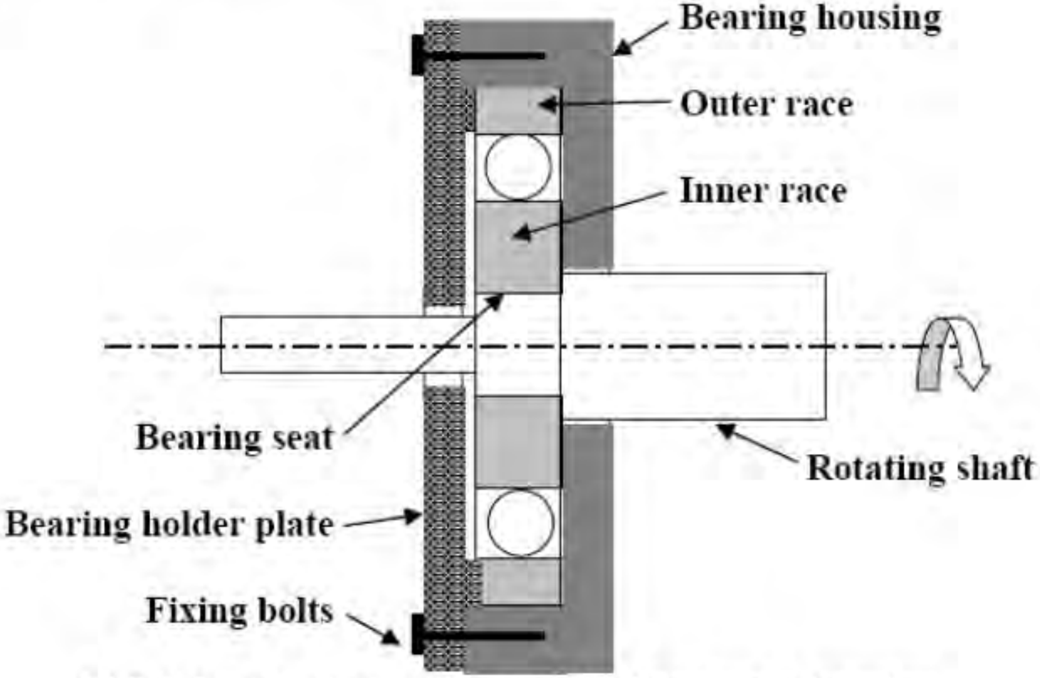
A typical ball bearing is shown the Fig.14.2.1. The figure shown on the right side, with nomenclature, is the schematic representation of the actual bearing.





14.2.1 A typical ball bearing

The bearing shown in the figure is called Single row deep groove ball bearing. It is used to carry radial load but it can also take up considerable amount of axial load. The retainer keeps the steel balls in position and the groove below the steel balls is the inner ring and over it is the outer ring. The outer ring, called outer race,is normally placed inside a bearing housing which is fixed, while the inner race holds the rotating shaft. Therefore, a seat of diameter d and width B is provided on the shaft to press fit the bearing. The arrangement for housing a bearing is shown through a schematic diagram, Fig.14.2.2.



14.2.2 A typical arrangement for housing a bearing

Single row Angular Contact Ball Bearing

The figure Fig.14.2.3 is a Single row Angular Contact Ball Bearing. It is mostly used for radial loads and heavy axial loads.

14.2.3

14.2.4

Double Row Angular Contact Bearing

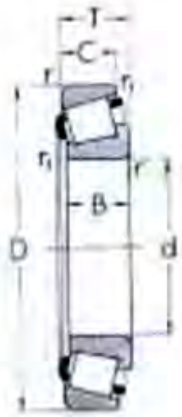
Double Row Angular Contact Bearing, shown in Fig.14.2.4, has two rows of balls.
Axial displacement of the shaft can be kept very small even for axial loads of varying magnitude.

Single thrust ball bearing

This Fig.14.2.5 shows a Single thrust ball bearing. It is mostly used for unidirectional axial load.

14.2.5

14.2.6



Taper Roller Bearing

A taper roller bearing and its nomenclature are shown in Fig.14.2.6 above. It is generally used for simultaneous heavy radial load and heavy axial load. Roller bearings has more contact area than a ball bearing, therefore, they are generally used for heavier loads than the ball bearings.

Spherical Roller Bearing

A spherical roller bearing, shown in the Fig.14.2.7, has self aligning property. It is mainly used for heavy axial loads. However, considerable amount of loads in either direction can also be applied.

14.2.7

14.2.8

Cylindrical Roller Bearing

For heavy radial load and high speed use, cylindrical roller bearings, shown in the Fig.14.2.8, are used. Within certain limit, relative axial displacement of the shaft and the bearing housing is permitted for this type of bearings.

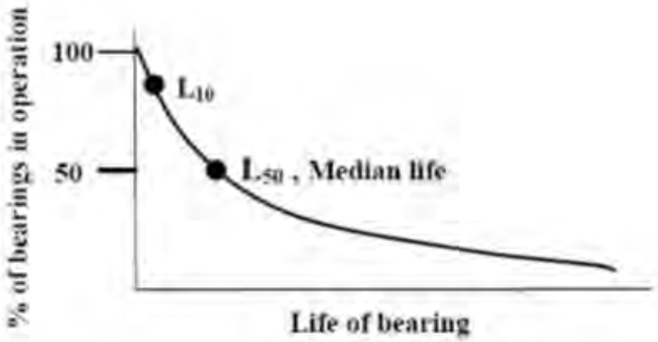
14.2.3 Rolling contact bearing selection

Some of the important terminologies which are required for selection of rolling contact bearing are given below.

Rating life:

Rating life is defined as the life of a group of apparently identical ball or roller bearings, in number of revolutions or hours, rotating at a given speed, so that 90% of the bearings will complete or exceed before any indication of failure occur.

Suppose we consider 100 apparently identical bearings. All the 100 bearings are put onto a shaft rotating at a given speed while it is also acted upon by a load. After some time, one after another, failure of bearings will be observed. When in this process, the tenth bearing fails, then the number of revolutions or hours lapsed is recorded. These figures recorded give the rating life of the bearings or simply L_{10} life (10 % failure). Similarly, L_{50} means, 50 % of the bearings are operational. It is known as median life. Fig.14.2.3 defines the life of rolling contact bearings.



14.2.9 Definition of life of rolling contact bearing

Bearing load

If two groups of identical bearings are tested underloads P_1 and P_2 for respective lives of L_1 and L_2 , then,

$$\frac{L_1}{L_2} = \left(\frac{P_2}{P_1}\right)^a \qquad (14.2.1)$$

Where, L is life in millions of revolution or life in hours

a is constant which is 3 for ball bearings and 10/3 for roller bearings

Basic load rating

It is that load which a group of apparently identical bearings can withstand for a rating life of one million revolutions.

Hence, in (15.3.66), if say, L_1 is taken as one million then the corresponding load is,

$$C = P(L)^{\frac{1}{a}} \qquad (14.2.2)$$

Where, C is the basic or dynamic load rating

Therefore, for a given load and a given life the value of C represents the load carrying capacity of the bearing for one million revolutions. This value of C, for the purpose of bearing selection, should be lower than that given in the manufacturer's catalogue. Normally the basic or the dynamic load rating as prescribed in the manufacturer's catalogue is a conservative value, therefore the chances of failure of bearing is very less.

Equivalent radial load

The load rating of a bearing is given for radial loads only. Therefore, if a bearing is subjected to both axial and radial load, then an equivalent radial load is estimated as,

$$P_e = VP_r \quad \text{or} \qquad (14.2.3)$$
$$P_e = XVP_r + YP_a$$

Where, P_e : Equivalent radial load

P_r : Given radial load

P_a : Given axial load

V : Rotation factor (1.0, inner race rotating; 1.2, outer race rotating)

X : A radial factor

Y : An axial factor

The values of X and Y are found from the chart whose typical format and few representative values are given below.

$\frac{P_a}{C_o}$	e	$\frac{P_a}{P_r} \leq e$ X	Y	$\frac{P_a}{P_r} \geq e$ X	Y
0.021	0.21	1.0	0.0	0.56	2.15
0.110	0.30	1.0	0.0	0.56	1.45
0.560	0.44	1.0	0.0	0.56	1.00

The factor, C_o is the bearing obtained from catalogue.

14.2.4 The selection procedure

Depending on the shaft diameter and magnitude of radial and axial load a suitable type of bearing is to be chosen from the manufacturer's catalogue, either a ball bearing or a roller bearing. The equivalent radial load is to be determined from equation (14.2.3). If it is a tapered bearing then manufacturer's catalogue is to be consulted for the equation given for equivalent radial load. The value of dynamic load rating C is calculated for the given bearing life and equivalent radial load. From the known value of C, a suitable bearing of size that conforms to the shaft is to be chosen. However, some augmentation in the shaft size may be required after a proper bearing is chosen.

Problems with Answers

Q1. A simply supported shaft, diameter 50mm, on bearing supports carries a load of 10kN at its center. The axial load on the bearings is 3kN. The shaft speed is 1440 rpm. Select a bearing for 1000 hours of operation.

A1. The radial load P_r = 5 kN and axial load P_r = 3 kN. Hence, a single row deep groove ball bearing may be chosen as radial load is predominant. This choice has wide scope, considering need, cost, future changes etc.

Millions of revolution for the bearing, $L_{10} = \frac{60 \times 1440 \times 1000}{10^6} = 86.4$

For the selection of bearing, a manufacturer's catalogue has been consulted.

The equivalent radial load on the bearing is given by,

$$P_e = XVP_r + YP_a$$

Here, V=1.0 (assuming inner race rotating)

From the catalogue, C_o = 19.6 kN for 50mm inner diameter.

$$\therefore \frac{P_a}{C_o} = \frac{3.0}{19.6} = 0.153,$$

Therefore, value of e from the table (sample table is given in the text above) and by linear interpolation = 0.327.

Here, $\frac{P_a}{P_r} = \frac{3}{5} = 0.6 > e$. Hence, X and Y values are taken from fourth column of the sample table. Here, X= 0.56 and Y= 1.356

Therefore, $P_e = XVP_r + YP_a =$ $0.56 \times 1.0 \times 5.0 + 1.356 \times 3.0 = 6.867\,\text{kN}$

$$\therefore \text{ basic load rating, } C = P\,(L)^{\frac{1}{3}} = 6.867 \times (86.4)^{\frac{1}{3}} = 30.36\,\text{kN}$$

Now, the table for single row deep groove ball bearing of series- 02 shows that for a 50mm inner diameter, the value of C = 35.1 kN. Therefore, this bearing may be selected safely for the given requirement without augmenting the shaft size. A possible bearing could be SKF 6210.

Q2. What is rating life of a rolling contact bearing?

A2. Rating life is defined as the life of a group of apparently identical ball or roller bearings, in number of revolutions or hours, rotating at a given speed, so that 90% of the bearings will complete or exceed before any indication of failure occur.

Q3. What is basic load rating of a rolling contact bearing?

A3. It is that load which a group of apparently identical bearings can withstand for a rating life of one million revolutions.

$$C = P(L)^{\frac{1}{a}}$$

Where, **C** is the basic load rating and P and L are bearing operating load and life respectively and **a** is a constant which is 3 for ball bearings and 10/3 for roller bearings.

Q4. Why determination of equivalent radial load is necessary?

A4. The load rating of a bearing is given for radial loads only. Therefore, if a bearing is subjected to both axial and radial loads, then equivalent radial load estimation is required.

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Часть 2

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