Determination of Flow Characteristics in Technological Processes with Controlled Pressure

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Abstract

Assessment of the environmental and economic performance of a safety valve requires information about the flow of the substance through the valve when it is actuated. The goal of this paper was to determine the flow rate of the discharged substance and the mass flow rate of the substance entering the safety valve when it is actuated.

Proposed a mathematical model to describe the processes occurring in the valve. The model includes a system of differential equations describing the physical laws of conservation in the internal volume of the valve and differential equations, which link the value of gas flow through the valve with the pressure and the amount of movement of the shut-off disk. Used a modified method by S.K. Godunov to solve gas-dynamic equations.

Established that the determination of the flow and power characteristics of the valve requires the preliminary construction of a mathematical model of the safety valve operation. Based on this, proposed a method for determining the flow rate of the discharged substance and the mass rate of the substance entering the safety valve when it is actuated.

Obtained the flow characteristics of the valves under review and the dynamics of movement of the shut-off disc of the valve, as well as the dependence of the pressure change on the opening time of the valve. Comparison of the calculated values with available experimental data gives good agreement of results (no more than 5.6 % for a gas flow rate, under 10 % for the movement of the valve and change the arrival of gas in time using the standard deviation function of the flow characteristics of 0.6 %), confirms the correctness of the defined mathematical model, used numerical schemes and algorithms, as well as the proposed method and recoverability of the arrival of gas in a pressure–time curve.

Keywords: safety valve, flow measurement, pressure monitoring, numerical simulation.

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Определение расходных характеристик в технологических процессах с контролируемым давлением

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Оценка экологичности и экономичности работы предохранительного клапана требует информации о расходе вещества через клапан при его срабатывании. Целью данной работы являлось определение величины расхода сбрасываемого вещества и массовой скорости поступления вещества в предохранительный клапан при его срабатывании.

Для описания процессов, протекающих в клапане, предложена математическая модель, включающая систему дифференциальных уравнений, описывающих физические законы сохранения во внутреннем объёме клапана и дифференциальные уравнения, связывающие величину расхода газа через клапан с давлением и величиной перемещения запорного диска. Для решения газодинамических уравнений применялся модифицированный метод С.К. Годунова.

Установлено, что определение расходной и силовой характеристик клапана требует предварительного построения математической модели функционирования предохранительного клапана. На основании чего предложена методика определения величины расхода сбрасываемого вещества и массовой скорости поступления вещества в предохранительный клапан при его срабатывании.

Получены расходные характеристики рассматриваемых клапанов и динамика перемещения запорного диска клапана, а также зависимость изменения давления от времени открытия клапана. Сопоставление расчётных значений с имеющимися экспериментальными данными даёт хорошее совпадение (не более 5,6 % для расхода газа, менее 10 % для перемещения клапана и изменения прихода газа во времени при среднеквадратичных отклонениях функции расходных характеристик 0,6 %) результатов, подтверждает корректность сформулированной математической модели, используемых численных схем и алгоритмов, предложенной методики и возможность восстановления прихода газа по кривой давление−время.

Ключевые слова: предохранительный клапан, измерение расходных характеристик, контроль давления, численное моделирование.

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Introduction

Chemical and technological processes, oil and gas pipeline systems, and high-pressure devices require the use of pressure level monitoring. The safety valve refers to the class of pipeline fittings designed to protect against mechanical destruction of equipment and pipelines, which is caused by overpressure, using automatically relief of excess working medium from systems and pressure vessels in excess of the set pressure. The valve should also ensure that the discharge of the medium is stopped when the operating pressure is restored [1–3]. Dangerous overpressure may occur in the system due to foreign factors (improper operation of equipment, heat transfer from exterior sources, incorrectly assembled thermal and mechanical circuit), as well as due to internal physical processes caused by an event beyond the normal operation.

In this case, the safety valve refers to quick-operation devices [4] that require the use of specialized quick-operation measuring instruments [5], the range of operation of which is quite limited [6]. However, we need to find the values of the discharge flow rate to assess the economic and environmental impacts. The determination of the mass rate of the substance input due to the process deviation from the normal technological mode is required to eliminate the causes of abnormal operation.

If there is a record of measurement of the current pressure in the tank, the value of gas arrival, G_P, and gas flow through the valve, G_v, can be determined based on the mathematical model of the safety valve operation. Therefore, the goal of this paper was to determine the flow rate of the discharged substance and the mass flow rate of the substance entering the safety valve when it is actuated.

Mathematical model of gas arrival calculation

To obtain the dependence between the flow characteristics and the pressure in the tank, let us consider the mass balance equation:

$$W \frac{d\rho}{dt} = G_p - G_v,$$

where: ρ is the gas density; W is the operation capacity of the tank; t is the time.

In the adiabatic approximation, we obtain an equation that links the pressure change, p, to the flow characteristics, G_P, G_v:

$$\frac{dp}{dt} = \frac{kRT(0)}{W} \left(G_p - G_v \right) \left(\frac{p}{p(0)} \right)^{\frac{k-1}{k}}, \qquad (1)$$

where k, R is the adiabatic exponent and the gas constant of the working medium; $T(0)$, $p(0)$ are the initial values of temperature and pressure in the tank.

The gas flow through the valve is a function dependant on the pressure and the value of the movement, X, of the shut-off disc: $G_v = G_v(p, X)$. We can obtain this dependence either experimentally or by calculation. We propose to find this dependence from the numerical solution of the equations of the mathematical model of valve operation [7, 8] with the verification using the results of experimental measurements for a specific type of valve. Equation (1) gives us the expression for the gas arrival:

$$G_p = G_v \left(X, p \right) + \frac{dp}{dt} \frac{W}{kRT(0)} \left(\frac{p(0)}{p} \right)^{\frac{k-1}{k}}. \qquad (2)$$

Expression (2) includes the value of the movement of the valve disc, $X(t)$. We find this dependence from the solution of the disk movement equation [7].

$$m_s \frac{d\eta}{dt} = F_f - F_s, \quad \frac{dX}{dt} = \eta. \qquad (3)$$

The movement of the disk in the axial direction is determined by the action of the force from the gas, F_f, and the elastic force of the spring, $F_s = K_s(X + X_0)$. Here: K_s is the spring stiffness coefficient; X_0 is the initial spring compression (preload): η is the speed of the disk movement. Initial conditions: $X(0) = 0$, $\eta(0) = 0$. The height of the disk lift is limited by the value X_k.

For the numerical solution of the disk movement equations, we introduce a difference grid $\left\{ t_0 < t_1 < ... < t_n < ... < t_q = t_k, h_n = t_{n+1} - t_n \right\}$ and a two-step difference scheme [8]:

To calculate the movement of the disk, we should define the dependence of the force acting on the disk from the gas on the movement and pressure, $F_f(X, p)$. To calculate dependencies, $G_v = G_v(p, X)$, $F_f(X, p)$, we use the following mathematical model.

$$X^{n+1} = \frac{\left(F_f \left(X^n, p(t_n) \right) - K_s X_0 \right) h_n + K_d X^n + m_s \left(\frac{X^n - X^{n-1}}{h_n} + \frac{X^n}{h_n} \right)}{K_s h_n + \frac{m_s}{h_n}}.$$

Mathematical model of safety valve operation

Processes occurring in the valve are described by a mathematical model in the form of a system of

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differential equations describing the physical laws of conservation in the internal volume of the valve. For a gas safety valve, we consider the processes of internal gas dynamics within the framework of the viscous compressible gas model. The change in the thermodynamic parameters of the gas in the tank is subject to the equation of state: $p = \rho RT$.

The continuity equation is written down in general form:

$$\frac{d\rho}{dt} + \rho \nabla \mathbf{U} = 0, \tag{4}$$

where $\mathbf{U}$ is the gas flow rate vector. The momentum conservation equations and the energy conservation equation have the following form:

$$\rho \frac{d\mathbf{U}}{dt} = -\nabla p + \mathrm{Div}P; \tag{5}$$

$$\rho \frac{d}{dt}\left(C_v T + \frac{\mathbf{U}^2}{2} \right) = \nabla(P\mathbf{U}) + \nabla\mathbf{q}, \tag{6}$$

where P is the tensor of viscous stresses; C_v is the specific heat capacity of the gas at a constant volume; $\mathbf{q}$ is the heat flow vector.

The system of equations (4)–(6) is supplemented by equations for the transfer of the kinetic energy of turbulence and the rate of turbulence dissipation [9]. The axial component of the force acting on the disk from the gas is determined by the integral on the disk surface, S_d:

$$F_f = \int\limits_{S_d} p\,ds. \tag{7}$$

The calculation area (Figure 1a) is divided into two parts.

The first part is axially-symmetric and it uses a cylindrical coordinate system (x, r, φ). The second part uses a rectangular coordinate system (x, y, z). We use the control volume method for the numerical solution of a system of gas-dynamic equations. We determine gas parameters at the boundaries of control volumes using the S.K. Godunov method [10]. To increase the order of approximation of the Godunov difference method, we use the *MUSCL* (*Monotone Upwind Schemes for Conservation Laws*) scheme. In accordance with this scheme, we define the values of gas-dynamic parameters for solving the problem of discontinuity decay using extrapolation with a limiter [11, 12]. For equations written down

in a cylindrical coordinate system, we construct a difference grid (Figure 1b) in the $\varphi = \mathrm{const}$ plane using the complex method of boundary elements [13]. For equations written down in a Cartesian coordinate system, the difference grid is unstructured. The gradients of variables included in the tensor components P are calculated in the middle of each face through the values of variables in the surrounding control volumes, as described in [14]. For time integration, we use a two-step Runge–Kutta scheme with second-order accuracy.
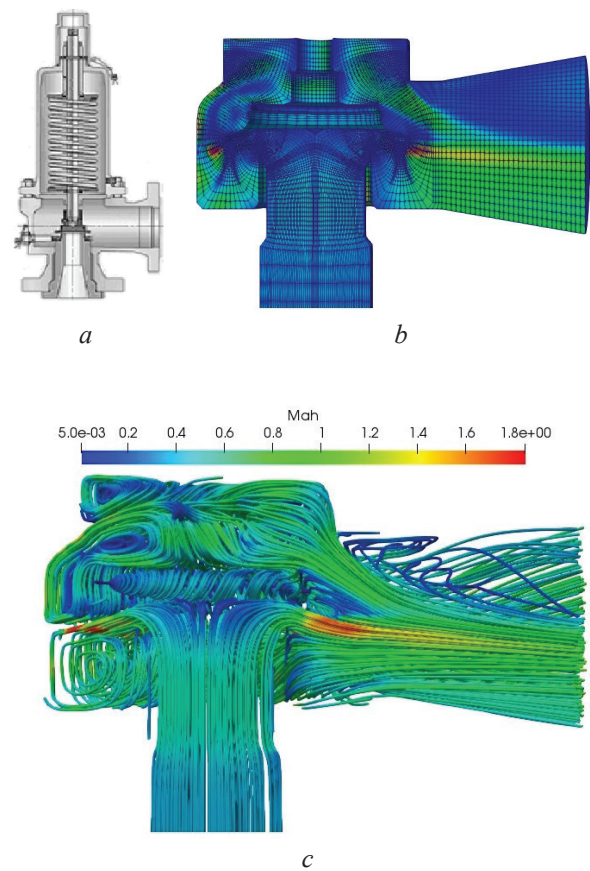

Figure 1 – Valve diagram (*a*), difference grid (*b*), and flow structure (*c*)

The gas-dynamic equations (4)–(7) are solved numerically together with the disk movement equations (3). The difference grid is adapted to the disk movement.

We consider two designs of the *LESER* safety valve, 2J3 and 441. The gas-dynamic force and flow through the valve are written down as: $F_f(X, p) = p\psi(X)$, $G_v(p, X) = p\gamma(X)$, where $\psi(X)$, $\gamma(X)$ functions are characteristics of the valve type. The calculated and experimental dependencies are shown in Figure 2.
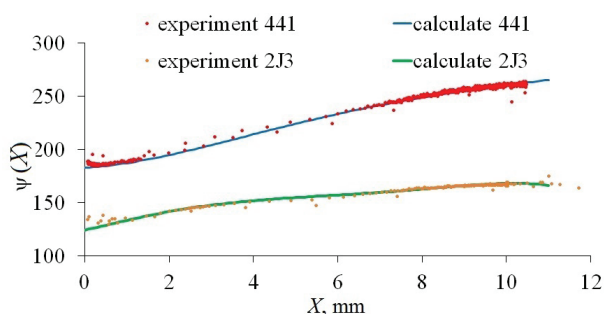
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Figure 2 – Valve characteristics

The calculated dependencies, $\psi(X)$, are approximated by 5th-degree polynomials, $\psi(X) = \sum_{k=0}^{5} \beta_k X^k$, with coefficients, β_k, in Table 1.

Table 1

Polynomial coefficients for $\psi(X)$

Valve	β_k					
441	182.5704	2.2713	2.4635	-0.3243	0.0155	-0.0002
2J3	123.8281	9.6135	0.1595	-0.3951	0.0570	-0.0024

The flow characteristics of valves obtained by calculation are approximated by 2nd-degree polynomials with coefficients from Table 2.

Table 2

Polynomial coefficients for $\gamma(X)$

Valve	β_k		
441	0	0.0345	-0.00071
2J3	0	0.0292	-0.001

The results of tests processing

The scheme for testing safety valves in accordance with the ASME PTC 25-2014 standard is shown in Figure 3.

The characteristics of the tested LESER 441 and 2J3 valves are shown in Table 3.

Table 3

Characteristics of valves

Valve	X_0, mm	K_s, N/m	L, m	D_t, m
441	0.017	54700	1.2	0.05
2J3	0.025	26000	1.2	0.05

The tests were carried out in the air. The *vessel*0 tank has a large volume under high pressure and

serves as a gas source for the *vessel* tank with pressure control. The intensity of the gas arrival is regulated by the operator during the test (regulator, R_1). One of the goals of the tests was to check the stability of the valves with a long pipe between the tank and the valve. A pipe with a length of L and a diameter of D_t connects the tank to the valve. Pressure and temperature are measured at the inlet and at the end of the pipe. The movement of the rod, X, is measured on the valve. The pressure is measured using a P900 series load cell with an error of 0.2 %. The force is measured using a U2B sensor with an accuracy class of 0.2. The movement of the disk is controlled by a WA standard displacement inductive transducer with an error of 1 %. Sensor readings are recorded in 0.000417 seconds.
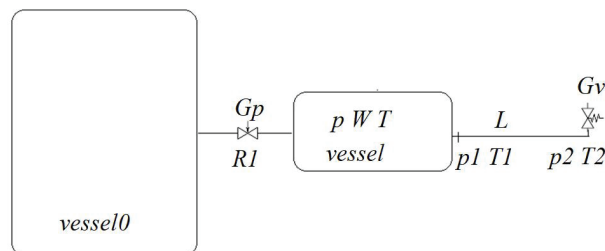

Figure 3 – Safety valve testing scheme: *vessel*0 – a tank with a large volume under high pressure, which is the source of gas for the *vessel* tank; *vessel* – a pressure-controlled tank with operating parameters P, T, W; R_1 – gas flow regulator; G_p – gas arrival; P_1, T_1 – gas pressure and temperature at the outlet of the tank with pressure control; L – length of a pipe with a diameter, D_t, connecting the tank to the valve; P_2, T_2 – gas pressure and temperature at the inlet to the valve; G_v – gas flow through the safety valve

The use of formula (2) for calculating the gas arrival value based on the results of experimental measurements of pressure, $p(t)$, and disk movement, $X(t)$, requires calculating the derivative $\frac{dp}{dt}$.

Differentiation of functions based on the results of experimental measurements is an incorrect operation. Figure 4 shows the measured dependence, $p(t)$ (red line).

The prese nce of acoustic oscillations and measurement errors leads to the occurrence of high-frequency oscillations. Therefore, the results of measuring all parameters were smoothed using smoothing cubic splines. The value of the derivative was determined by the value of the relevant spline coefficient. The smoothed dependency, $p(t)$, is shown in Figure 4 with a black line.
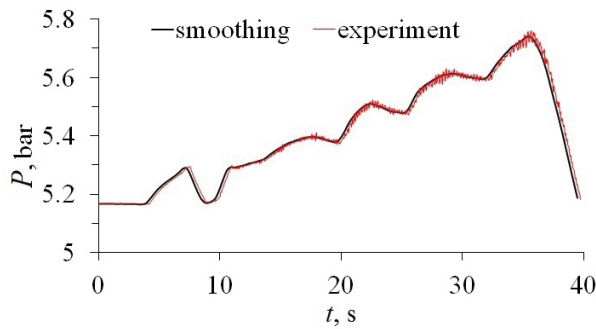
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Figure 4 – Pressure versus time (valve 441)

The time change of the value of air arrival in the *vessel* tank, which is calculated using the formula (2), equations (3), and pre-calculated functions, $\psi(X)$, $\gamma(X)$, is shown in Figure 5.
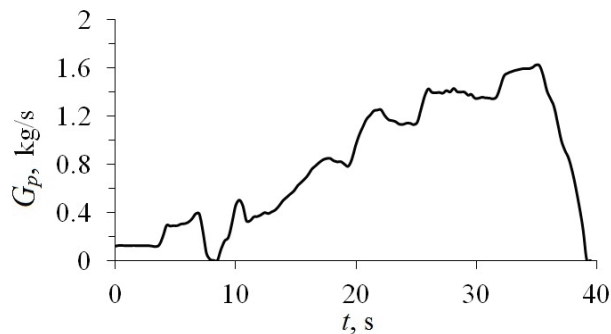

Figure 5 – Dependence of gas intake on time

The scheme of the valve tests under review provided for measuring the movement of the valve disc, as well as the pressure and temperature in two sections of the pipe (Figure 1). Based on this information, we can determine the value of gas flow through the valve during testing and compare it with the calculated dependence, $G_v(p, X) = p\gamma(X)$. If p_2, T_2 are the pressure and temperature at the end of the pipe before the valve, we determine the mass flow of gas through the pipe using the following expression:

$$G_{\exp} = \varphi_2 \pi \frac{D_t^2}{4} \sqrt{2\left(p - p_2\right)\frac{p_2}{RT_2}},$$

where φ_2 is the flow coefficient related to a sharp narrowing at the inlet to the pipe from the tank (Figure 6) and friction in the pipe. We determine the flow coefficient when solving the gas-dynamic problem in an axially-symmetric setting in the calculated area (Figure 6).

The value of the flow coefficient, $\varphi_2 = 0.814$. The measured movement of the valve disc is shown in Figure 7.
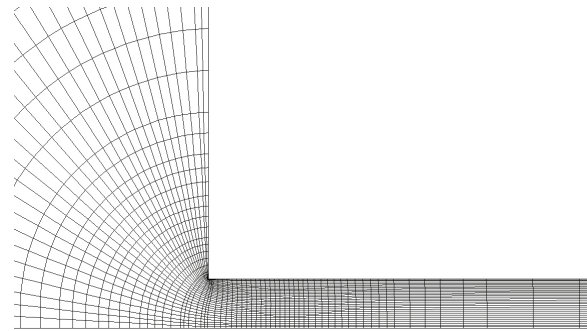

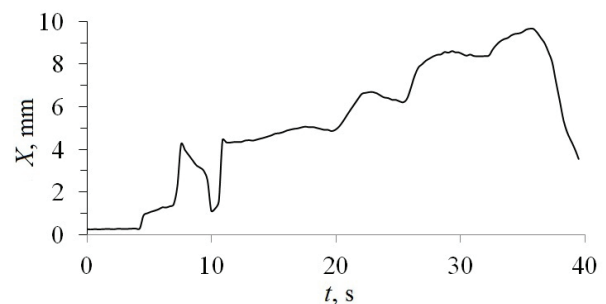**Figure 6** – Fragment of the vessel – pipe design area

Figure 7 – Moving the valve disc

The calculated and experimental flow values corresponding to these values, X, are shown in Figure 8.
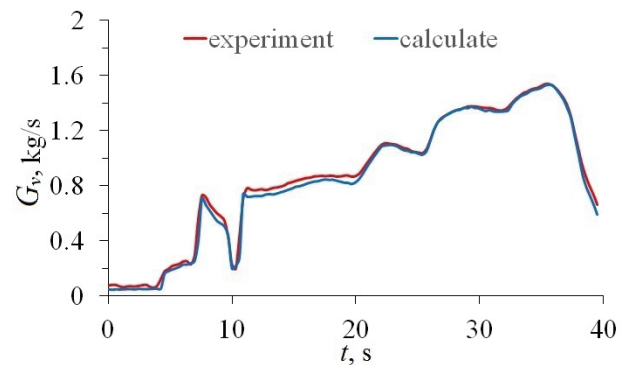

Figure 8 – Calculated and experimental dependencies for gas flow

The experimental and calculated, $\gamma(X) = 0.0345X - 0.00071X^2$, flow characteristics of the 441 valve are shown in Figure 9.

The average square deviation is 0.006. We may conclude from the presented comparison of the results of calculations and experiments that the design characteristics of the valve provide the determination of gas arrival in the controlled tank only through the pressure–time dependence.

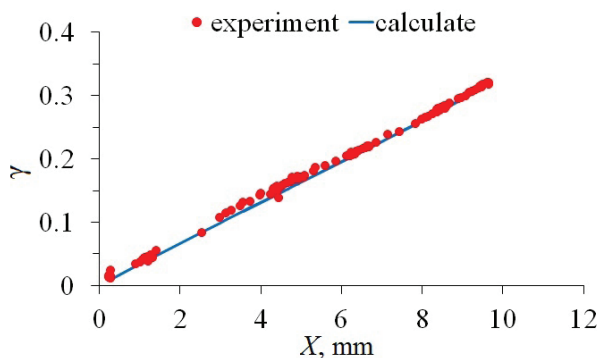Let us review the test results of the 2J3 valve. The time change in pressure is shown in Figure 10.

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Figure 9 – Calculated and experimental flow characteristics
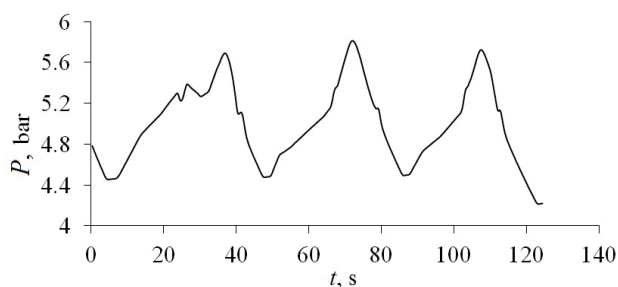

Figure 10 – Time dependence of pressure (2J3 valve)

During this time, the three-time actuation of the valve occurs, as shown in Figure 11.
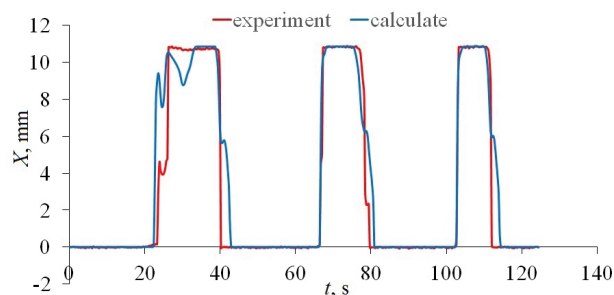

Figure 11 – Measured and calculated movement of the 2J3 valve disc

The time interval under review corresponds to the dependence of the gas arrival on time, which is obtained by the calculation and confirmed in experiments (Figure 12).
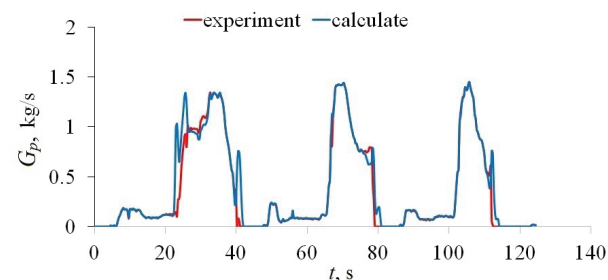

Figure 12 – Change of gas arrival in time (valve 2J3)

The comparison of the change in the value of the disk movement obtained in the experiment and by the calculated method (solution of equations (3) using the power characteristic of the valve 2J3, $\psi(X) = 123.8281 + 9.6135X + 0.159X^2 - 0.3951X^3 + 0.0570X^4 - 0.0024X^5$), which is shown in Figure 11, demonstrates that the power characteristic of the valve was obtained correctly.

Conclusion

We have shown that in order to find the arrival of a gas mass in a controlled technological tank, which leads to increase in pressure, it is required to have the following information: the pressure-time dependence recorded in the technological process; calculated flow and power characteristics of the valve. We confirmed during our work that the determination of the flow and power characteristics of the valve requires the preliminary construction of a mathematical model of the safety valve operation. Based on this, we proposed a method for determining the flow rate of the discharged substance and the mass rate of the substance entering the safety valve when it is actuated.

Following the numerical simulation, we obtained the flow characteristics of the valves under review and the dynamics of movement of the shut-off disc of the valve, as well as the dependence of the pressure change on the opening time of the valve. Comparison of the calculated values with available experimental data gives good agreement of results (no more than 5.6 % for a gas flow rate, under 10 % for the movement of the valve and change the arrival of gas in time using the standard deviation function of the flow characteristics of 0.6 %), confirms the correctness of the defined mathematical model, used numerical schemes and algorithms, as well as the proposed method in general and recoverability of the arrival of gas in a pressure-time curve in particular.

Acknowledgments

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