Equistrong Tower Design

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Abstract: *The September 11, 2001 events attracted the attention of civil engineers of the world to the problem of safety of towers and skyscrapers. According to recent studies, the WTC collapse was caused by man-made explosions on a critical floor located much lower than the floors hit by terrorist planes which supports the wide-spread rumor in the U.S. that MASAD agents used the aftermath mess-up after the crush of the planes, put some explosives in the critical floor and detonated them in a while. These or any other possible calamities must be technically treated which can be achieved by drastic increase of safety factor of tower design. In the present paper a new approach to the optimal design of towers is advanced based on the principle of equistrength. Introductive Section 1 provides a short overview of the theory of equistrong structures given in the book by Cherepanov and Ershov (1977). In Section 2, a continuum model is used for the design of equistrong towers. The same problem is studied in Section 3 with an account of the discrete, floor-by-floor structure of equistrong towers. Conclusion emphasizes that the equistrong tower designs allow developers to achieve the maximum possible safety factor. Current designs of skyscrapers including WTC towers are as primitive as the Tower of Babylon.*

1. Introduction

The optimization methods currently used for the design of optimal engineering structures are based on the comparison of many direct solutions and selection of one of them having the desired optimum property. This approach is likable by engineers because the existence theorem is valid for the direct solutions which allows to easily computerize it. However, if optimal means perfect, ideal, non-improvable, then the "optimal" design derived by this approach is, in fact, never optimal because there are no sound physical basis in this approach.

The theory of equistrong structures uses the following physical principle of equistrength: A structure is referred to as the equistrong one if it is designed in such a way that the failure criterion is achieved simultaneously at all points of the structure, or, if it is impossible, in a maximum-possible volume or area of the structure. *All parts of the equistrong structure are* loaded to one and same extent; and so, all parts are equally stress-loaded and safe. In practical terms, this principle is reduced to the requirement of uniform maximum stresses throughout the whole structure or along some area with inevitable stress concentration. In the future, the principle of equistrength will allow to create perfect designs of structures of minimum weight and structures with maximum safety factor.

The first equistrong design appeared simultaneously with the notion of stress, when Galileo Galilei illustrated this then-unknown notion by a beam of a hyperbolic variable cross-section bent by a force, with maximum stresses being the same along the beam. Since then, several equistrong designs were occasionally found and used in practice. However, this subject has never been given a serious consideration by engineers and because of lack of computerizability it even has not been ever presented in any numerical optimization codes, numerous , very comprehensive and detailed.

The principle of equistrength was formulated in the book by Cherepanov and Ershov (1977) based on several dozens of their publications on the subject. Some equistrong designs of chains, beams, blades, disks, plates, bridges, shells, pipelines, pressure vessels, laminated composites, holes of equistrong shape, etc can be found in this book. The main difficulty in the quest for equistrong designs is the existence problem because there are no existence theorems for the inverse, strongly non-linear problems of this kind with unknown boundaries to be found as the main goal of such studies. However, if an equistrong design is found, it can become the ideal design pattern.

Since the time immemorial, humans have tried to come closer to the heaven by building tall and taller structures. The Tower of Babylon had collapsed long ago but pyramids of Egypt still stand. On September 11, 2001 the towers of the World Trade Center in New York, the tallest and most majestic buildings in the world, had suddenly collapsed into dust and debris, with almost three thousand humans including 330 firemen being perished. According to recent studies, the collapses were caused by some explosions on a critical floor located much lower than the floors hit by terrorist planes, which supports the rumor wide-spread in the United States that MASAD agents used the mess-up after the plane crush, put explosives in a critical floor below those under fire and detonated explosives in a while . The towers were poor-designed and built on cheap. Surely, the collapses would not have happened if the safety factor of their design were significantly greater. The towers collapsed in free fall regime as if disintegrated just before the fall, which supports the same rumor. In Cherepanov (2006) it was proven that the official version was built on miscalculations and therefore IS wrong.

The problem of optimal design of towers is studied below based on the physical principle of equistrength.

2. The Continuum Model

Let us consider the vertical tower of height H. The x-axis is assumed to be directed upward along the vertical so that $x = 0$ is the tower basement and $x = H$ is the coordinate of the roof. Let us designate by: $S(x)$, the area of the horizontal cross-section of the tower; $\gamma(x)$, the weight of unit volume of the tower; and $F(x)$, the equivalent compression force from gravitation in the cross-section $x = \text{const}$. It is assumed that the tower height is much greater than any width of its cross-section. Under this assumption the mean stress $\sigma_x = \sigma(x)$ in the cross-section $x = \text{const}$ can be introduced as:

$$\sigma(x) = \frac{F(x)}{S(x)}. \tag{2.1}$$

The equilibrium equation is as follows:

$$\frac{dF(x)}{dx} = \gamma(x)S(x). \qquad (2.2)$$

In what follows we consider only the towers of the following two designs:

(i) Most primitive one when

$$\gamma(x) = \gamma_o = \text{const}, \qquad S(x) = S_o = \text{const}; \qquad (2.3)$$

The design of the WTC towers as well as most of other buildings is close to this type of designs.

(ii) Most perfect one, the equistrong design, when

$$\sigma(x) = -\sigma_o = \text{const}. \qquad (2.4)$$

Here σ_o is the maximum value of safe stresses.

The solution to the equation system (2.1) and (2.2) for both designs is as follows:

(i) $$\sigma = -\gamma_o(H - x) - \frac{R}{S_o}, \qquad F = \sigma S_o, \qquad |\sigma| \leq \sigma_o. \qquad (2.5)$$

(ii) $$S = \frac{R}{\sigma_o}\exp\left(\frac{1}{\sigma_o}\int_x \gamma(x)dx\right), \qquad F = \sigma_o S. \qquad (2.6)$$

Here R is the weight of the tower roof.

Let us apply this calculation to homogeneous, vertical, solid columns of height H and compare these two designs provided that the total weight W of the column is one and the same for both designs. One can find:

(i) $$H = \frac{\sigma_o}{\gamma_o}\frac{1}{1 - \frac{R}{W}}, \qquad (W > R); \qquad (2.7)$$ *(ii)*

$$H = \frac{\sigma_o}{\gamma_o}\ln\left(1 + \frac{W}{R}\right), \qquad (W > R). \qquad (2.8)$$

For very tall columns $W >> R$ and these equations become even simpler:

(i) $$H = \frac{\sigma_o}{\gamma_o}, \qquad (W >> R); \qquad (2.9)$$ *(ii)*

$$H = \frac{\sigma_o}{\gamma_o}\ln\left(\frac{W}{R}\right), \qquad (W >> R). \qquad (2.10)$$

As seen, for one and same weight the equistrong column $\ln(W/R)$ times taller than the column of uniform cross-section. The safety of both columns is about the same since the maximum stress is the same in both columns. An account of the column stability with respect to horizontal displacement and forces will evidently be in favor of the equistrong column.

The value of σ_o / γ_o for some materials is brought below.

Material	γ_o, g/cm^3	σ_o, N/mm^2	$\dfrac{\sigma_o}{\gamma_o}$, m
Wood (White Oak)	0.7	5	700
Steel	7.9	120	1500
Aluminum	2.7	20	750
Window glass	2.4	50	2000
Epoxy	1.3	20	1500

The value of σ_o is taken about seven times less than the ultimate compression stress.

Suppose, now, that the height and total weight of the equistrong column is the same as for the column of uniform cross-section. Compare the maximum stresses σ_o in both designs:

(i) $$\sigma_o = \gamma_o H, \quad \text{and} \tag{2.11}$$

(ii) $$\sigma_o = \frac{\gamma_o H}{\ln\left(\dfrac{W}{R}\right)}. \tag{2.12}$$

Hence, the maximum stress in the equistrong column is $\ln(W/R)$ times less than the maximum stress in the column of uniform cross-section. And so, the equistrong column is many times safer. For example, if W/R = 10 the safety factor of equistrong column is 2.3 times greater than the safety factor of common primitive design.

These calculations and estimates for solid columns can be applied to real towers of complex structure using the continuum approach. (As a reminder, each material we consider a homogeneous solid represents a complex, discrete system of atoms, grains, fibers, etc.). By this way, γ_o is equal to the tower weight divided by the tower volume, and the stress in the tower cross-section is equal to the equivalent force in bearing walls and columns of the tower cross-section $x = \text{const}$ divided by the area of this cross-section.

3. The Discrete Model

Let us consider the vertical tower of N stories. Let us designate by: S_n, the area of cross-section of bearing columns and walls of the n^{th} floor, and σ_n, the mean compressive stress in these structural elements bearing the weight of the building above this floor. The equilibrium equation is as follows:

$$-\sigma_{n+1}S_{n+1} + \sigma_n S_n = -G_n - \gamma h_n S_n \qquad (3.1)$$

$$(n = 1,2,3,....,N-1)$$

$$\sigma_n S_n = -R - \gamma h_n S_N \qquad (n = N). \qquad (3.2)$$

Here: G_n, the weight of the floor-ceiling structure between the $(n+1)^{th}$ and n^{th} floors plus the passive operation weight of the n^{th} floor including equipment, people, non-bearing walls, etc; h_n, the height of the n^{th} floor; γ, the weight of unit volume of the material of the bearing columns and walls; and R, the weight of the tower roof plus the passive weight of the N^{th} floor.

Let us require that the stress σ_n be one and same for all floors

$$\sigma_n = -\sigma \qquad (n = 1,2,3,....,N-1). \qquad (3.3)$$

Here σ is the safe stress for the given material and structure. The condition of equistrength (3.3) provides for the ideal using of all of the material of bearing structural elements and thus allows to make a perfect design from the point of view of maximum safety and minimum amount of the bearing material.

Equations (3.1) and (3.2) for the equistrong towers become:

$$S_{n+1} - S_n + \frac{\gamma}{\sigma} h_n S_n = -\frac{1}{\sigma} G_n, \qquad (3.4)$$

$(n = 1,2,3,....,N-1)$

$$S_N = \frac{1}{\sigma} R + \frac{1}{\sigma} \gamma h_N S_N \qquad (n = N). \qquad (3.5)$$

The recurrent solution to these equations is:

$$S_N = \frac{R}{\sigma - \gamma h_n} \qquad (n = N), \qquad (3.6)$$

$$S_n = \frac{\sigma S_{n+1} + G_n}{\sigma - \gamma h_n} \qquad (n = N-1, N-2, ..., 3, 2, 1). \quad (3.7)$$

This solution determines the floor-by-floor distribution of the bearing material in the equistrong towers. Evidently, numerous architectural designs not considered here are possible within the framework of this solution. The consideration of maximum useful volume of the tower as well as its maximum stability with respect to horizontal random loads leads to the conclusion that the tower should be a vertical cylinder of circular cross-section with bearing columns uniformly concentrated along the perimeter of the cross-section creating a tube, see Cherepanov and Ershov (1977). This shape allows also to use minimum amount of building materials.

In the particular case when $h_n = h = \text{const}$ and $G_n = G = \text{const}$ the equations system (3.4) and (3.5) has the following analytical solution for all n:

$$S_n = \frac{G}{\gamma h} + \left(\frac{R}{\sigma - \gamma h} + \frac{G}{\gamma h}\right)\left(1 - \frac{\gamma h}{\sigma}\right)^{n-N} \qquad (n = 1, 2, 3, ..., N). \qquad (3.8)$$

In this case the total weight of the tower without basement is equal to:

$$W = R + G(N-1) + \gamma h \sum_{n=1}^{N} S_n = \\ = R + NG + \left[R + G\left(\frac{\sigma}{\gamma h} - 1\right)\right]\left[\left(1 - \frac{\gamma h}{\sigma}\right)^{-N} - 1\right] \qquad (3.9)$$

For the towers and skyscrapers, $N >> 1$ and $\sigma >> \gamma h$, so that after using the asymptotic equality

$$\left(1 - \frac{1}{\lambda}\right)^N = e^{-N/\lambda} \qquad (\lambda >> 1, \;\; N >> 1) \qquad (3.10)$$

$$\left(\lambda = \frac{\sigma}{\gamma h}\right),$$

which is valid for finite values of N/λ, not very large and not very small, equation (3.9) becomes

$$W = R + G\left(N - \lambda + \lambda e^{N/\lambda}\right) \qquad \left(\lambda = \frac{\sigma}{\gamma h}\right). \qquad (3.11)$$

For the most primitive designs when $S_n = S_o = \text{const}$, the solution to equations (3.4) and (3.5) is:

$$\sigma_n = -\frac{R}{S_o} - \left(\frac{G}{S_o} + \gamma h\right)(N - n) \qquad (n = 1,2,3,\ldots,N),$$
$$W = R + N(G + \gamma h), \qquad (3.12)$$
$$\sigma = \frac{W}{S_o} = \frac{R}{S_o} + N\left(\frac{G}{S_o} + \gamma h\right).$$

The design of WTC towers is close to this primitive design commonly used as well in most of other buildings. As seen, the stresses in equistrong towers are several times smaller than maximum stresses in common primitive designs of the same weight and height and , hence, equistrong towers are much safer.

4. Conclusions

The equistrong design approach allows to ultimately use the bearing capacity of the material and hence to: (*i*) build the towers of maximum height for the given weight and stress in bearing columns and walls, or, (*ii*) build the towers of maximum safety, that is with minimum stresses in bearing columns and walls of the tower, for the given weight and height of the building. Equistrong designs will allow to far surpass today's height of skyscrapers and their safety.

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Reference

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Автореферат[1]

С незапамятных времен люди старались подняться выше и выше, строя все более высокие здания. Вавилонская башня развалилась давным-давно, а египетские пирамиды стоят много тысячелетий. Недавно, а точнее 11-ого сентября 2001-ого года, рассыпались в пыль и мелкие обломки башни всемирного торгового центра в Нью-Йорке, считавшиеся самыми высокими зданиями в мире (свыше 400 метров). Рассыпались от пожара, потому что были плохо спроектированы и построены задешево. Под обломками погибло почти три тысячи человек, в том числе 330 пожарников и несколько россиян…

Представляет интерес оценить максимальную высоту идеально спроектированных башен.

[1] Черепанов Г.П. Равнопрочная башня. Вестник ЧГПУ им. Яковлева. 2006. №1(48). Стр.32-39.